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Supplement of

A Bayesian approach to evaluation of soil biogeochemical models

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Table S1: List of CON and AWB model parameters.

Model	Parameter	Value, if not varied	Units	Parameter Description
CON/AWB	I_S	0.0009	mg C g^{-1} soil h^{-1}	External SOC input rate
CON/AWB	I_D	0.0001	mg C g^{-1} soil h^{-1}	External DOC input rate
CON	$k_{S_{ref}}$	Dependent	mg C mg^{-1} C h^{-1}	SOC decay constant
CON	$k_{D_{ref}}$	Dependent	mg C mg^{-1} C h^{-1}	DOC decay constant
CON	$k_{M_{ref}}$	Dependent	mg C mg^{-1} C h^{-1}	MIC decay constant
CON	u_M	0.002	mg C mg^{-1} C h^{-1}	DOC uptake rate of microbes
CON	Ea_S	Fitted by HMC	kJ mol^{-1}	SOC decomposition activation energy
CON	Ea_D	Fitted by HMC	kJ mol^{-1}	DOC decomposition activation energy
CON	Ea_M	Fitted by HMC	kJ mol^{-1}	MIC decomposition activation energy
CON	a_{DS}	Fitted by HMC		DOC to SOC transfer coefficient
CON	a_{SD}	Fitted by HMC		SOC to DOC transfer coefficient
CON	a_M	Fitted by HMC		MIC to SOC transfer coefficient
CON/AWB	a_{MS}	Fitted by HMC		Fraction of dead MIC transferred
AWB	K_{ref}	Dependent	mg C g^{-1} soil	SOC reference K_M
AWB	$K_{U_{ref}}$	Dependent	mg C g^{-1} soil	DOC uptake into MIC reference K_M
AWB	V_{ref}	Fitted by HMC	mg C mg^{-1} C h^{-1}	SOC reference V_{max}
AWB	$V_{U_{ref}}$	Fitted by HMC	mg C mg^{-1} C h^{-1}	DOC uptake into MIC reference V_{max}
AWB	Ea_K	Fitted by HMC	kJ mol^{-1}	SOC K_M activation energy
AWB	Ea_{KU}	Fitted by HMC	kJ mol^{-1}	DOC uptake K_M activation energy
AWB	Ea_V	Fitted by HMC	kJ mol^{-1}	SOC V_{max} activation energy
AWB	Ea_{VU}	Fitted by HMC	kJ mol^{-1}	DOC uptake V_{max} activation energy
AWB	r_E	Dependent	mg C mg^{-1} C h^{-1}	Enzyme production rate
AWB	r_L	0.0005	mg C mg^{-1} C h^{-1}	Enzyme loss rate
AWB	r_M	Dependent	mg C mg^{-1} C h^{-1}	MIC death rate
AWB	$E_{C_{ref}}$	Fitted by HMC	mg C mg^{-1} C	Reference temperature C use efficiency (CUE)
AWB	m_t	Fitted by HMC	$°C^{-1}$	CUE temperature change slope

Section S1

(a) CON ODE system equations

The conventional (CON) model consists of three C pools in SOC, DOC, and MIC. The mass transfer of C between these pools is represented as first-order linear decay processes. The CON model obeys the following dynamics:

$$\frac{dS}{dt} = I_S + a_{DS}k_D D + a_M a_{MS} k_M M - k_S S \quad \text{(S1)}$$

$$\frac{dD}{dt} = I_D + a_{SD}k_S S + a_M(1 - a_{MS})k_M M - u_M D - k_D D \quad \text{(S2)}$$

$$\frac{dM}{dt} = u_M D - k_M M \quad \text{(S3)}$$

The decay constants k_I vary from their reference values k_{Iref} based on the Arrhenius equation of temperature dependence,

$$k_I = k_{I_{ref}} exp\left[-\frac{Ea_I}{R}\left(\frac{1}{T} - \frac{1}{T_{ref}}\right)\right] \quad \text{(S4)}$$

where R is the ideal gas constant 8.314 J mol^{-1} K^{-1} and the reference temperature T_{ref} used was 283.15 K.

CO_2 soil flux is calculated from the CON model by summing the proportion of fluxes that do not enter soil C pools at each time step:

$$\text{CON flux } = k_S S(1 - a_{SD}) + k_D D(1 - a_{DS}) + k_M M(1 - a_M) \quad \text{(S5)}$$

Response ratios are then calculated from the model output flux by dividing the flux calculated at a given time point by the pre-warming steady state flux.

(b) AWB ODE system equations

The Allison-Wallenstein-Bradford (AWB) model consists of four C pools in SOC, DOC, MIC, and ENZ (representing the extracellular enzyme C mass). In the AWB model, MIC accumulation and SOC decomposition follow a non-linear Michaelis-Menten function. Other processes, including ENZ production, ENZ loss, and MIC death still follow a first-order linear decay process. The AWB system equations are as follows:

$$\frac{dS}{dt} = I_S + a_{MS} r_M M - \frac{VES}{K + S} \quad \text{(S6)}$$

$$\frac{dD}{dt} = I_D + (1 - a_{MS})r_M M + \frac{VES}{K + S} + r_L E - \frac{V_U MD}{K_U + D} \quad \text{(S7)}$$

$$\frac{dM}{dt} = E_C \frac{V_U MD}{K_U + D} - r_M M - r_E M \quad \text{(S8)}$$

$$\frac{dE}{dt} = r_E M - r_L E \quad \text{(S9)}$$

Similar to the CON decay constants, the Michaelis-Menten function parameters K, K_U, V, and V_U vary from their reference values based on the Arrhenius equation. E_C, the AWB microbial C use efficiency parameter, depends linearly on temperature, following Li et al., 2014, and operates under the simplifying assumption that higher temperatures make C use slightly less efficient:

$$E_C = E_{C_{ref}} - m_t(T - T_{ref}) \quad \text{(S10)}$$

The loss rate parameters r_I were not made to be temperature dependent.

AWB CO_2 flux is calculated as the proportion of the C transfer out of the DOC pool that is not partitioned into the MIC pool:

$$\text{AWB flux} = (1 - E_C)\frac{V_U MD}{K_U + D} \tag{S11}$$

Section S2

(a) Re-arranged CON steady state equations

The steady state solutions for the C pools in CON are as follows:

$$D_0 = \frac{a_{SD} I_S + I_D}{u_M + k_D + u_M a_M (a_{MS} - a_{MS} a_{SD} - 1) - a_{DS} k_D a_{SD}} \tag{S12}$$

$$M_0 = \frac{u_M}{k_M} D_0 \tag{S13}$$

$$S_0 = \frac{I_S + D_0 (a_{DS} k_D + u_M a_M a_{MS})}{k_S} \tag{S14}$$

To set pre-warming steady state soil C densities to desired values, we re-arranged the steady state equations into the following forms to solve for the steady state values of parameters that depend on the soil C densities:

$$k_{Mref} = \frac{u_M D_0}{M_0} \tag{S15}$$

$$k_{Dref} = \frac{-I_D - a_{SD} I_S + u_M D_0 - a_M D_0 u_M + a_M a_{MS} u_M D_0 - a_M a_{MS} a_{SD} u_M D_0}{(a_{DS} a_{SD} - 1) D_0} \tag{S16}$$

$$k_{Sref} = \frac{I_S + D_0 \left(a_{DS} k_{Dref} + u_M a_M a_{MS}\right)}{S_0} \tag{S17}$$

(b) Re-arranged AWB steady state equations

The steady state solutions for the C pools in AWB are as follows:

$$S_0 = \frac{-r_L K \left(I_S \left(r_M \left(1 + E_C (a_{MS} - 1)\right) + r_E (1 - E_C) \right) + E_C I_D a_{MS} r_M \right)}{I_S \left(r_M \left(r_L \left(1 + E_C (a_{MS} - 1)\right)\right) + r_E (r_L (1 - E_C) - E_C V) \right) + E_C I_D (a_{MS} r_M r_L - r_E V)} \tag{S18}$$

$$M_0 = \frac{E_C (I_D + I_S)}{(1 - E_C)(r_M + r_E)} \tag{S19}$$

$$D_0 = \frac{-K_U (r_M + r_E)}{r_M + r_E - E_C V_U} \tag{S20}$$

$$E_0 = \frac{r_E M_0}{r_L} \tag{S21}$$

To set pre-warming steady state soil C densities to desired values, we re-arranged the steady state equations into the following forms:

$$r_E = \frac{r_L E_0}{M_0} \tag{S22}$$

$$r_{\mathrm{M}} = \frac{-E_{C_{ref}} (I_D + I_S) + M_0 r_E \left(1 - E_{C_{ref}}\right)}{M_0 \left(E_{C_{ref}} - 1\right)} \tag{S23}$$

$$K_{U_{ref}} = \frac{-D_0 \left(r_M + r_E - E_{C_{ref}} V_{U_{ref}} \right)}{r_M + r_E} \tag{S24}$$

$$K_{ref} = \frac{-S_0\left(-I_S r_E r_L + E_{C_{ref}} I_S r_E r_L - a_{MS} E_{C_{ref}} I_D r_L r_M - I_S r_L r_M + E_{C_{ref}} I_S r_L r_M - a_{MS} E_{C_{ref}} I_S r_L r_M + E_{C_{ref}} I_D r_E V_{ref} + E_{C_{ref}} I_S r_E V_{ref}\right)}{r_L\left(-I_S r_E + E_{C_{ref}} I_S r_E - a_{MS} E_{C_{ref}} I_D r_M - I_S r_M + E_{C_{ref}} I_S r_M - a_{MS} E_{C_{ref}} I_S r_M\right)} \tag{S25}$$

Figure S1: Distribution of CON and AWB fits to meta-analysis data (Romero-Olivares et al., 2017) with CON fits at **(a)** MIC = 1 mg C g^{-1} soil; and **(b)** MIC = 8 mg C g^{-1} soil.; and AWB fits at **(c)** MIC = 1 mg C g^{-1} soil; and **(d)** MIC = 8 mg C g^{-1} soil. Open circles show the meta-analysis data points. Blue vertical lines mark the 95% confidence interval for each data point calculated from the pooled standard deviation. The black line indicates the mean posterior predictive model fit. The orange shading marks the 95% posterior predictive interval for the fit. Non-MIC pre-warming steady state soil C densities were set at SOC = 100 mg C g^{-1} soil, DOC = 0.2 mg C g^{-1} soil, and ENZ = 0.1 mg C g^{-1} soil.

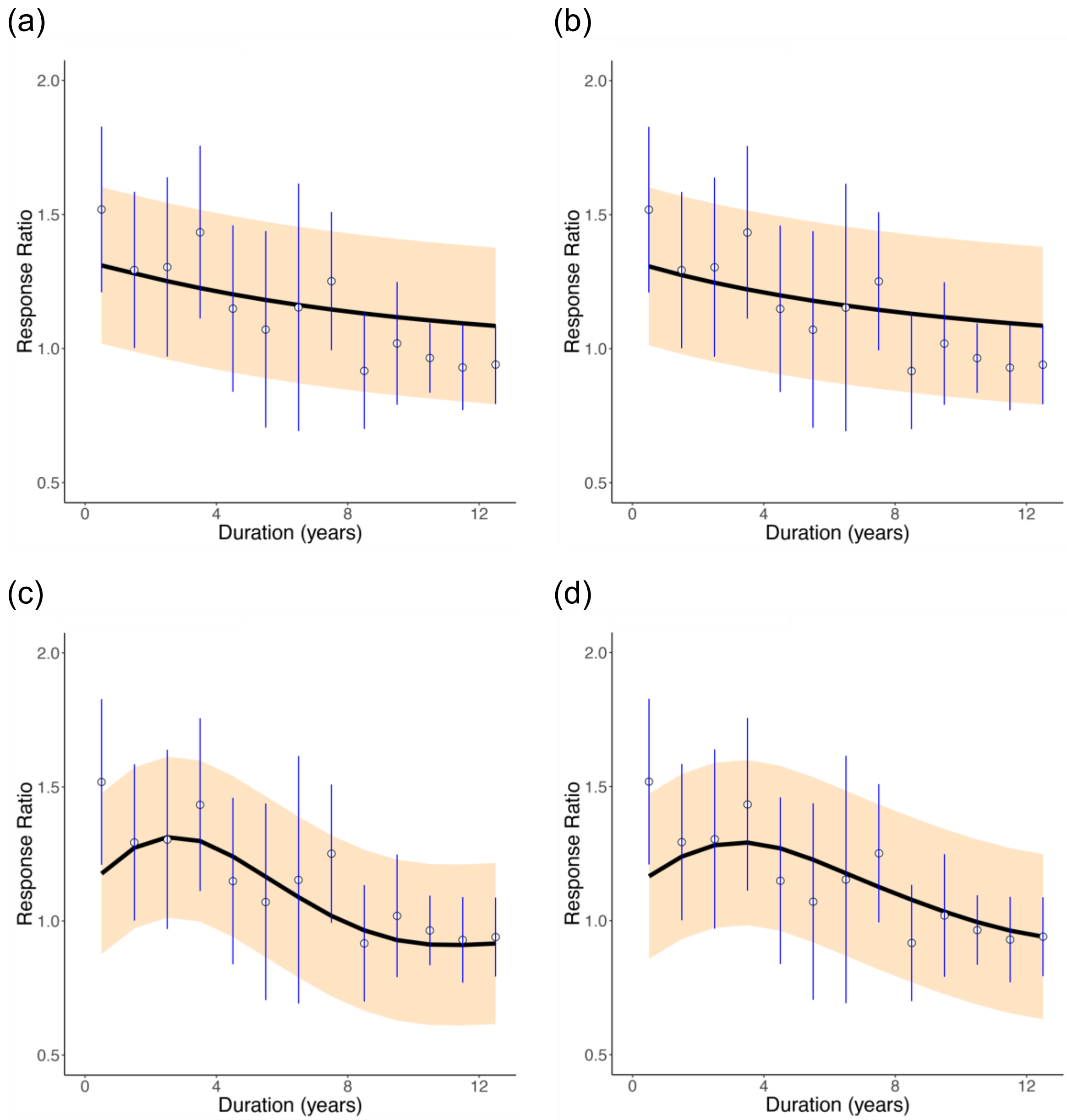

Figure S2: 95% probability density credible areas for **(a)** AWB $V_{U_{ref}}$; and **(b)** m_t parameters corresponding to pre-warming steady state SOC = 100 mg C g^{-1} soil, DOC = 0.2 mg C g^{-1} soil, MIC = 2 mg C g^{-1} soil, and ENZ = 0.1 mg C g^{-1} soil. Yellow shaded regions represent 80% credible areas and vertical purple lines indicate distribution mean.

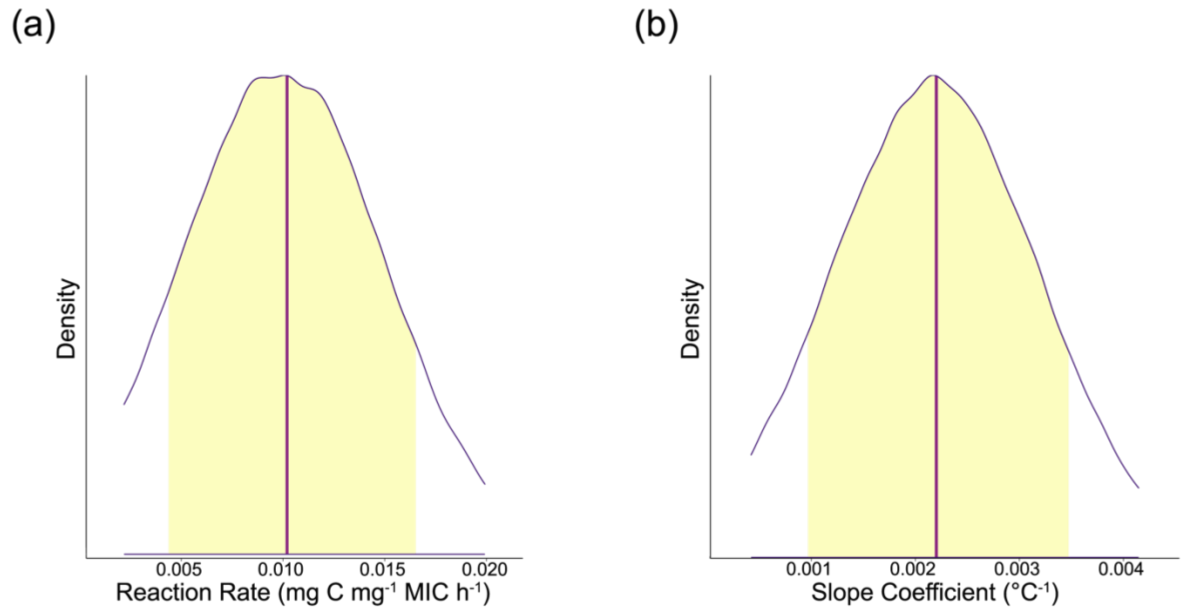

Figure S3: Response ratios of model SOC stocks after 12.5 years of warming in AWB and CON simulations. **(a)** Pre-warming steady state SOC varied from 50 to 200 mg C g^{-1} soil, with pre-warming MIC, DOC and ENZ held constant respectively at 2 mg C g^{-1} soil, 0.2 mg C g^{-1} soil, and 0.1 mg C g^{-1} soil; **(b)**, Pre-warming MIC varied from 1 to 8 mg C g^{-1} soil, with pre-warming SOC, DOC and ENZ held constant, respectively, at 100 mg C g^{-1} soil, 0.2 mg C g^{-1} soil, and 0.1 mg C g^{-1} soil.

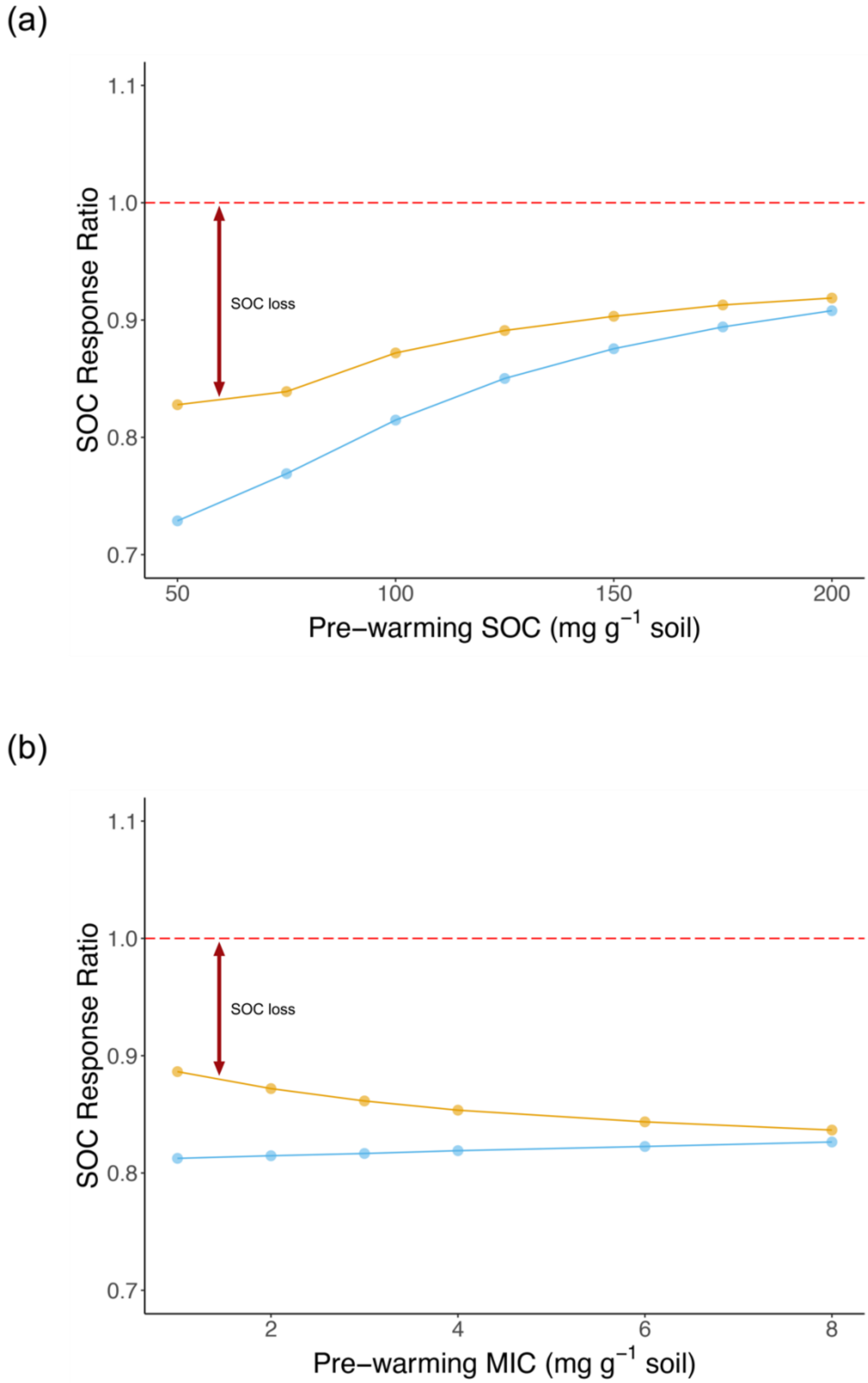

Table S2: (a) AWB; and **(b)** CON prior distribution tables. Including σ, the residual error scale term, we fit 10 parameters in our AWB runs and 8 parameters in our CON runs. Normal, Gaussian priors were used for all fitted ODE model parameters. The notation we use for our normal distributions follows an N(mean, standard deviation) format. The Markov chain guess-scaling parameter, σ, was drawn from a more weakly informative half-Cauchy distribution per recommendations from literature (Gelman, 2006).

(a) CON priors

Parameter	Distribution	Parameter Description
Ea_S	N(50,25)	SOC activation energy
Ea_D	N(50,25)	DOC activation energy
Ea_M	N(50,25)	MIC activation energy
a_{DS}	N(0.3,0.15)	DOC to SOC transfer coefficient
a_{SD}	N(0.3,0.15)	SOC to DOC transfer coefficient
a_M	N(0.3,0.15)	MIC to SOC transfer coefficient
a_{MS}	N(0.5,0.25)	Fraction of dead MIC transferred
σ	Cauchy(0,1)	Residual Error Scale

(b) AWB priors

Parameter	Distribution	Parameter Description
V_{ref}	N(0.4,0.2)	SOC reference V_{max}
$V_{U_{ref}}$	N(0.01,0.005)	DOC reference V_{max}
Ea_V	N(50,25)	SOC V_{max} activation energy
Ea_{VU}	N(50,25)	DOC V_{max} activation energy
Ea_K	N(50,25)	SOC K_M activation energy
Ea_{KU}	N(50,25)	DOC K_M activation energy
$E_{C_{ref}}$	N(0.4,0.2)	Reference C use efficiency (CUE)
m_t	N(0.002,0.001)	CUE slope
a_{MS}	N(0.5,0.25)	Fraction of dead MBC transferred to SOC
σ	Cauchy(0,1)	Residual Error Scale

Table S3: Posterior means calculated for parameters that were fit in HMC runs are displayed in the following tables. Tables are presented in the order of **(a)** CON SOC-varied runs; **(b)** AWB SOC-varied runs; **(c)** CON MIC-varied runs; and **d)** AWB MIC-varied runs**.**

(a) CON posterior distribution means for SOC-varied runs

Parameter	SOC50	SOC75	SOC100	SOC125	SOC150	SOC175	SOC200
Ea_S	77.565	73.582	66.578	60.796	56.695	53.602	51.347
Ea_D	50.24	50.17	50.133	50.211	50.174	50.25	50.246
Ea_M	52.612	52.531	52.143	52.109	52.07	51.989	51.836
a_{DS}	0.324	0.325	0.327	0.327	0.327	0.327	0.326
a_{SD}	0.334	0.336	0.336	0.337	0.337	0.337	0.337
a_M	0.338	0.34	0.336	0.334	0.333	0.332	0.331
a_{MS}	0.504	0.504	0.502	0.5	0.498	0.497	0.497
σ	0.134	0.139	0.155	0.168	0.176	0.183	0.187

(b) AWB posterior distribution means for SOC-varied runs

Parameter	SOC50	SOC75	SOC100	SOC125	SOC150	SOC175	SOC200
V_{ref}	0.383	0.405	0.41	0.414	0.418	0.42	0.423
$V_{U_{ref}}$	0.0104	0.0104	0.0104	0.0104	0.0104	0.0104	0.0104
Ea_V	74.791	70.071	65.21	62.218	60.447	59.174	58.475
Ea_{VU}	50.201	50.486	50.669	50.823	51.104	51.015	51.106
Ea_K	25.846	30.357	35.532	38.395	40.214	41.404	42.364
Ea_{KU}	49.791	49.671	49.318	49.265	49.014	49.013	48.964
$E_{C_{ref}}$	0.204	0.253	0.337	0.406	0.454	0.49	0.513
m_t	0.00184	0.00214	0.00222	0.00225	0.00227	0.00229	0.00232
a_{MS}	0.495	0.498	0.5	0.507	0.514	0.52	0.526
σ	0.152	0.151	0.16	0.165	0.17	0.174	0.178

(c) CON posterior distribution means for MIC-varied runs

Parameter	MIC1	MIC2	MIC3	MIC4	MIC6	MIC8
Ea_S	67.472	66.578	65.991	65.34	64.076	62.972
Ea_D	49.941	50.133	50.337	50.388	50.608	50.801
Ea_M	50.909	52.143	53.063	53.862	54.861	55.681
a_{DS}	0.327	0.327	0.326	0.327	0.327	0.327
a_{SD}	0.332	0.336	0.339	0.342	0.344	0.346
a_M	0.336	0.336	0.336	0.336	0.336	0.335
a_{MS}	0.503	0.502	0.499	0.498	0.496	0.495
σ	0.155	0.155	0.155	0.155	0.156	0.156

(d) AWB posterior distribution means for MIC-varied runs

Parameter	MIC1	MIC2	MIC3	MIC4	MIC6	MIC8
V_{ref}	0.403	0.41	0.413	0.417	0.422	0.424

$V_{U_{ref}}$	0.0105	0.0104	0.0104	0.0104	0.0104	0.0104
Ea_V	65.571	65.21	65.207	65.308	65.689	66.111
Ea_{VU}	50.719	50.669	50.67	50.601	50.56	50.461
Ea_K	34.939	35.532	35.585	35.467	35.071	34.936
Ea_{KU}	49.441	49.318	49.4	49.403	49.608	49.495
$E_{C_{ref}}$	0.26	0.337	0.395	0.437	0.495	0.5343
m_t	0.00218	0.00222	0.00223	0.00224	0.00226	0.00227
a_{MS}	0.487	0.5	0.516	0.534	0.564	0.581
σ	0.16	0.16	0.16	0.161	0.163	0.164

Table S4: Numerical LOO, WAIC, LPML, and R^2 results are displayed in the following tables. Lower LOO and WAIC values are preferred. Higher LPML values are preferred. Tables are presented in the order of **(a)** CON SOC-varied runs; **(b)** AWB SOC-varied runs; **(c)** CON MIC-varied runs; and **d)** AWB MIC-varied runs**.**

(a) CON goodness-of-fit metrics for SOC-varied runs

Parameter	SOC50	SOC75	SOC100	SOC125	SOC150	SOC175	SOC200
LOO	-15.704	-14.929	-11.918	-9.844	-8.51	-7.574	-6.891
WAIC	-15.818	-15.002	-11.992	-9.92	-8.58	-7.639	-6.966
LPML	7.849	7.465	5.959	4.92	4.256	3.788	3.439
R^2	0.627	0.596	0.496	0.413	0.351	0.304	0.269

(b) AWB goodness-of-fit metrics for SOC-varied runs

Parameter	SOC50	SOC75	SOC100	SOC125	SOC150	SOC175	SOC200
LOO	-11.028	-10.633	-8.388	-7.732	-7.084	-6.525	-5.97
WAIC	-11.379	-11.123	-9.284	-8.446	-7.733	-7.114	-6.579
LPML	5.499	5.312	4.252	3.9	3.547	3.254	3.002
R^2	0.572	0.585	0.528	0.492	0.463	0.435	0.406

(c) CON goodness-of-fit metrics for MIC-varied runs

Parameter	MIC1	MIC2	MIC3	MIC4	MIC6	MIC8
LOO	-11.963	-11.918	-11.931	-11.887	-11.808	-11.731
WAIC	-12.035	-11.992	-12.004	-11.966	-11.881	-11.802
LPML	5.982	5.959	5.966	5.943	5.904	5.865
R^2	0.498	0.496	0.496	0.496	0.493	0.49

(d) AWB goodness-of-fit metrics for MIC-varied runs

Parameter	MIC1	MIC2	MIC3	MIC4	MIC6	MIC8
LOO	-8.63	-8.388	-8.587	-8.462	-8.219	-8.181
WAIC	-9.302	-9.284	-9.213	-9.079	-8.863	-8.711
LPML	4.314	4.252	4.204	4.203	4.116	4.088
R^2	0.526	0.528	0.525	0.521	0.516	0.513

Figure S4: Change in fit metrics for AWB and CON as pre-warming steady state MIC is varied from 1 to 8 mg C g^{-1} soil. **(a)** LOO; **(b)** WAIC; **(c)** LPML; **(d)** R^2

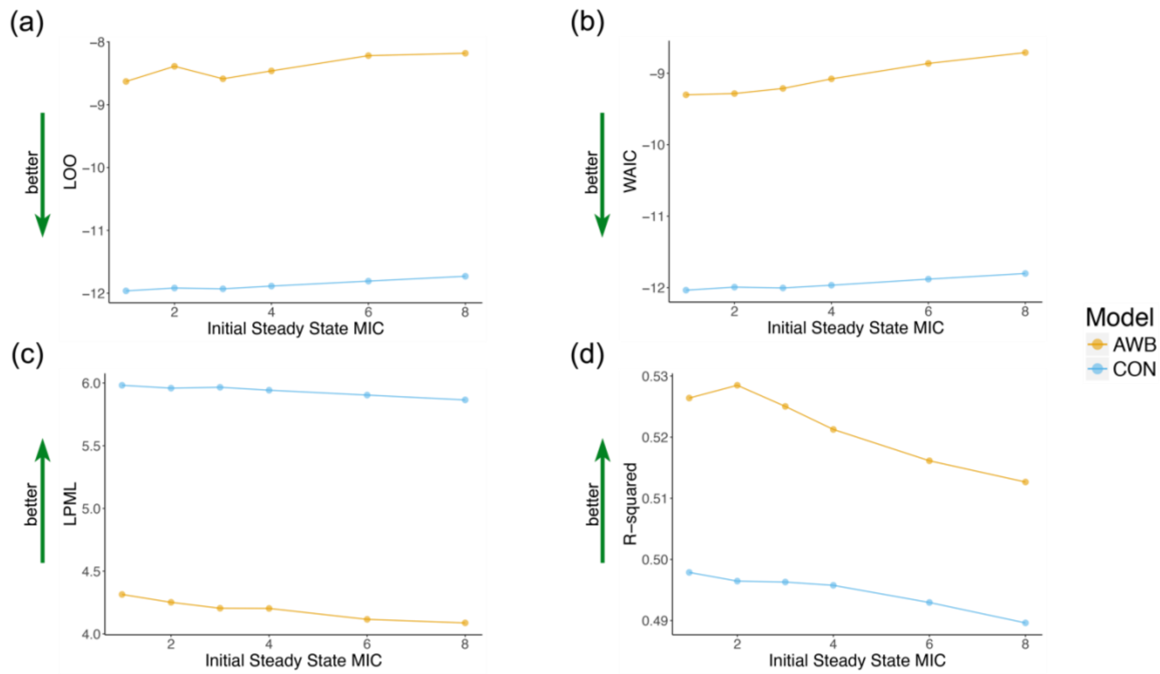

Figure S5: Trace plots for AWB and CON parameters indicate that the Markov chains were well-mixed with appropriate burn-in. Example trace plots depicted in which pre-warming SOC = 100 mg C g^{-1} soil, MIC = 2 mg C g^{-1} soil, DOC = 0.2 mg C g^{-1} soil, and (for AWB) ENZ = 0.1 mg C g^{-1} soil. **(a)** CON Ea parameters; **(b)** CON partition fraction parameters; **(c)** AWB Ea parameters; **(d)** AWB parameters V_{ref}, $E_{C_{ref}}$, a_{MS}, $V_{U_{ref}}$, and m_t. The red ticks at the bottom of the AWB panels indicate divergent transitions on one out of the four chains.

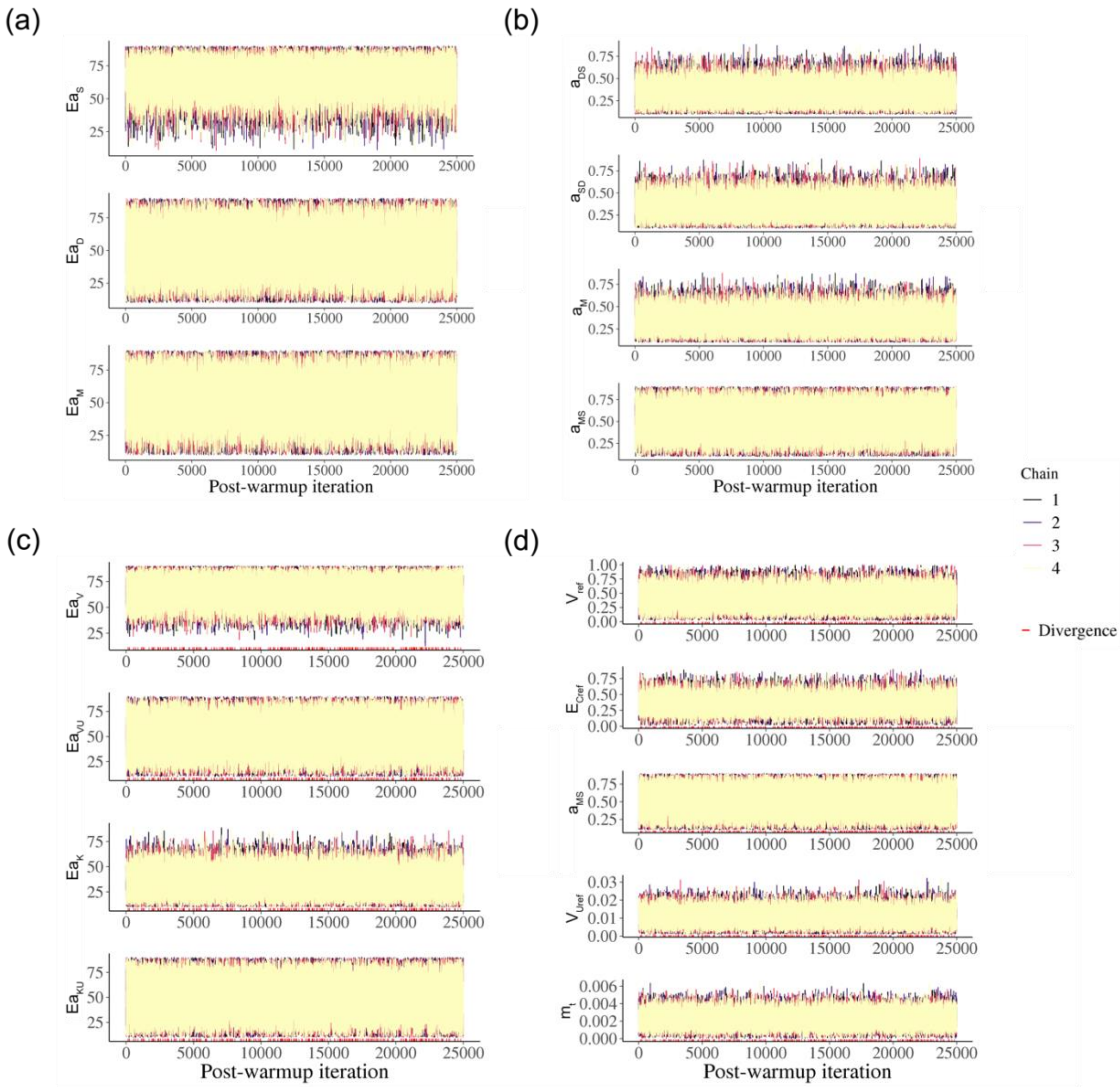

Figure S6: Autocorrelation plots for pre-warming SOC = 100 mg C g^{-1} soil, MIC = 2 mg C g^{-1} soil, DOC = 0.2 mg C g^{-1} soil, and (for AWB) ENZ = 0.1 mg C g^{-1} soil indicate effective sample collection. For all fitted AWB and CON parameters, autocorrelation, or the dependence between values of the same parameter accepted by Markov chains, tends to drop as lag, the distance between MCMC iterations increases. Low autocorrelation indicates more independence between samples and more efficient collection of effective samples for inference. **(a)** CON Ea parameters; **(b)** CON partition fraction parameters; **(c)** AWB Ea parameters; **(d)** AWB parameters V_{ref}, $E_{C_{ref}}$, a_{MS}, $V_{U_{ref}}$, and m_t.

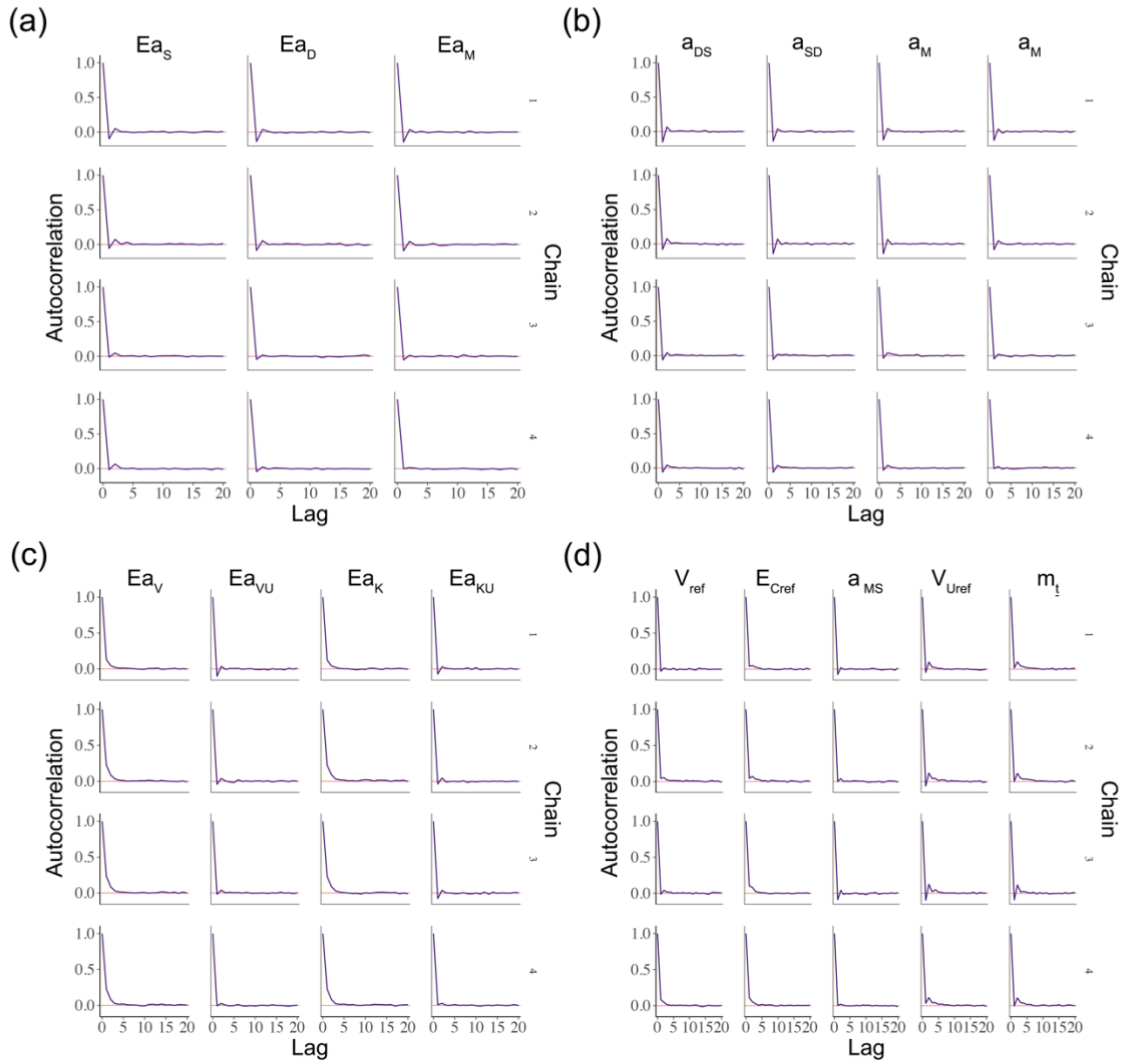

Figure S7: $\hat{R}$ is a Bayesian diagnostic measure that estimates the degree of convergence between multiple Markov chains. An $\hat{R}$ value that approaches 1 as the number of Markov chain iterations increase is ideal. Plots demonstrating convergence of $\hat{R}$ values to 1 are presented for **(a)** CON; and **(b)** AWB parameters corresponding to simulations using pre-warming SOC = 100 mg C g^{-1} soil, MIC = 2 mg C g^{-1} soil, DOC = 0.2 mg C g^{-1} soil, and (for AWB) ENZ = 0.1 mg C g^{-1} soil.

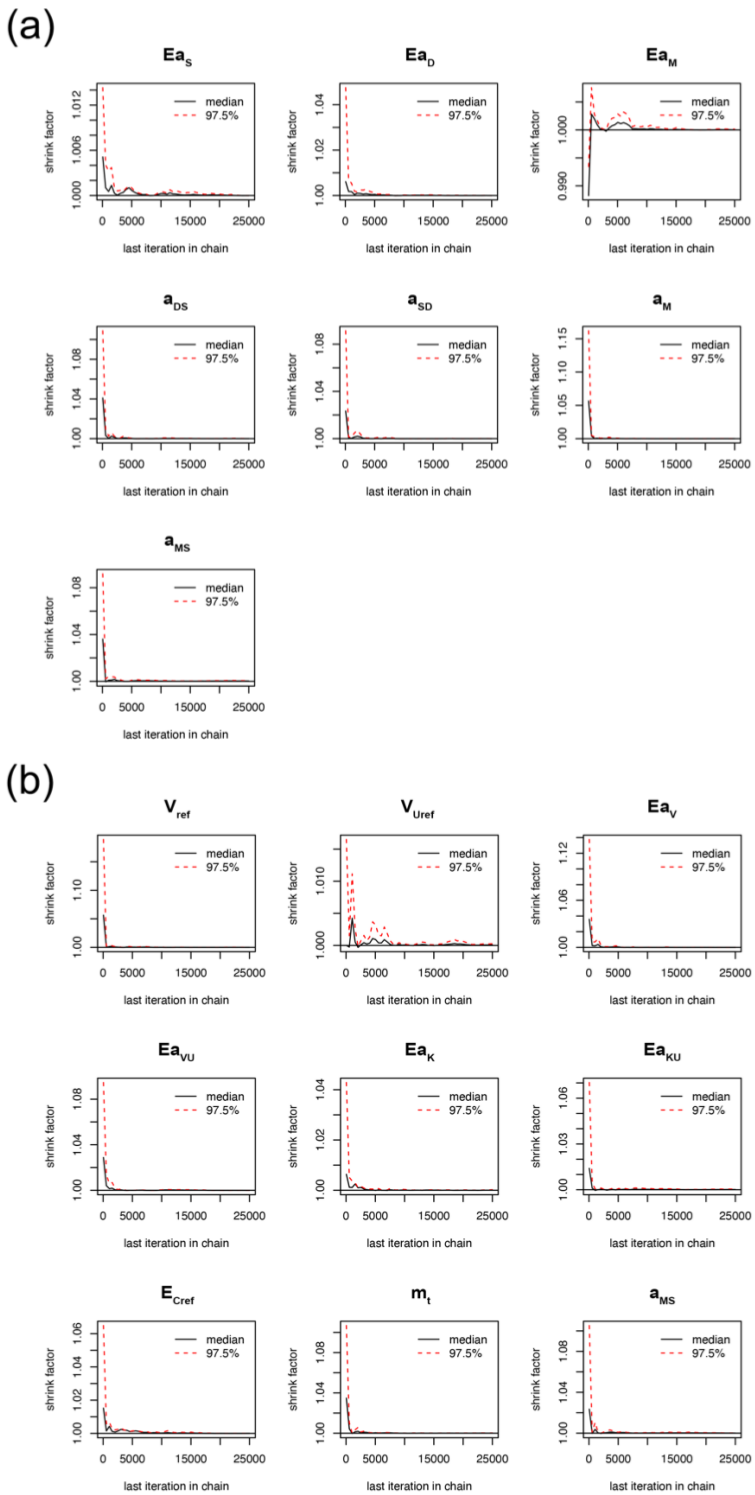

Table S5: CON and AWB model parameter effective sample size fractions N_{eff} / N (ratio of effective posterior samples to total posterior samples where N = 100,000) calculations from posterior. Ratios that are closer to 1.0 are preferred. Tables are presented in the order of **(a)** CON SOC-varied runs; **(b)** AWB SOC-varied runs; **(c)** CON MIC-varied runs; and **d)** AWB MIC-varied runs**.**

(a) CON SOC-varied runs

Parameter	SOC50	SOC75	SOC100	SOC125	SOC150	SOC175	SOC200
Ea_S	0.877	0.848	0.935	0.924	0.867	0.884	0.87
Ea_D	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Ea_M	1.0	1.0	1.0	1.0	1.0	1.0	1.0
a_{DS}	0.974	0.949	0.974	1.0	1.0	0.953	1.0
a_{SD}	0.95	0.957	1.0	1.0	0.925	1.0	1.0
a_M	0.93	0.919	1.0	1.0	0.987	0.987	0.955
a_{MS}	1.0	1.0	1.0	1.0	1.0	1.0	1.0
σ	0.686	0.653	0.719	0.732	0.693	0.689	0.691

(b) AWB SOC-varied runs

Parameter	SOC50	SOC75	SOC100	SOC125	SOC150	SOC175	SOC200
V_{ref}	0.594	0.883	0.81	0.914	0.959	0.816	0.707
$V_{U_{ref}}$	0.699	0.705	0.704	0.723	0.694	0.64	0.629
Ea_V	0.568	0.687	0.579	0.561	0.511	0.413	0.423
Ea_{VU}	0.961	1.0	1.0	1.0	1.0	0.95	0.845
Ea_K	0.463	0.687	0.599	0.527	0.519	0.424	0.428
Ea_{KU}	0.995	1.0	1.0	1.0	1.0	0.949	0.828
$E_{C_{ref}}$	0.439	0.635	0.698	0.65	0.458	0.339	0.261
m_t	0.625	0.668	0.679	0.722	0.625	0.638	0.556
a_{MS}	1.0	1.0	1.0	1.0	1.0	0.879	0.793
σ	0.529	0.638	0.612	0.65	0.681	0.579	0.533

(c) CON MIC-varied runs

Parameter	MIC1	MIC2	MIC3	MIC4	MIC6	MIC8
Ea_S	0.863	0.935	0.879	0.881	0.943	0.984
Ea_D	1.0	1.0	1.0	1.0	1.0	1.0
Ea_M	1.0	1.0	1.0	1.0	1.0	1.0
a_{DS}	0.968	0.974	0.964	0.966	0.936	1.0
a_{SD}	0.971	1.0	0.947	0.935	0.958	1.0
a_M	0.995	1.0	1.0	0.917	0.957	0.97
a_{MS}	1.0	1.0	1.0	1.0	1.0	1.0
σ	0.68	0.719	0.699	0.659	0.689	0.747

(d) AWB MIC-varied runs

Parameter	MIC1	MIC2	MIC3	MIC4	MIC6	MIC8
V_{ref}	0.673	0.81	0.84	0.761	0.851	0.907

$V_{U_{ref}}$	0.705	0.704	0.693	0.62	0.722	0.702
Ea_V	0.536	0.579	0.611	0.563	0.621	0.632
Ea_{VU}	0.968	1.0	1.0	0.935	1.0	1.0
Ea_K	0.44	0.599	0.403	0.523	0.622	0.609
Ea_{KU}	0.985	1.0	1.0	0.921	1.0	1.0
$E_{C_{ref}}$	0.486	0.698	0.432	0.582	0.534	0.547
m_t	0.539	0.679	0.739	0.632	0.641	0.729
a_{MS}	0.973	1.0	1.0	0.898	0.895	0.954
σ	0.564	0.612	0.697	0.548	0.579	0.621

Table S6: Pareto *k* diagnostic counts for CON and AWB simulations sourced from LOO metric computations for CON and AWB model fits. *k* diagnostics are calculated by fitting each set of leave-one-out importance ratios used to evaluate LOO to the shape parameter of a Pareto distribution. In our case, each simulation will produce 13 *k* parameters since there are 13 meta-analysis data points and consequently the same number of holdout sets. *k* gives an indication on model quality and LOO result reliability. Having a higher proportion of lower *k* values are preferred; samples are suitable if they fall within the intervals of (-∞, 0.5) and (0.5, 0.7) and questionable if they are higher. Having a high proportion of *k* values greater than 1 should raise concerns of unreliable information criteria and cross validation results stemming from model specification issues. The presence of a diagnostic for LOO renders it a superior Bayesian predictive metric; other predictive metrics lack reviewable diagnostics. Tables are presented in the order of **(a)** CON SOC-varied runs; **(b)** AWB SOC-varied runs; **(c)** CON MIC-varied runs; and **d)** AWB MIC-varied runs**.**

(a) CON SOC-varied runs

k interval	SOC50	SOC75	SOC100	SOC125	SOC150	SOC175	SOC200
$(-\infty, 0.5]$	13	13	13	13	13	13	12
(0.5,0.7]	0	0	0	0	0	0	1
(0.7,1]	0	0	0	0	0	0	0
$(1, \infty]$	0	0	0	0	0	0	0

(b) AWB SOC-varied runs

k interval	SOC50	SOC75	SOC100	SOC125	SOC150	SOC175	SOC200
$(-\infty, 0.5]$	11	11	12	12	12	11	12
(0.5,0.7]	2	2	0	0	0	2	1
(0.7,1]	0	0	1	1	1	0	0
$(1, \infty]$	0	0	0	0	0	0	0

(c) CON MIC-varied runs

k interval	MIC1	MIC2	MIC3	MIC4	MIC6	MIC8
$(-\infty, 0.5]$	13	13	13	13	13	13
(0.5,0.7]	0	0	0	0	0	0
(0.7,1]	0	0	0	0	0	0
$(1, \infty]$	0	0	0	0	0	0

(d) AWB MIC-varied runs

k interval	MIC1	MIC2	MIC3	MIC4	MIC6	MIC8
$(-\infty, 0.5]$	12	12	12	11	12	12
(0.5,0.7]	0	0	1	2	0	1
(0.7,1]	1	1	0	0	1	0
$(1, \infty]$	0	0	0	0	0	0

Figure S8: Plots of effective parameter counts for CON and AWB in SOC-varied and MIC-varied HMC runs. Decreasing SOC in AWB and CON runs increased effective parameter count and over-fitting punishment in the LOO and WAIC calculations. Effective parameter counts computed as part of **(a)** LOO for SOC-varied runs; **(b)** WAIC for SOC-varied runs; **(c)** LOO for MIC-varied runs; and **(d)** WAIC for MIC-varied runs.

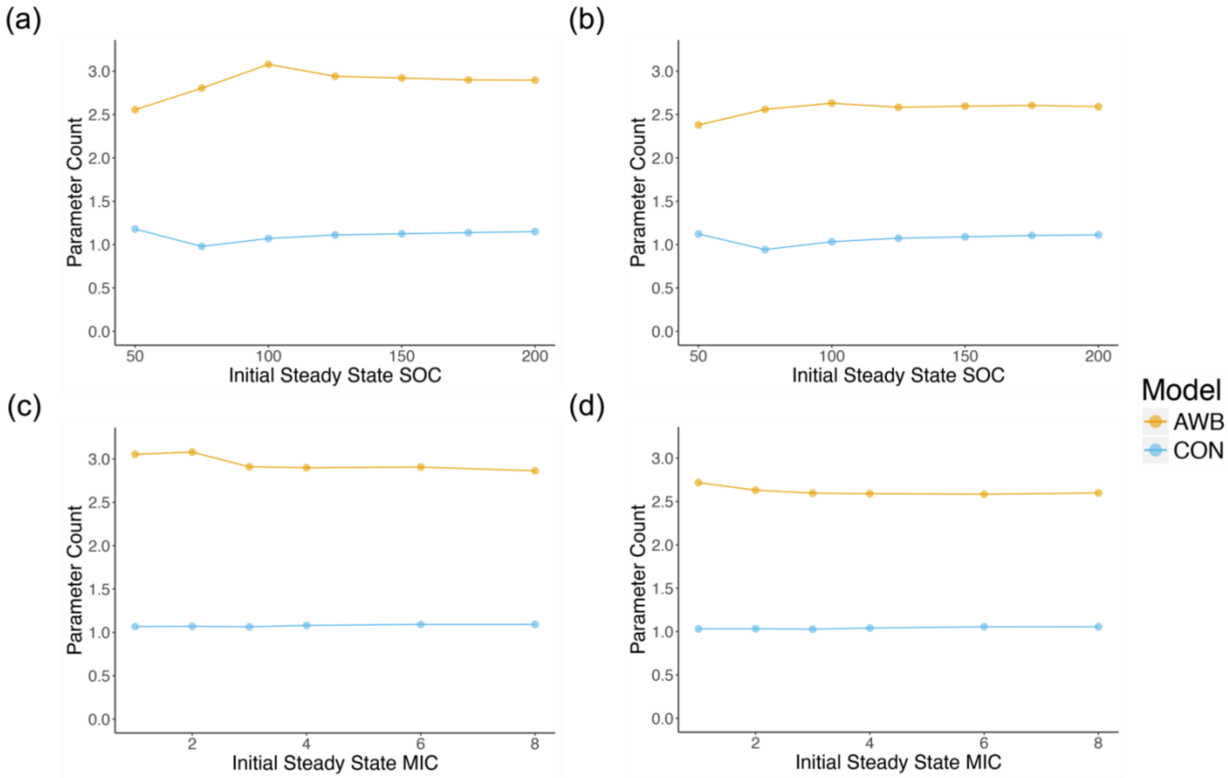

Figure S9: Ratio of divergent transitions to total posterior samples collected in AWB runs. Decreasing the MIC-to-SOC ratio in AWB runs corresponded to an increase in the number of divergent transitions. Divergent transition frequencies per 100,000 posterior samples in **(a)** SOC-varied runs; and **(b**) in MIC-varied runs.

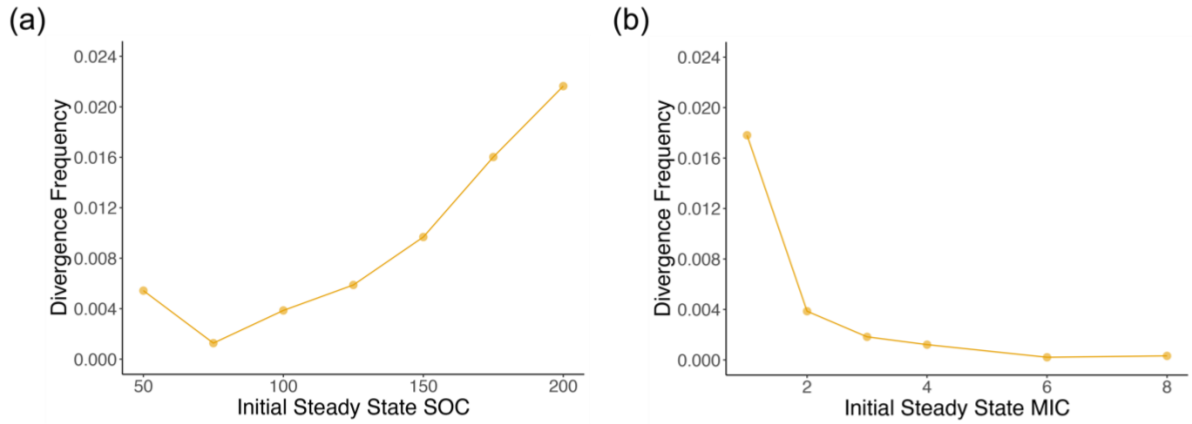

Figure S10: Response ratios of empirical SOC stocks from 143 field warming studies (van Gestel et al., 2018) plotted against study duration. A statistical analysis not accounting for sample size of each study found that the effect of study duration on response ratio was not significant (p = 0.7822). Response ratios ranged from 0.544 to 1.9. Mean response ratio was 1.03, not accounting for sample sizes. The red dashed line at 1.0 divides studies in which an increase in SOC was observed from those in which a decrease was ultimately observed.

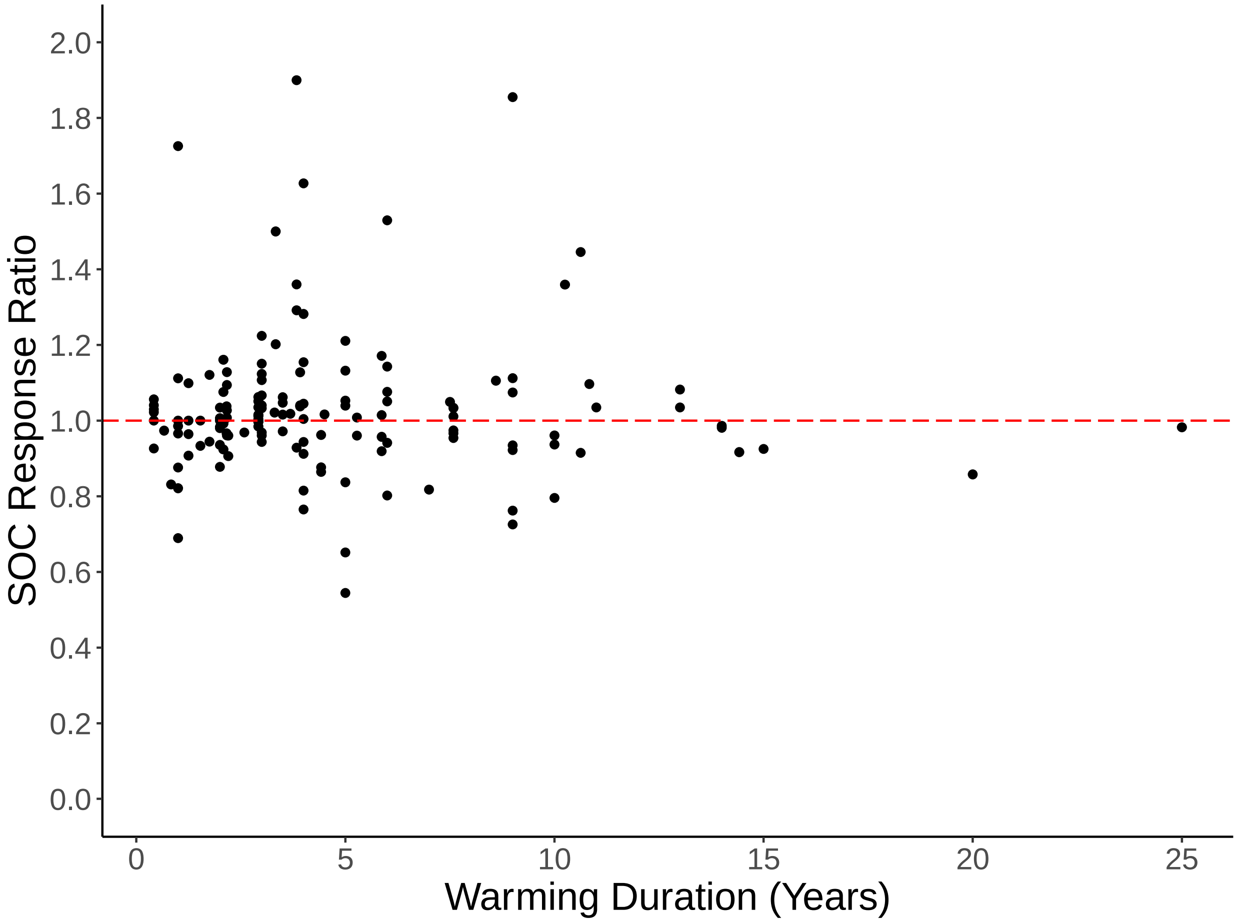

Figure S11: 95% credible areas for some **(a)** AWB and **(b)** CON parameters corresponding to pre-warming steady state SOC = 50 mg C g^{-1} soil, DOC = 0.2 mg C g^{-1} soil, MIC = 2 mg C g^{-1} soil, and ENZ = 0.1 mg C g^{-1} soil. Yellow shaded regions represent 80% credible areas and vertical purple lines indicate distribution mean. Note the deformity of the Ea_S and $E_{C_{ref}}$ densities. Parameter units are displayed in Supplemental Table 1.