The complete physical processes of the ROSE module

The ROSE model is a mechanistic model based on the physical processes of soil erosion (Rose et al., 1983; Stewart, 1985), which predicts the intensity of water-induced soil erosion by conceiving and simulating three continuous and simultaneous physical processes of rainfall detachment (*e*), sediment entrainment (*r*) and sediment deposition (*d*).

First of all, the eroded sediment particles are divided into a number of classes (*I*) based on their size ranges. Assuming no flux of water entering the up-slope edge of the planar land unit, the mass conservation of sediment in an elementary unit can be provided as Eq. S1. The first and second terms on the left-hand side in Eq. S1 represents the sediment amount flowing accompany with and remaining in the surface runoff, respectively. *D* (m) and *q* (m^2 s^{-1}) are referred to the water depth and the flux of the surface runoff, respectively. And *q* is calculated from the product of the surface runoff rate per unit area (*Q*, m s^{-1}) and the distance down the planar land unit (*x*, m). Assuming the equal mass of each range class of the sediment particles, the sediment concentration (c_i) of size class *i* can be calculated from the total sediment concentration (*c*) and *I*, as shown in Eq. S2.

$$\frac{\partial}{\partial_x}(qc_i) + \frac{\partial}{\partial_t}(Dc_i) = e_i - d_i + r_i \quad \text{S1}$$

$$c_i = c/I \quad \text{S2}$$

The detachment rate (e_i) is proportional to the fraction of soil exposed to the raindrops (C_e). Assuming that the rainfall detachment process is unselective with respect to the size of the detached sediment, then e_i by the rainfall (P) for the sediment size range class i can be calculated as Eq. S3. The detachability of soil by rainfall (α) in Eq. S3 is related to soil type, management practices and the depth of surface runoff.

$$e_i = \alpha C_e P^2 / I \tag{S3}$$

The sediment deposition rate (d_i) is greatly influenced by the sizes of the sediment particles, which is calculated as Eq. S4 based on the settling velocity (v_i) of the sediment particle size class i and c_i.

$$d_i = v_i c_i \tag{S4}$$

The rate of sediment entrainment (r_i) is linearly related to the stream power (Ω, kg m^{-2} s^{-1} m^{-1}), which is calculated by Eq. S5. ρ and g are referred to the water density with the value of 1.0×10^3 kg m^{-3} and the acceleration of gravity with the value of 9.8 m s^{-2}, respectively. S (dimensionless) is referred to the sine of the land slope in the radian value. Then the entrained sediment mass flux (q_s) and the sediment entrainment efficiency (K) can be computed by Eq. S6 and Eq. S7, respectively. The efficiency of bed-load transport (η) in Eq. S7 varies with the soil types, management practices and surface runoff. The value of 0.276 (in kg m^{-3}) in Eq. S7 is assumed as the specific gravity of sediment. x_* is the distance down the planar land unit where the entrainment occurrence. The stream power at the place of the entrainment occurrence (Ω_0) is calculated using Eq. S8. The c can be obtained by substituting Eq. S5 and Eq. S8 into Eq. S6, as shown in Eq. S9. According to the law of mass conservation, r_i should be the sum of the d_i, the sediment amount flowing accompany with and remaining in surface runoff, as shown in Eq. S10.

$$\Omega = \rho gSQx \tag{S5}$$

$$q_s = qc = K(\Omega - \Omega_0) \tag{S6}$$

$$K = 0.276\eta \tag{S7}$$

$$\Omega_0 = \rho gSQ(1 - x_*) \tag{S8}$$

$$c = \rho gSK(1 - x_*/x) \tag{S9}$$

$$r_i = d_i + \frac{\partial_{qc_i}}{\partial_x} + \frac{\partial}{\partial_t}(Dc_i) \tag{S10}$$

Equation S11 is obtained by substituting Eq. S4 and Eq. S9 into Eq. S10 to calculate r_i. For an individual planar land unit, a portion of the soil is exposed to the entrainment by surface runoff (C_r), while the remaining soil is protected by the land covers (e.g., vegetation, buildings, rocks, and other soil cover materials). If C_r is introduced as a multiplier into the right-hand side of Eq. S9 to affect c_i and into the second term on the right-hand side of Eq. S11. Then, Eq. S9 can be rearranged as Eq. S12. β_i in Eq. S12 can be calculated by Eq. S13.

$$r_i = v_i c_i + \rho gSKQ/I + \frac{\partial}{\partial_t}(Dc_i) \tag{S11}$$

$$r_i = C_r(\rho gSKQ/I)(\beta_i - \frac{v_i x_*}{Qx}) + \frac{\partial}{\partial_t}(Dc_i) \qquad \text{S12}$$

$$\beta_i = 1 + \frac{v_i}{Q} \qquad \text{S13}$$

Substituting Eq. S3, S4 and S12 into Eq. S1, the partial differentiation term $\frac{\partial}{\partial_t}(Dc_i)$ on the right-hand side in Eq. S12 can be reduced. The c_i of all sediment particle size classes are summed up to obtain the total sediment concentration ($c(L, t)$) of a planar land unit with the slope length of L as a function of time t, as shown in Eq. S14.

$$c(L, t) = (\alpha C_e P^2/QI)\sum_{i=1}^{I}(1/\beta_i) + \rho gSKC_r(1 - \frac{x_*}{L}) \qquad \text{S14}$$

The larger the surface runoff and the sediment yield, the smaller the first term on the right-hand side of Eq. S14 is compared with the second term. For example, in a typical surface runoff event with 50 mm h^{-1} rainfall and 30 mm h^{-1} surface runoff for a typical planar land unit with a slope of 6%, the second term on the right-hand side of Eq. S14 is approximately 1000 times larger than the first term. Although the ratios of the two terms on the right-hand side of Eq. S14 vary rapidly with the soil and hydraulic conditions, this example does justify the neglect of the first term on the right-hand side of Eq. S14 during a typical soil erosion period. In fact, even for a fairly small erosion event, neglecting the first term on the right-hand side of Eq. (14) still provides a good approximation whose accuracy is usually higher than the inherent uncertainty in the observations used to calibrate it. Thus, Eq. S14 can be simplified to

$$c(L,t) = \rho g S K C_r(1 - \frac{x_*}{L}) = 2700 S\eta C_r(1 - \frac{x_*}{L}) \quad \text{S15}$$

However, if x_* is replaced by the time mean value of $\overline{x}_*$, and assuming η is a constant, c is a function of the slope length L.

$$c(L,t) = 2700 S\eta C_{\mathrm{r}}(1 - \frac{\overline{x}_*}{L}) \quad \text{S16}$$

When the slope length of L is not less than approximately 30 m, x_* is much smaller than the slope length of L during most of the whole water-induced soil erosion processes. During the early and late periods of a surface runoff event with the small surface runoff volume, x_* is non-negligible compared with L, but the sediment yield is only a small fraction of that lost in the whole soil erosion event. That is, the sediment entrainment by the surface runoff dominates the overall water-induced soil erosion process, while the two processes of rainfall detachment and sediment deposition are usually negligible. Therefore, c can be approximated by Eq. S17.

$$c = 2700 S\eta C_r \quad \text{S17}$$

The upgraded CNMM-DNDC introduced the total sediment concentration (c) calculated by the simplified ROSE model shown in Eq. S17 into its hydrological framework. Then the yield of the water-induced sediment (Y_{s}, kg ha^{-1}) for an individual surface runoff event was calculated from the surface runoff amount (R_s, m) and c over the simulation time step, as shown in Eq. S18.

$$Y_{\mathrm{s}} = cR_{\mathrm{s}} \quad \text{S18}$$

Reference

Rose, C., Williams, J., Sander, G., and Barry, D.: A mathematical model of soil erosion and deposition processes: I. theory for a plane land element, Soil Science Society of America Journal, 47, 991–995, https://doi.org/10.2136/sssaj1983.03615995004700050030x, 1983.

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