

Figure S1. (a) *Neogloboquadrina pachyderma* calibration dataset of Tierney et al 2019 (showing the temperature sensitivity determined in that study) and **(b)** covariance between temperature sensitivity and the ‘intercept’ term, implemented in MgCaRB.

MgCaRB requires a covariance matrix of parameter uncertainty in order to implement the uncertainty propagation (Gray and Evans, 2019). In order to generate the covariance matrix, we performed a bootstrap of the Tierney et al. (2019) *Neogloboquadrina pachyderma* dataset, following the method outlined in Gray and Evans (2019), however prescribing the calibration sensitivities and uncertainties from Tierney et al 2019. This allows us to implement the *N. pachyderma* calibration of Tierney et al (2019) within the MgCaRB script. The calibration dataset and uncertainty covariance are shown in Supplementary Figure 1 above. Given the carbonate system sensitivity for *Neogloboquadrina pachyderma* is poorly constrained, we performed the same analysis but prescribing the ‘generic’ pH sensitivity of Gray and Evans (2019) as a sensitivity experiment. The version of MgCaRB with the implementation of these *Neogloboquadrina pachyderma* calibrations is available at <https://github.com/dbjevals/MgCaRB/releases/tag/v1.3>.

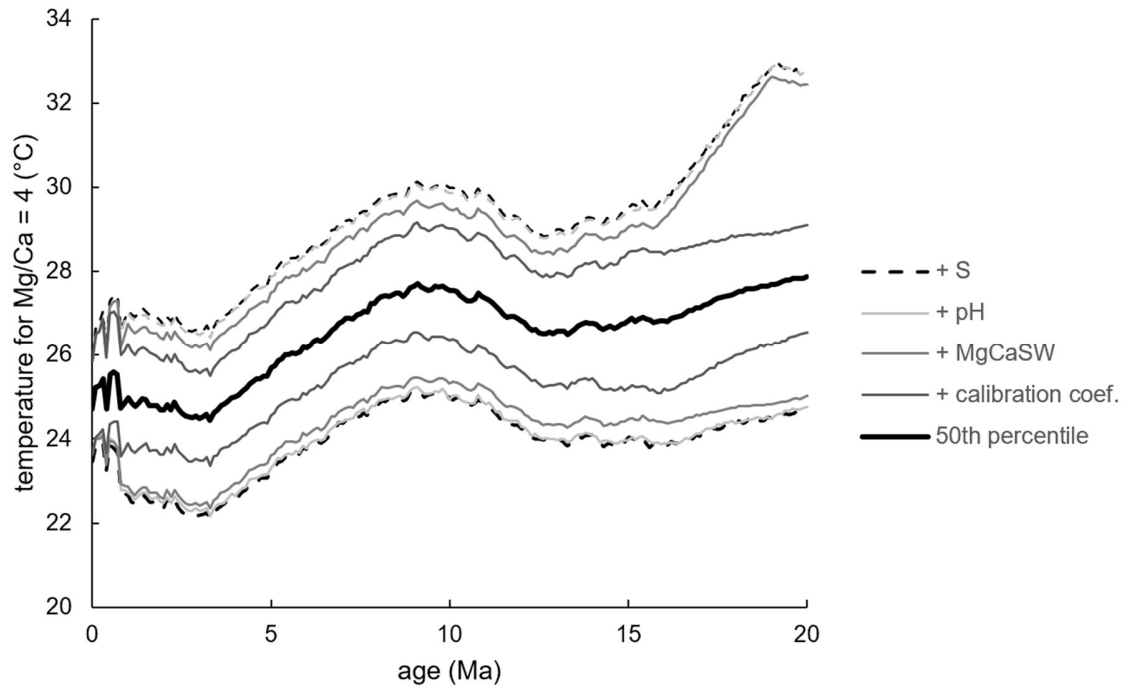


Figure S2. The relative contributions of the various nonthermal factors impacting Mg/Ca-derived temperature, calculated by reconstructing temperature through time using a hypothetical sample set characterised by a constant Mg/Ca = 4 mmol/mol. Before ~1 Ma, total uncertainty derives from (in order of importance): calibration coefficients (38%), Mg/Ca_{sw} (38%), pH (20%), and salinity (4%). In the case of samples <1 Ma, Mg/Ca_{sw} is not a significant source of uncertainty. We note that pH contributes to uncertainty to a relatively minor degree because whole-ocean pH changes are relatively well constrained over the last 20 Ma, as is the response of foraminifera Mg/Ca to pH (where this is relevant). There is, however, a much larger uncertainty surrounding whether or not a pH correction should be applied in the case of extinct species. Omitting this correction would have the effect of shifting reconstructed temperature to values ~2-3°C warmer during the interval of peak Miocene warmth.

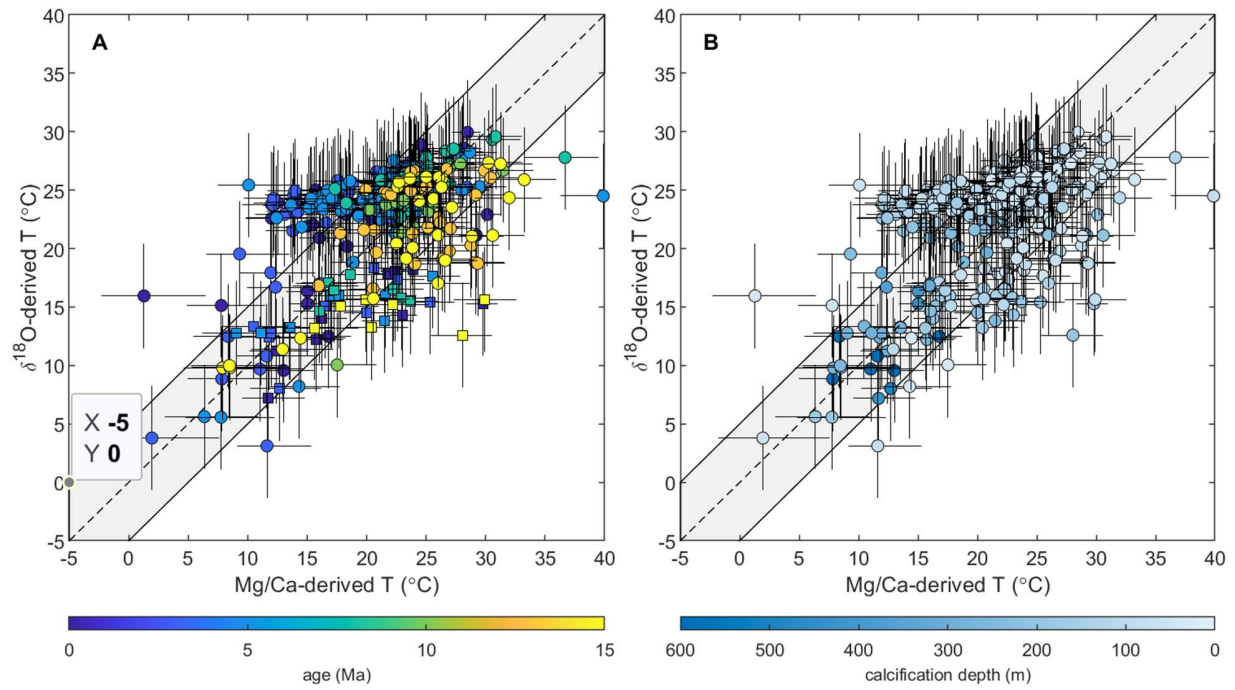


Figure S3. Main text Fig. 3 except using the sea level and deep ocean temperature record of Miller et al. (2020) to derive $\delta^{18}\text{O}_{\text{sw}}$ following the approach of Gaskell et al. (2023).