

# 1 **Supplementary Material**

## 2 3 **Simultaneous assimilation of satellite and eddy covariance data for improving** 4 **terrestrial water and carbon simulations at a semi-arid woodland site in Botswana**

### 5 6 **Appendix A: Gap-filling procedure for LHF**

7 The gap-filling procedure is done in the following steps (Appendix A of Reichstein et al.,  
8 2005): (1) In cases where only data of direct interest are missing, but all meteorological  
9 data are available, the missing values are replaced by the average values under similar  
10 meteorological conditions within a time-window of  $\pm 7$  days. If no similar meteorological  
11 conditions are present within the time window, the averaging window is increased to  $\pm$   
12 14 days. (2) In cases where auxiliary data such as air temperature or VPD are missing,  
13 but radiation is available, the same approach is taken, but similar meteorological  
14 conditions can only be defined via shortwave radiation deviation less than  $50 \text{ Wm}^{-2}$  and  
15 the window size is not further increased. (3) In cases where radiation data are also  
16 missing, the missing values are replaced by the average values at the same time of the  
17 day ( $\pm 1$  h), i.e. by the mean diurnal course. In this case, the window size starts with  $\pm$   
18 0.5 days, i.e. adjacent hours. If after these steps the values could not be filled, the  
19 procedure is repeated with increased window sizes until the values can be filled.

20

## 1 **Appendix B: Model description of BETHY**

### 2 **Overview**

3 BETHY is a process-based model of the terrestrial biosphere (Knorr, 1997, 2000). It  
4 simulates carbon assimilation and plant and soil respiration embedded within a full  
5 energy and water balance. Hence it is structured into four compartments: (1) energy  
6 and water balance, (2) photosynthesis, (3) phenology, and (4) carbon balance. BETHY is  
7 driven by observed climate, and is run on a specific site, or on a regional to global spatial  
8 scales. Each grid cell is designed to contain up to three different plant functional types  
9 (PFTs) out of a total of 13 PFTs, with the amount specified by their fractional coverage.  
10 Time steps are 1 hour for the energy balance, photosynthesis and autotrophic  
11 respiration part and 1 day for the phenology, hydrology and heterotrophic respiration  
12 part. Control parameters in the set-up used here affect the photosynthesis, phenology  
13 and hydrology scheme. The actual parameters, which are optimized in this study, are  
14 listed with their a priori values and uncertainties in Table 2.

15

### 16 **Photosynthesis**

17 At each model grid cell, photosynthesis is simulated following the formulations of  
18 Farquhar et al. (1980) or Collatz et al. (1992) for C<sub>3</sub> or C<sub>4</sub> metabolism, respectively. In  
19 the case of C<sub>3</sub> photosynthesis, gross primary productivity (GPP) is calculated as the  
20 minimum of an electron transport limited rate,  $J_E$ , and a rate,  $J_C$ , limited by the  
21 carboxylation enzyme Rubisco from which the leaf or dark respiration,  $R_d$ , is subtracted,

22

$$23 \quad \text{GPP} = \min[J_C; J_E] - R_d, \quad [\text{A1}]$$

1

2 with

3

$$4 \quad J_C = V_{\max} \frac{C_i - \Gamma_*}{C_i + K_C(1 + O_x/K_O)} \quad [\text{A2}]$$

$$5 \quad J_E = J \frac{C_i - \Gamma_*}{4(C_i + 2\Gamma_*)} \quad [\text{A3}]$$

6

7 where

8

$$9 \quad J = \frac{\alpha_q I J_{\max}}{\sqrt{J_{\max}^2 + \alpha_q^2 I^2}} \quad [\text{A4}]$$

10

11 with the parameter  $\alpha_q$  the quantum efficiency,  $C_i$  the leaf-internal CO<sub>2</sub> concentration,  $I$   
 12 the photosynthetically active radiation (PAR) absorption rate, and  $O_x$  the O<sub>2</sub> partial  
 13 pressure. The maximum electron transport,  $J_{\max}$ , varies linearly with the vegetation  
 14 temperature,  $T_v$  in °C. The temperature sensitivity  $a_{J,T}$  (Farquhar, 1988) is a control  
 15 parameter,

16

$$17 \quad J_{\max}(T_v) = J_{\max}^{25} \times a_{J,T} \times T_v, \quad [\text{A5}]$$

18

19 with

20

$$1 \quad J_{\max}^{25} = a_{J,V} \times V_{\max}^{25} \quad [A6]$$

2

3 Equation (A6) is a simple recasting of the usual photosynthesis equations to use the  
 4 parameters  $a_{J,V}$  and  $V_{\max}(25^{\circ}\text{C})$  rather than the usual  $J_{\max}(25^{\circ}\text{C})$  and  $V_{\max}(25^{\circ}\text{C})$ , and  
 5 thus taking into account their correlation for the optimization. Initial values of  $a_{J,V}$  are  
 6 calculated from the a priori values of  $J_{\max}$  and  $V_{\max}$  at  $25^{\circ}\text{C}$ . The  $\text{CO}_2$  compensation  
 7 point,  $\Gamma^*$ , depends linearly on the vegetation temperature with the parameter  $a_{\Gamma,T}$  being  
 8 the dependency factor (Farquhar, 1988),

9

$$10 \quad \Gamma^* = a_{J,T} \times T_v \quad [A7]$$

11

12 The temperature dependency of further enzyme kinetic rates such as the  
 13 Michaelis-Menten constants  $K_O$  and  $K_C$  and the maximum carboxylation rate,  $V_{\max}$ , are  
 14 computed from the following equation with  $E$  the respective activation energy (with  $T_v$   
 15 in  $^{\circ}\text{C}$ ):

16

$$17 \quad r(T_v) = r^{25} \exp\left\{ \frac{(T_v - 25^{\circ}\text{C})E}{298KR(T_v - 273^{\circ}\text{C})} \right\}, \quad [A8]$$

18

19  $R$  is the general gas constant in  $\text{JK}^{-1}\text{mol}^{-1}$ ,  $r$  stands for the rate in question, and  $r^{25}$  the  
 20 respective rates at  $25^{\circ}\text{C}$ , i.e.,  $K_O^{25}$ ,  $K_C^{25}$  and  $V_{\max}^{25}$  (the values of  $K_O$ ,  $K_C$ , and  $V_{\max}$  at  
 21  $25^{\circ}\text{C}$ ). These rates and the respective activation energies ( $E_{K_O}$ ,  $E_{K_C}$ , and  $E_{V_{\max}}$ ) are  
 22 taken as parameters in this study.

1 For C<sub>4</sub> photosynthesis, there are three limiting rates (Collatz et al., 1992),

$$2 \quad GPP = \min[J_C; J_E; J_i] - R_d, \quad [A9]$$

$$3 \quad J_C = k \times Ci, \quad [A10]$$

$$4 \quad J_E = V_{\max}, \quad [A11]$$

$$5 \quad J_i = \alpha_i I, \quad [A12]$$

6 where in the case of C<sub>4</sub> photosynthesis,

$$7 \quad k^{25} = a_{J,V} \times V_{\max}^{25}, \quad [A13]$$

8 with  $k^{25}$  being the PEPcase (the initial CO<sub>2</sub> fixating enzyme in C<sub>4</sub> plants) CO<sub>2</sub> specificity  
 9 at 25°C. The temperature dependency of  $k$  is then also calculated by equation (9) with  
 10 the activation energy  $E_k$  as a parameter. Here  $\alpha_i$  is the integrated C<sub>4</sub> quantum efficiency,  
 11 and  $a_{J,V}$  and  $\alpha_i$  are both parameters. The dark respiration  $R_d$  is calculated as (Farquhar  
 12 et al., 1980; Knorr, 1997)

$$13 \quad R_d(25^\circ C) = 0.011V_{\max}^{25}(C_3) \text{ or } 0.042V_{\max}^{25}(C_4), \quad [A14]$$

14 The temperature dependency of  $R_d$  is also given by equation (A8) with the  
 15 activation energy  $E_R$  as a controlling parameter.

16 **Carbon balance**

1 The net primary productivity (NPP) is calculated as gross uptake of CO<sub>2</sub> by the  
 2 leaves (GPP) minus total autotrophic respiration which includes plant maintenance  
 3 respiration  $R_M$  and growth respiration  $R_G$ . Following Knorr (2000),  $R_M$  is calculated  
 4 from the leaf respiration as

$$6 \quad R_M = R_d / f_{R,leaf}, \quad [A15]$$

7  
 8 with  $f_{R,leaf}$  the leaf fraction of the maintenance respiration. Growth respiration is itself  
 9 proportional to NPP and calculated as follows:

$$11 \quad R_G = (f_{R,growth} - 1)NPP = (f_{R,growth} - 1)(GPP - R_M - R_G), \quad [A16]$$

12  
 13 where  $f_{R,growth}$  is the amount of carbon to be produced for a unit gain in vegetation  
 14 biomass.

15 The net ecosystem productivity (NEP) is now defined as

$$17 \quad NEP = NPP - R_s = NPP - R_{S,f} - R_{S,s}, \quad [A17]$$

18  
 19 where the heterotrophic soil respiration ( $R_s$ ) is composed of respiration from a  
 20 short-lived litter pool with time-varying size and a long-lived soil carbon pool (Knorr,  
 21 2000). Input to the litter pool is parameterized by the annual course of LAI in the case of  
 22 deciduous PFTs or, in the case of evergreen PFTs, as a constant fraction of the leaf  
 23 carbon pool. Soil respiration is assumed to be temperature and soil moisture dependent

1 and calculated from the following equations:

2

$$3 \quad R_S = (1 - f_S)k_f C_f + k_S C_S, \quad [\text{A18}]$$

4

5 with the sizes of the fast or litter pool,  $C_f$ , and the slow pool,  $C_S$ .  $f_S$  is the fraction of

6 decomposition from the fast pool that goes to the long-lived soil carbon pool. The rate

7 constants  $k_f$  and  $k_S$  for fast and slow pools are

8

$$9 \quad k_f = \alpha^\kappa Q_{10f}^{T_a/10} / \tau_f, \quad [\text{A19}]$$

$$10 \quad k_S = \alpha^\kappa Q_{10S}^{T_a/10} / \tau_S, \quad [\text{A20}]$$

11

12 where  $T_a$  is the air temperature,  $\alpha^\kappa$  the soil moisture dependence parameter,  $Q_{10,f}$  and

13  $Q_{10,s}$  temperature dependence parameters, and  $\tau_f$  and  $\tau_S$  the pool turnover times at 25°C.

#### 14 **Stomatal control**

15 The model of stomatal control follows the assumption that, in the absence of water

16 stress, leaf-level photosynthesis operates at a standard ratio between the leaf internal

17 CO<sub>2</sub> concentration,  $C_i$ , and the CO<sub>2</sub> concentration of free air,  $C_a$ . This value is given by

18

$$19 \quad C_{i,0} = f_{C_i} C_a, \quad [\text{A21}]$$

20

21 with two values for  $f_{C_i}$ , one for C3 and one for C4 vegetation. In order to determine the

22 demand for CO<sub>2</sub> uptake,  $A_n$  is first calculated as  $A_n, 0$  for  $C_i = C_{i,0}$ , and  $T_c = T_a$ . Inversion

1 of the diffusion equation for CO<sub>2</sub> at the stomatal boundary is then used to compute  
 2 stomatal conductance in the absence of water stress at each canopy layer (in ms<sup>-1</sup>):

$$3 \quad g_{s,0} = \frac{1.6A_{n,0}}{C_a - C_{i,0}} \frac{RT_k}{p}, \quad [A22]$$

5  
 6  $p$  is air pressure (in Pa). If at the time of highest demand,  $D$ , transpiration rates exceed  
 7 a root water supply rate,  $g_s$ , stomatal conductance at each canopy layer is reduced  
 8 according to

$$9 \quad g_s = \frac{g_{s,0}}{1 + b_e D_a}, \quad [A23]$$

11  
 12 The factor  $b_e$  is assumed to change with soil water status in such a way that during the  
 13 course of a day, the transpiration rate,  $E_t$ , does not exceed a root supply rate,  $S$ ,  
 14 described by Federer (1982),

$$15 \quad S = C_w \frac{W}{W_{max}}, \quad [A24]$$

17  
 18  $W$  is the soil water content, adjusted to take soil freezing into account (Knorr, 1997),  
 19 and  $C_w$  an empirical parameter representing root density.  $W_{max}$  is the maximum plant  
 20 available soil water and used as a parameter here.



1 Next, the canopy temperature,  $T_c$ , is computed consistent with the energy balance  
 2 after integrating  $g_s$  over the canopy to obtain the canopy conductance used in the  
 3 Penman–Monteith equation. Then, the photosynthesis model is run again, but at a fixed  
 4 stomatal conductance,  $g_s$ , which yields the final GPP.

### 6 Latent heat flux

7 Latent heat flux (LHF) is the sum of two terms,

$$9 \quad LHF = \lambda E_v + \lambda E_s, \quad [A25]$$

10  
 11 where  $\lambda$  is latent heat,  $E_v$  is evapotranspiration from the vegetation and  $E_s$  is  
 12 evaporation from the soil.

### 14 Evapotranspiration from the vegetation

15  $E_v$  is primarily driven by the net radiative balance of the vegetation ( $R_{n,v}$ ) and is limited  
 16 by the available amount of soil ( $W_s$ ) and skin or intercepted water ( $W_i$ ). If the vegetation  
 17 surfaces are wet ( $W_i > 0$ ), the canopy conductance is infinite ( $G_c \rightarrow \infty$ ) so that  
 18 evaporation follows its maximum rate,  $E_{v,max}$ , with the evaporated water coming from  
 19 the skin or intercepted reservoir:

$$21 \quad E_v = E_i = E_{v,max} = \frac{sR_{n,v} + \rho C_p (e_s(T) - e_a) G_a}{s + \gamma} \quad (W_i > 0), \quad [A26]$$

1 Here, it is allowed to assume negative values during dew formation. When the  
 2 vegetation is dry ( $W_i = 0$ ), evapotranspiration is determined by  $G_c$ , which is the  
 3 combined conductance of all stomata within the plant canopy,:

$$5 \quad G_c = \int_0^{\Lambda} g_s(l) dl, \quad [\text{A27}]$$

6  
 7  $\Lambda$  denotes the leaf area index (LAI) of the canopy, and  $dl$  its differential element. The  
 8 transpiration rate is then calculated from the Penman-Monteith formula (Monteith  
 9 1965):

$$11 \quad E_v = E_t = \frac{sR_{n,v} + \rho C_p (e_s(T) - e_a) G_a}{s + \gamma(1 + G_a/G_c)} \quad (W_i = 0), \quad [\text{A28}]$$

12  
 13 An additional condition is given by  $E_t \geq 0$ , i.e. transpiration is only allowed from the  
 14 vegetation to the atmosphere.

15 The daily integral of the transpiration rate  $E_{v,\text{daily}}$  also depends on the size of the  
 16 skin reservoir,  $W_i$ , and the rain input:

$$18 \quad E_{v,\text{daily}}(t)\Delta t = \min \left\{ \int_{1\text{day}} E_{v,\text{max}}(t') dt'; W_i(t - \Delta t) + (P_i(t) - P_v(t))\Delta t \right\}, \quad [\text{A29}]$$

19

1 with a one-day time step  $\Delta t$ . In the model, this and all other daily values are  
 2 approximated by integrating over the instantaneous rates given at the hourly time  
 3 steps.

4 According to equation 29, transpiration can only happen when the vegetation is  
 5 dry. To account for this fact when calculating the daily rate  $E_{v,\text{daily}}(t)\Delta t$ , a time average  
 6 wetness fraction is defined:

7

$$8 \quad F_{\text{daily}} = \frac{E_{v,\text{daily}}(t)\Delta t}{\int_{1\text{day}} E_{v,\text{max}}(t')dt'}, \quad [\text{A30}]$$

9 and daily transpiration is reduced accordingly:

10

$$11 \quad E_{v,\text{daily}}(t)\Delta t = (1 - F_{\text{daily}}) \int_{1\text{day}} \frac{sR_{n,v} + \rho C_p \Delta e G_a}{s + \gamma(1 + G_a/G_c)}, \quad [\text{A31}]$$

12

13 The aerodynamic exchange between the canopy and the free air is described as

14

$$15 \quad G_a = g_{a,v} u, \quad [\text{A32}]$$

16

17 with wind speed,  $u$ , and a proportionality factor serving as a free model parameter.

18 Wind speeds below  $1 \text{ ms}^{-1}$  are uniformly set to  $1 \text{ ms}^{-1}$  to avoid unrealistically high canopy  
 19 temperature under conditions of extremely still air and high incoming radiation. The  
 20 prior value of  $g_{a,v}$  is determined from the following formula:

21

$$1 \quad g_{a,v} = \frac{k^2}{\left[\ln\left(h_{ref}/r_z h_c + a_z\right)\right]^2} \quad [A33]$$

2

3  $h_{ref}$  is the reference height above canopy (10 m),  $h_c$  the canopy height,  $k = 0.41$ ,  $r_z = 0.1$   
4 and  $a_z = 1$ .

5

### 6 **Evaporation from the soil**

7 Evaporation from the soil,  $E_s$ , is calculated by Ritchie's method (Ritchie, 1972),  
8 which describes soil evaporation in two phases: it starts with phase 1 at the potential  
9 rate up to a total evaporated amount of  $\Sigma E_1$ , while in the ensuing phase 2, the  
10 cumulative evaporation is proportional to the square root of time, with a proportionality  
11 factor  $\kappa_s$ , called "desorptivity". Following is a description of how the Ritchie model has  
12 been adopted to fit into the structure of the vegetation model developed in this study.  
13 The two status variables of the Ritchie model are evaporation time,  $t_s$ , and the total  
14 evaporation,  $\Sigma E_s$ . Evaporation time is the time that has passed without additional  
15 rainfall since the beginning of phase 2, with a total phase 2 evaporation of  $\Sigma E_s - \Sigma E_1$ .  
16 Daily soil evaporation is at first calculated as follows:

17

$$18 \quad E_{s,0}(t)\Delta t = \begin{cases} E_{s,\max}(t)\Delta t & \text{if } \sum E_s(t-\Delta t) - P'_s(t)\Delta t < \sum E_1 \\ 0.8P'_s(t)\Delta t & \text{if } \sum E_s(t-\Delta t) - P'_s(t)\Delta t \geq \sum E_1 \\ \kappa_s(\sqrt{t_s + \Delta t} - \sqrt{t_s}) & \text{and } P'_s(t)\Delta t < 1.25\kappa_s(\sqrt{t_s + \Delta t} - \sqrt{t_s}) \\ & \text{else,} \end{cases} \quad [A34]$$

19

1 where  $t_s$  is taken at the time step  $t\Delta t$ , i.e.  $t_s \equiv t_s(t\Delta t)$ . The precipitation rate arriving at  
 2 the ground is  $P'_s = P_s + P_v$ . The first case represents phase 1 evaporation at the potential  
 3 rate,  $E_{s,\max}$ , and the last case phase 2 evaporation, controlled by desorptivity,  $\kappa_s$ . In  
 4 between there is another case with additional rain input.

5 In a second step, the evaporation rate in phase 2 is limited by the atmospheric  
 6 demand, i.e. the potential rate, and a case is included where evaporation is at the  
 7 transition from phase 1 to phase 2. Actual soil evaporation is thus calculated from:

$$8 \quad E_s(t)\Delta t = \begin{cases} E_{s,\max}(t)\Delta t - 0.4\left[\sum \tilde{E}_s(t-\Delta t) - \sum E_1\right]\Delta t & \text{if } \sum E_s(t-\Delta t) - P'_s(t)\Delta t < \sum E_1 \\ & \text{and } \sum \tilde{E}_s(t-\Delta t) < \sum E_1 \\ \min\{E_{s,0}(t); E_{s,\max}(t)\}\Delta t & \text{else,} \end{cases} \quad [A35]$$

10

11 with

12

$$13 \quad \sum E_s(t-\Delta t) = \max\{\sum E_s(t-\Delta t) - P'_s\Delta t; 0\} + \min\{E_{s,0}(t); E_{s,\max}(t)\}\Delta t \quad [A36]$$

14

15 At the end of the time step  $\Delta t$ , the two status variables are advanced:

16

$$17 \quad \sum E_s(t) = \max\{\sum E_s(t-\Delta t) - P'_s\Delta t; 0\} + E_s(t)\Delta t \quad [A37]$$

18

19 and

20

$$t_s(t) = \begin{cases} 0 & \text{if } \sum E_s(t) < \sum E_1 \\ \left( \sum E_s(t) - \sum E_1 \right)^2 / \kappa_s^2 & \text{else,} \end{cases} \quad [\text{A38}]$$

2

3 The parameters desorptivity,  $\kappa_s$ , and phase 1 evaporation  $\sum E_1$ , are set dependig on the  
4 sand fraction of the uppermost soil layer,  $f_1^{\text{sand}}$ , following the values given in Ritchie  
5 (1972):

6

$$\kappa_s = 5.62 - 2.56 f_1^{\text{sand}} \quad [\text{A39}]$$

8

9 in  $\text{kg m}^{-2} \text{day}^{-0.5}$  and

10

$$\sum E_1 = 14.29 - 0.23 f_1^{\text{sand}} \quad [\text{A40}]$$

12

13 in  $\text{kg m}^{-2}$ .

14 Potential soil evaporation  $E_{s,\text{max}}$  is assumed as an equilibrium evaporation (Philippe,  
15 1957), reduced by the rate of snow evaporation,  $E_{sn}$ ;

16

$$E_{s,\text{max}} = \frac{s(R_{n,s} + G)}{\lambda(s + \gamma)} - E_{sn}, \quad [\text{A41}]$$

18

19 where  $E_{sn}$  is snow evaporation, calculated as a follow,

20

$$E_{sn}(t)\Delta t = \min \{ E_{s,\text{max}}(t)\Delta t; \quad W_{sn}(t - \Delta t) + P_{sn}(t)\Delta t \}, \quad [\text{A42}]$$

21

1

2 where  $W_{sn}$  is snow amount in mm,  $P_{sn}$  is snow fall, and  $E_{sn, \max}$  is potential snow  
 3 evaporation, determined primarily by net radiation,  $R_{ns}$ , and depends to a large extent  
 4 on snow albedo as a follow,

5

$$6 \quad E_{sn, \max} = \frac{s(R_{n,s} + G)}{\lambda(s + \gamma)} \quad [A43]$$

7

8

9

## 10 **Air humidity**

11 Since no reliable data of near-surface air humidity exist for purposes of global modeling,  
 12 this quantity has to be estimated. It is often assumed that the daily mean of the vapor  
 13 pressure is equal to the saturation vapor pressure at the daily minimum temperature.  
 14 Friend (1998) has checked this assumption with climate data by Müller (1982) and has  
 15 found a good agreement for Europe and North America. For weather stations in arid  
 16 regions, however, agreement is much less satisfactory, resulting in an overall  
 17 correlation coefficient of = 0.87 for 805 stations. An overestimate of the vapor pressure  
 18 occurs, when the air is not saturated at the minimum temperature,  $T_{\min}$ , as under  
 19 severe drought (Running et al. 1987), and an underestimate, when the vapor pressure  
 20 rises during the day because of evapotranspiration (Rosenberg, 1974).

21 In order to account for such findings, the daily course of the vapor pressure,  $e_s(t)$ ,  
 22 is calculated from instantaneous saturation vapor pressure,  $e_s(T)$ , saturation vapor

1 pressure at sunrise,  $e_s(T_{\min})$ , and the ratio of daily mean evapotranspiration and daily  
 2 mean evaporative demand (cf. subsection “soil water balance”). Variation of the  
 3 dependence is achieved through the parameters  $h_0$  (relative humidity at sunrise, when  
 4  $T=T_{\min}$ , and total drought, i.e.  $f_e = 0$ ) and  $\hat{h}$  (daily amplitude of the vapour pressure  
 5 under moist conditions, i.e.  $= 1$ , as a fraction of the amplitude at constant saturation):

$$7 \quad e_a = e_{a0} + f_e \hat{h} (e_s(T) - e_{a0}), \quad [\text{A44}]$$

8 where

$$9 \quad e_{a0} = (h_0 + (1 - h_0) f_e) e_s(T_{\min}), \quad [\text{A45}]$$

10 and

$$11 \quad f_e(t) = \frac{E(t - \Delta t)}{E_{t,\max}(t - \Delta t) + E_{s,\max}(t - \Delta t)}, \quad [\text{A46}]$$

12

13  $f_e$  is defined as the ratio of actual to potential evapotranspiration from vegetation and  
 14 soil. For the computation of  $e_a$ , the value of the preceding time step  $\Delta t$  of one day is  
 15 taken. The saturation vapor pressure over water or ice,  $e_s(T)$ , is calculated from Murray  
 16 (1967):

17

$$18 \quad e_s(T) = \begin{cases} 610.78 \exp(17.269T/(237.3 + T)) & \text{for } T > 0 \\ 610.78 \exp(17.269T/(271.15 + T)) & \text{for } T < 0 \end{cases} \quad [\text{A47}]$$

19

20 **Energy and radiation balance**



1 PAR absorption is calculated according to the two-flux scheme by Sellers (1985) with  
 2 three vertical layers of equal LAI. The diffuse fraction of PAR is calculated according to  
 3 a procedure by Weiss and Norman (1985). Leaf-angle distribution is assumed to be  
 4 uniform, and the only free parameters for this scheme is  $\omega$ , the leaf single-scattering  
 5 albedo. To determine evapotranspiration rates from the Penman–Monteith formula,  
 6 BETHY computes net radiation balance of the canopy,  $R_{n,c}$ , according to the following  
 7 equation:

$$9 \quad R_{n,c} = (1 - t_{l,v}) \left[ (\varepsilon_a - \varepsilon_{sfc}) \sigma T_{K,a}^4 - G \right] - (1 - a_C - a_S) f_{PAR} R_S, \quad [A48]$$

10

11  $\varepsilon_a$  and  $\varepsilon_{sfc} = 0.97$  are sky and surface emissivity, respectively,  $T_{K,a}$  air temperature in  
 12 Kelvin,  $\sigma = 5.6703 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$  the Stefan–Boltzmann constant, and  $t_{l,v}$  the longwave  
 13 transmissivity of the vegetation, assumed  $t_{l,v} = f_c \exp(-0.5A/f_c) + (1 - f_c)$ .  $f_c$  is the fraction  
 14 of soil covered by vegetation. For the shortwave part,  $R_S$  is incoming solar radiation  
 15 ( $\text{W m}^{-2}$ ),  $f_{PAR}$  is the fraction of PAR absorbed by the vegetation and computed by the  
 16 two-flux scheme,  $a_v$  the albedo of the vegetation at the limit of high LAI and closed  
 17 canopy, and as the amount of solar radiation absorbed by the soil under the canopy at  
 18 the same limit.  $G$  is the ground heat flux, assumed to be a fixed fraction of total net  
 19 radiation. (Contrary to  $R_{n,c}$ , total net radiation does not depend on  $G$ , so there is no  
 20 implicit equation to be solved.) The sky emissivity is computed from

21

$$22 \quad \varepsilon_a = \varepsilon_s \left( \frac{e_a}{T_{K,a}} \right)^{1/7} (1 + 0.22n_c^2), \quad [A49]$$

1

2 with the cloud cover fraction  $n_c$ . If no separate radiation data for PAR and solar  
 3 radiation are available,  $R_s$  is calculated from PAR according to Weiss and Norman  
 4 (1985).

5

## 6 Phenology

7 We assume that spatial variability within a grid cell is entirely the result of differences  
 8 in the response among the existing PFTs, which are defined by threshold parameter for  
 9 each PFT as the trigger. This parameter is assumed to have a Gaussian probability  
 10 distribution in space. There are two of those threshold parameters:  $\tilde{T}_\phi$  and  $\tilde{t}_c$ . It is  
 11 important to note that the transition to the active state requires both  $\tilde{T}_\phi > T$  and  $\tilde{t}_c > t_d$ ,  
 12 where  $T$  is a temperature and  $t_d$  length of day. The tilde ( $\tilde{\cdot}$ ) denotes that these are  
 13 parameters. As shown next, these parameters are integrated over their probability  
 14 distribution, replacing the integration across the space of the grid cell.

15 Before proceeding to the spatial integration, we define a generic differential  
 16 equation in time for the LAI of individual plants,  $\tilde{\Lambda}(t)$ :

17

$$18 \quad \frac{d\tilde{\Lambda}(t)}{dt} = \begin{cases} f_1, & \text{if } T \geq \tilde{T}_\phi \text{ and } t_d \geq \tilde{t}_c, \\ f_2, & \text{else,} \end{cases} \quad [\text{A50}]$$

19

20 where  $f_1$  and  $f_2$  are some arbitrary functions of the state of the vegetation. In this  
 21 discrete formulation, the response of LAI to changes in  $\tilde{T}_\phi$  or  $\tilde{t}_c$  is usually  
 22 nondifferentiable at the threshold. The continuous version of equation (A39), which is

1 valid for the spatially integrated LAI,  $\Lambda(t)$ , resolves to an integral over the Gaussian  
 2 probability density functions (PDF),  $p$  and  $q$ , of the two trigger variables.

3

$$4 \quad \frac{d\Lambda(t)}{dt} = f_1 \int_{-\infty}^T \int_{-\infty}^{t_d} p(\tilde{T}_\phi) q(\tilde{t}_c) d\tilde{T}_\phi d\tilde{t}_c + f_2 \left( 1 - \int_{-\infty}^T \int_{-\infty}^{t_d} p(\tilde{T}_\phi) q(\tilde{t}_c) d\tilde{T}_\phi d\tilde{t}_c \right) \quad [\text{A51}]$$

5

6 The spatial PDF  $p$  is characterized by a mean  $T_\phi$  and its standard deviation  $T_r$ , while  
 7 the mean of  $q$  is  $t_c$  and the standard deviation  $t_r$ . All four are CCDAS control  
 8 parameters. Note the distinction between these two spatial PDFs and the fact that their  
 9 four parameters have again PDFs in the Bayesian sense in the same way as all other  
 10 control parameters.

11 The previous expression simplifies to

$$12 \quad \frac{d\Lambda(t)}{dt} = ff_1 + f_2(1-f) \quad [\text{A52}]$$

13

14 With

15

$$16 \quad f = \int_{-\infty}^T p(\tilde{T}_\phi) d\tilde{T}_\phi \int_{-\infty}^{t_d} q(\tilde{t}_c) d\tilde{t}_c = \Phi\left(\frac{T-T_\phi}{T_r}\right) \Phi\left(\frac{t_d-t_c}{t_r}\right) \quad [\text{A53}]$$

17

18 where  $\Phi$  is the cumulative normal distribution.  $f$  is the fraction of plants within the  
 19 proportion of a grid cell occupied by each PFT that are actively growing or maintaining  
 20 leaves.

1

## 2 4.3. Time evolution of LAI

3 Describing the generic formulation for the time evolution of LAI of a single plant –  
 4 which is then integrated spatially via equation (A41) – requires definition of  $f_1$  (for  
 5 plants in their growing season) and  $f_2$  (for senescent plants). For  $f_1$ , we assume the  
 6 simplest formulation that satisfies the following two conditions: leaf growth starts  
 7 immediately and is not limited by substrate availability, such as LAI itself; and growth  
 8 stops if a target LAI is reached that is in balance with the environmental limitations,  
 9 described as  $\Lambda_{\max}$ . These conditions are met by the following formulation:

10

$$11 \quad f_1 = \xi(\Lambda_{\max} - \Lambda), \quad [A54]$$

12

13 where  $\xi$  is a linear growth constant describing the increase in LAI per time unit shortly  
 14 after bud burst. This rate is chosen to be independent of carbon gains (NPP), because  
 15 initial leaf development relies on buds and reserves from the previous year (Kaduk and  
 16 Heimann, 1996). This formulation differs from those used in similar applications, such  
 17 as the logro-P phenology model (C. Reick, personal communication, 2010, implemented  
 18 by Raddatz et al., 2007) or the one by Liu et al. (2008), where the initial growth is  
 19 exponential resulting in a logistic function for the time integral under constant  
 20 conditions. Equation (A43) results in a time dependence described by  $\Lambda(t)/\Lambda_{\max} = 1 -$   
 21  $\exp(-\xi t)$  for  $\Lambda(0) = 0$ , which is linear for small  $t$ . The advantage of this approach is that it  
 22 does not require setting a minimum LAI to set off growth, which was 0.1 in the work by  
 23 Liu et al. (2008). This would not work here, because  $\Lambda_{\max}$  might be less than such a

1 minimum value. The difference in approach can be explained by the fact that the work  
 2 just cited is restricted only to temperature controlled phenology and does not include  
 3 situations where either the temperature or the water balance only allows small values  
 4 of LAI. For those plants that are outside their growth stage, we again chose the simplest  
 5 formulation that allows accommodating both deciduous and evergreen phenology:

$$7 \quad f_2 = \Lambda / \tau_L, \quad [A55]$$

8  
 9 The new parameter  $\tau_L$ , which is related to leaf longevity, describes how quickly leaves  
 10 are shed, or whether they stay inactive until the next growing season. Deciduous  
 11 vegetation will normally shed leaves (which includes leaves turning brown, see above)  
 12 within days to weeks. Evergreen vegetation on the other side should have values at the  
 13 order of a year or more. We now consider evolution of the spatially integrated grid-cell  
 14 average LAI. Inserting equation (A43) and (A44) into equation (A41) yields:

$$16 \quad \frac{d\Lambda(t)}{dt} = \xi[\Lambda_{\max} - \Lambda(t)]f - \frac{\Lambda(t)}{\tau_L}(1-f), \quad [A56]$$

17  
 18 In order to find a convenient form for integrating this expression, we define

$$20 \quad r = \xi f + (1-f)/\tau_L, \quad [A57]$$

21  
 22 and

1

$$2 \quad \Lambda_{\text{lim}} = \xi \Lambda_{\text{max}} f / r, \quad [\text{A58}]$$

3

4 so that equation (A52) takes the form:

5

$$6 \quad \frac{d\Lambda(t)}{dt} = \xi \Lambda_{\text{max}} f - r\Lambda(t), \quad [\text{A59}]$$

$$7 \quad = r[\Lambda_{\text{lim}} - \Lambda(t)], \quad [\text{A60}]$$

8

9 As long as  $f$  and  $\Lambda_{\text{max}}$  (and therefore  $r$  and  $\Lambda_{\text{min}}$ ) do not depend on  $t$ , the equation above

10 has the following solution:

11

$$12 \quad \Lambda(t + \Delta t) = \Lambda_{\text{lim}} - [\Lambda_{\text{lim}} - \Lambda(t)]e^{-r\Delta t}, \quad [\text{A61}]$$

13

14 Here it is sufficient to state that  $\Lambda_{\text{max}}$  depends on quantities that are updated either  
 15 daily or every few days, while  $f$  depends on daily values of temperature and day length.

16 Therefore, the last equation can be used to integrate over a single daily time step of the  
 17 phenology scheme,  $\Delta t$ . This mixture of analytical and numerical integration is not only  
 18 highly efficient, it also ensures stability, i.e. it avoids negative  $\Lambda$  as long as  $\Lambda_{\text{lim}}$  and  $\Lambda(0)$   
 19 are non-negative.

20

21 4.5. Water and structural limitations

1 On a global scale, the main limiting factor on terrestrial plant growth is not  
2 temperature, but water (Woodward, 1987). Whenever photosynthesizing, plants lose  
3 water by transpiration through pore openings (“stomata”) in their leaves. This  
4 limitation together with any other limitation on leaf growth, here considered  
5 “structural”, is described by a single state variable,  $A_{\max}$ . If soil water is limiting, an  
6 increasingly negative soil water potential leads to a falling leaf potential in a  
7 complicated process involving root water uptake, xylem resistance to flow, and  
8 transpiration through the leaf stomata. If stomata close, leaves can retain water, but  
9 only to a degree that depends on its cuticular resistance, which in itself is dependent on  
10 the plant functional type (PFT). However, it is not possible to represent those complex  
11 mechanisms in a model designed for global-scale applications. The scheme chosen here  
12 goes back to Woodward (1987), who used annual potential evapotranspiration and  
13 precipitation to derive water limited LAI on a global scale. To accommodate the shorter  
14 time scale of our model, we have modified his scheme by using daily actual  
15 transpiration and soil moisture instead of annual potential evapotranspiration and  
16 precipitation. However, to have LAI react not to rapidly changing daily conditions but to  
17 the longer-term climatic state, the water limited LAI is averaged back in time using the  
18 same approach as for  $T$ . Generally speaking, leaf development will stop and leaves will  
19 be shed if there is insufficient soil water for transpiration. At which level this happens  
20 exactly will be a function of various drought adaptations of the PFT concerned.  
21 Independent of the details, however, adaptation will determine how long the plant at a  
22 given LAI  $A$  can survive with a given amount of soil moisture without rain. This time

1 scale,  $\tau_w$ , can serve as a universal parameter of water limitation. This defines a  
 2 water-limited LAI,  $\Lambda_w$  through

$$3 \quad E(\Lambda_w)\tau_w = W, \quad [A62]$$

4 where  $W$  is plant-available soil moisture. What is needed then is the total water loss  
 5 after time  $\tau_w$  as a function of leaf area.

6 To compute this water loss, we linearize the potential rate of transpiration,  $E$ , as a  
 7 function of the LAI,  $\Lambda$ :

$$8 \quad E(\Lambda) \approx \frac{\tilde{E}}{\tilde{\Lambda}} \Lambda, \quad [A63]$$

9  $\tilde{E}$  is the daily mean potential rate of transpiration last computed by the model at a LAI  
 10 of  $\tilde{\Lambda}$ . This approximation is most accurate at low values of  $\Lambda$  and  $\tilde{\Lambda}$ , where net  
 11 radiation of the leaf canopy, which drives evapotranspiration (Jarvis and McNaughton,  
 12 1986), can be assumed to scale linearly with LAI.

13 Combining Equ. (A62) and Equ. (A63) yields

$$14 \quad \Lambda_w = \frac{W\tilde{\Lambda}}{\tilde{E}\tau_w}. \quad [A64]$$

15



1 The parameter  $\tau_w$  represents the expected length of drought periods. For  $\tau_w \rightarrow 0$  the  
 2 plant “expects” its water reserves to always be sufficient for continuing survival. In this  
 3 case,  $\Lambda_w \rightarrow \infty$ , meaning the plant has no explicit drought adaptation in its phenology.  
 4 This is assumed for the cold-deciduous and cold-evergreen PFTs. For warm-evergreen  
 5 plants, we expect the value for  $\tau_w$  in the region of 1 year, and for grasses and  
 6 warm-deciduous plants between one and two months. Water limitation is implemented  
 7 separately for each PFT to reflect differences in the water use strategy, defined mainly  
 8 by  $\tau_w$ .

9 Observe also that for  $\tilde{E} \rightarrow 0$  we have  $\Lambda_w \rightarrow \infty$ , since without evaporative demand  
 10 the leaf area is not water limited, as it is the case for  $\tau_w \rightarrow 0$ . Since the LAI cannot grow  
 11 indefinitely, it must be limited by other factors, such as light availability, nutrients and  
 12 structure. These additional limitations are summarized into a single universal  
 13 parameter  $\hat{\Lambda}$  (cf., Knorr (2000)) and incorporated into the model via:

$$15 \quad \tilde{\Lambda}_{\max} = \nu(\hat{\Lambda}, \Lambda_w). \quad [A65]$$

16

17  $\nu(x, y)$  is a smoothed minimum function defined by

18

$$19 \quad \nu(x, y) = \frac{x + y - \sqrt{(x + y)^2 - 4xy}}{2\eta} \quad [A66]$$

20

21 with  $\eta = 0.99$ .  $\tilde{\Lambda}_{\max}$  is recomputed daily with daily values of the soil moisture,  $W$ ,

22 whereas  $\tilde{E}$  may be recomputed at longer intervals. This allows avoiding re-computing

1 the diurnal cycle of photosynthesis and energy balance for every simulated day, while  
 2 keeping a daily time step of phenology and water balance, to save computing time with  
 3 the full BETHY model (Knorr, 2000).

4 Instead of  $\tilde{\Lambda}_{\max}$ , equation (A43) uses  $\Lambda_{\max}$ , its weighted time integration  
 5 computed in the same way as  $T$  from  $T_{2m}$  (equation (A53)), with the analogous  
 6 definition:

7

$$8 \quad \Lambda_{\max}(t) = \frac{1}{\tau_s} e^{-t/\tau_s} \int_{-\infty}^t \tilde{\Lambda}_{\max}(t') e^{t'/\tau_s} dt' . \quad [\text{A67}]$$

9

10 It is computationally favourable to bring equation (A67) to an incremental form:

11

$$12 \quad \begin{aligned} \Lambda_{\max}(t + \Delta t) &= \frac{1}{\tau_s} e^{-(t+\Delta t)/\tau_s} \int_{-\infty}^{t+\Delta t} \tilde{\Lambda}_{\max}(t') e^{t'/\tau_s} dt' \\ &= \frac{1}{\tau_s} e^{-\Delta t/\tau_s} e^{-t/\tau_s} \left( \int_{-\infty}^t \tilde{\Lambda}_{\max}(t') e^{t'/\tau_s} dt' + \int_t^{t+\Delta t} \tilde{\Lambda}_{\max}(t') e^{t'/\tau_s} dt' \right). \\ &= e^{-\Delta t/\tau_s} \Lambda_{\max}(t) + \frac{1}{\tau_s} e^{-(t+\Delta t)/\tau_s} \int_t^{t+\Delta t} \tilde{\Lambda}_{\max}(t') e^{t'/\tau_s} dt' \end{aligned} \quad [\text{A68}]$$

13

14 If  $\Delta t$  is very short and thus the time incremental of  $\Lambda_{\max}$  is assumed to be constant, and  
 15 then is expressed simply:

16

$$17 \quad \Lambda_{\max}(t + \Delta t) = e^{-\Delta t/\tau_s} \Lambda_{\max}(t) + \hat{\Lambda}_{\max}(t) (1 - e^{-\Delta t/\tau_s}), \quad [\text{A69}]$$

18

1 The advantage of this scheme is that it has only one free parameter,  $\tau_w$ , in addition to  $\tau_s$ .  
2 Since changing  $\tau_s$  might lead to instabilities of the optimization in a way similar to  $\tau_m$ ,  
3 this parameter is also held constant at value of 30 days. Detailed information for this  
4 phenology part is to be referred to Knorr et al. (2010).

5

6

## 7 **References**

- 8 Collatz, G. J., Ribas-Carbo, M., and Berry, J. A.: Coupled photosynthesis-stomatal  
9 conductance model for leaves of C<sub>4</sub> plants, *Australian Journal of Plant Physiology*, 19,  
10 519-538, 1992.
- 11 Farquhar, G. D., von Caemmerer, S., and Berry, J. A.: A Biochemical Model of  
12 Photosynthetic CO<sub>2</sub> Assimilation in Leaves of C<sub>3</sub> species, *Planta*, 149, 78-90, 1980.
- 13 Farquhar, G. D.: Models relating subcellular effects of temperature to whole plant  
14 response, *Symp. Soc. Exper. Biol.*, 42, 395-409, 1988.
- 15 Federer, C. A.: Transpirational supply and demand: plant, soil, and atmospheric effects  
16 evaluated by simulation, *Water Resources Research*, 18, 355-362, 1982.
- 17 Friend, A. D.: Parameterisation of a global daily weather generator for terrestrial  
18 ecosystem and biogeochemical modelling, *Ecological Modelling*, 109, 121-140, 1998.
- 19 Jarvis, P. G., and McNaughton K. G.: Stomatal control of transpiration: scaling up from  
20 leaf to region, *Advances in Ecological Research*, 15, 1-49, 1986.
- 21 Kaduk, J., and Heimann, M.: A prognostic phenology scheme for global terrestrial carbon  
22 cycle models, *Clim. Res.*, 6, 1-19, 1996.

- 1 Knorr, W.: Satellitengestützte Fernerkundung und Modellierung des globalen  
2 CO<sub>2</sub>-Austauschs der Landvegetation: Eine Synthese, Ph.D. thesis, Max-Planck-Inst.  
3 für Meteorol., Hamburg, Germany, 1997.
- 4 Knorr, W.: Annual and interannual CO<sub>2</sub> exchanges of the terrestrial biosphere: Process  
5 based simulations and uncertainties, *Glob. Ecol. Biogeogr.*, 9(3), 225–252, 2000.
- 6 Knorr, W., Kaminski, T., Scholze, M., Gobron, N., Pinty, B., Giering, R., and Mathieu,  
7 P.-P.: Carbon Cycle Data Assimilation with a Generic Phenology Model, *J. Geophys.*  
8 *Res. Atmos.*, 115, G04017, doi:10.1029/2009JG001119, 2010.
- 9 Liu, Q., Gu, L., Dickinson, R. E., Tian, Y., Zhou, L., and Post, W. M.: Assimilation of  
10 satellite reflectance data into a dynamical leaf model to infer seasonally varying  
11 leaf areas for climate and carbon models, *J. Geophys. Res.*, 113, D19113,  
12 doi:10.1029/2007JD009645, 2008.
- 13 Müller, M. J.: Selected Climatic Data for a Global Set of Standard Stations for  
14 Vegetation Science. 306 pp. Dr. W. Junk Publishers, The Hague, 1982.
- 15 Murray, F. W.: On the computation of saturation vapour pressure, *J. Applied*  
16 *Meteorology*, 6, 203-204, 1967.
- 17 Philip, J.R.: Evaporation, and moisture and heat fields in the soil. *J. Meteorol.* 14,  
18 354–366. 19, 1957.
- 19 Raddatz, T. J., C. Reick, W. Knorr, J. Kattge, E. Roeckner, R. Schnur, K.-G. Schnitzler,  
20 P. Wetzler, and J. Jungclaus (2007), Will the tropical land biosphere dominate the  
21 climate carbon cycle feedback during the 21st century?, *Clim. Dyn.*,  
22 doi:10.1007/s00382-00007-00247-00388.
- 23 Reichstein, M., Falge, E., Baldocchi, D., Papale, D., Aubinet, M., Berbigier, P.,

- 1 Bernhofer, C., Buchmann, N., Gilmanov, T., Granier, A., Grunwald, T., Havrankova,  
2 K., Ilvesniemi, H., Janous, D., Knohl, A., Laurila, T., Lohila, A., Loustau, D.,  
3 Matteucci, G., Meyers, T., Miglietta, F., Ourcival, J.M., Pumpanen, J., Rambal, S.,  
4 Rotenberg, E., Sanz, M., Tenhunen, J., Seufert, G., Vaccari, F., Vesala, T., Yakir, D.,  
5 and Valentini, R.: On the separation of net ecosystem exchange into assimilation  
6 and ecosystem respiration: review and improved algorithm, *Global Change Biology*  
7 11, 1424-1439, 2005.
- 8 Ritchie, J. T.: Model for predicting evaporation from a row crop with incomplete cover.  
9 1204-1213, 1972.
- 10 Rosenberg, N. J.: *Microclimate. The biological environment*. Ed. John Wiley sons, New  
11 York, 315 p, 1974.
- 12 Running, S., Nemani, R., and Hungerford R.: Extrapolation of synoptic meteorological  
13 data in mountainous terrain, and its use for simulating forest evapotranspiration  
14 and photosynthesis, *Canadian Journal of Forest Research*, 17, 472-483, 1987.
- 15 Sellers, P. J.: Canopy reflectance, photosynthesis and transpiration, *International*  
16 *Journal of Remote Sensing*, 6, 1335–1372, 1985.
- 17 Weiss, A., and Norman, J.A.: Partitioning solar radiation into direct and diffuse, visible  
18 and near-infrared components, *Agricultural and Forestry Meteorology*, 34,  
19 205–213, 1985.
- 20 Woodward, F. I.: *Climate and Plant Distribution*, Cambridge Univ. Press, Cambridge, U.  
21 K., 1987.
- 22