



# *Supplement of*

# Efficiency metrics for ocean alkalinity enhancements under responsive and prescribed atmospheric  $pCO<sub>2</sub>$  conditions

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## S1 Analytical equality of  $\eta(t)$  and direct atmosphere removal

Consider a simple box model comprising an atmosphere box of finite volume, and an ocean mixed layer box, as shown in Figure [S1.](#page-2-0) The intent is not to create a model with detailed realism but to demonstrate the consequences of modelling the atmosphere to be finite, and therefore change its  $pCO_2$  in response to fluxes of  $CO_2$  across the air-sea interface.

Firstly, the flux of  $CO_2$  across the air-water interface is typically modelled as proportional to the partial pressure difference

$$
F_{CO_2} = k_w \alpha (pCO_2^{atm} - pCO_2^{ocn}) = k_w \alpha \Delta p CO_2,\tag{S1}
$$

where  $\alpha$  is the solubility of CO<sub>2</sub> in seawater [ $\approx$  34 mol/m<sup>3</sup>/atm] and  $k_w$  is the gas transfer velocity [m/s]. Thus, the total amount of dissolved inorganic carbon (DIC) in the mixed layer and the atmosphere respectively change according to

<span id="page-1-0"></span>
$$
\frac{dC^{ocn}}{dt} = -\frac{dC^{atm}}{dt} = F_{CO_2}A = k_w \alpha A \Delta p CO_2 = \Psi \Delta p CO_2 \tag{S2}
$$

where A is the surface area over which gas exchange occurs. For later convenience,  $\Psi$  is defined here as  $k_w \alpha A$ . The atmospheric partial pressure  $pCO_2^{atm}$  is directly proportional to the total amount of  $CO_2$  in the atmosphere  $C^{atm}$  in [mol]

$$
pCO_2^{atm} = V_m p \frac{C^{atm}}{V^{atm}},\tag{S3}
$$

where  $V_m$  is the molar volume of gas (0.024  $m^3$ /mol) and p is the atmospheric pressure. Or as a differential:

$$
\frac{\mathrm{d}pCO_2^{atm}}{\mathrm{d}C^{atm}} = \frac{V_m p}{V_{atm}}\tag{S4}
$$

In the ocean (specifically in the mixed layer, "ml") the differential relationship between  $pCO_2$  and C, for small changes in either, is

$$
\frac{\mathrm{d}pCO_2^{cen}}{\mathrm{d}C^{ocn}} = \frac{1}{\alpha V_{ml}} \frac{\partial [CO_2]}{\partial [DIC]} \tag{S5}
$$

where [DIC] is the concentration of total dissolved carbon and  $\alpha$  is the solubility of CO<sub>2</sub> in seawater [\(Zeebe](#page-8-0) [and Wolf-Gladrow, 2001\)](#page-8-0). This relation follows from the following differential relationships d[ $DIC$ ] =  $dC/V_{ml}$ and  $dpCO_2 = \frac{1}{\alpha}d[CO_2].$ 

Now the changes in partial pressure in the atmosphere and ocean mixed layer can be written using the chain rule  $\left(\frac{\text{d}pCO_2}{\text{d}t} = \frac{\text{d}C}{\text{d}t} \cdot \frac{\text{d}pCO_2}{\text{d}C}\right)$  as:

$$
\frac{\mathrm{d}pCO_2^{atm}}{\mathrm{d}t} = -\Psi \Delta p CO_2 \frac{\mathrm{d}p CO_2^{atm}}{\mathrm{d}C^{atm}}\tag{S6}
$$

$$
\frac{\mathrm{d}pCO_2^{oce}}{\mathrm{d}t} = \Psi \Delta p CO_2 \frac{\mathrm{d}p CO_2^{cen}}{\mathrm{d}C^{ocn}}\tag{S7}
$$

<span id="page-2-0"></span>

Figure S1. Two simple box models, one with a static, prescribed atmosphere and one with a reactive finite-volume atmosphere. The resulting decay laws for an initial pulse of CO<sub>2</sub> added to to ocean ( $\Delta C(0)$ ) and initial pulse of  $pCO_2(0)$  are given below.

The partial pressure difference between the two reservoirs thus evolves as

$$
\frac{\mathrm{d}\Delta pCO_2}{\mathrm{d}t} = \frac{\mathrm{d}pCO_2^{atm}}{\mathrm{d}t} - \frac{\mathrm{d}pCO_2^{oce}}{\mathrm{d}t} = -\Psi \left[ \frac{\mathrm{d}pCO_2^{atm}}{\mathrm{d}C^{atm}} + \frac{\mathrm{d}pCO_2^{ocn}}{\mathrm{d}C^{ocn}} \right] \Delta pCO_2 \tag{S8}
$$

<span id="page-2-2"></span><span id="page-2-1"></span>
$$
=-\tau^{-1}\Delta pCO_2\tag{S9}
$$

This ODE in  $\Delta pCO_2$  is readily solved - any partial pressure differences evolve in an exponential fashion

$$
\Delta p CO_2(t) = \Delta p CO_2(0) exp(-t\tau^{-1})
$$
\n
$$
(S10)
$$

with a characteristic timescale

$$
\tau = \Psi^{-1} \left( \frac{\mathrm{d}p C O_2^{atm}}{\mathrm{d}C^{atm}} + \frac{\mathrm{d}p C O_2^{ocn}}{\mathrm{d}C^{ocn}} \right)^{-1} \tag{S11}
$$

Note that the frequently taken assumption that atmospheric  $pCO_2$  is constant, corresponds to the assumption that  $V_{atm} = \infty$ , in which case  $\frac{dpCO_2^{atm}}{dC^{atm}} = 0$ , i.e. it assumes an atmosphere which can accommodate arbitrary movement of  $CO_2$  in or out of it, without a change in  $pCO_2^{atm}$ . In this case, the e-folding time simplifies to

$$
\tau_c = \Psi^{-1} \left( \frac{\mathrm{d}p C O_2^{ocn}}{\mathrm{d}C^{ocn}} \right)^{-1} \tag{S12}
$$

denoted here as  $\tau_c$  for "constant". It is worth noting that the timescale obtained under constant atmosphere assumptions  $\tau_c$  is always a little larger than  $\tau$  due to the missing  $\frac{dpCO_2^{atm}}{dC^{atm}}$  term. The ODE for the  $pCO_2$  difference simplifies to the often used gas-exchange expression [\(Zeebe and Wolf-Gladrow, 2001\)](#page-8-0) after substitution of Ψ and  $\frac{\mathrm{d}pCO_2^{ocn}}{\mathrm{d}C^{ocn}}$ :

<span id="page-3-1"></span>
$$
\frac{\mathrm{d}\Delta pCO_2}{\mathrm{d}t} = -\frac{k_w}{z_{ml}} \frac{\partial [CO_2]}{\partial [DIC]} \Delta p CO_2 = -\tau_c^{-1} \Delta p CO_2 \tag{S13}
$$

where the depth of the mixed layer  $z_{ml} = V_{ml}/A$ .

### S1.1 Differences between reference and perturbed simulations

In general, the state of the mixed layer will not be in equilibrium with the atmosphere. Some areas may be taking in CO<sub>2</sub>, some may be outgassing. The  $pCO_2^{atm}$  may be changing. However, the effects of a small perturbation can be studied by taking the difference between a reference simulation and an otherwise identical simulation that has a perturbation applied to it.

Since the ODEs describing changes in  $\Delta pCO_2$  in both simulations (let's say A and B) are linear, the same kinetics also describe any differences between two simulations, say,  $\Delta p CO_2^A$  and  $\Delta p CO_2^B$ :

$$
\frac{\mathrm{d}}{\mathrm{d}t} \left( \Delta p C O_2^A - \Delta p C O_2^B \right) = \frac{\mathrm{d}\Delta p C O_2^A}{\mathrm{d}t} - \frac{\mathrm{d}\Delta p C O_2^B}{\mathrm{d}t} = -\tau^{-1} (\Delta p C O_2^A - \Delta p C O_2^B) \tag{S14}
$$

Therefore, from here on I assume that  $\Delta pCO_2$  and  $\Delta C$  refer to the difference between a perturbed simulation and a reference simulation, for example:

$$
\Delta p CO_2 = (\Delta p CO_2^A - \Delta p CO_2^B) = (\Delta p CO_2^{atm,A} - \Delta p CO_2^{ml,A}) - (\Delta p CO_2^{atm,B} - \Delta p CO_2^{ml,B})
$$
(S15)

# ${\bf S1.2}$   $\;$  Equilibration of  ${\rm CO_2}$  inventories under reactive atmospheres

How does a small CO<sub>2</sub> deficit of size  $\Delta C(0)$  induced in the ocean reservoir evolve in time under finite, responsive atmospheres ? If the perturbation is small, then an initial perturbation  $\Delta C(0)$  causes a initial change in  $pCO_2^{ocn}$ of

$$
\Delta p CO_2^{ocn}(0) = \frac{\mathrm{d}p CO_2^{ocn}}{\mathrm{d}C^{ocn}} \Delta C(0)
$$
\n
$$
(S16)
$$

This initial partial pressure difference will then cause transfer of  $CO<sub>2</sub>$  from the atmosphere to the ocean. The  $\Delta pCO_2$  will equilibrate with the exponential law given by equation [S10.](#page-2-1) The total rate of transfer of CO<sub>2</sub> between the atmosphere and the ocean (Equation [S2\)](#page-1-0) is proportional to  $\Delta pCO_2$ , given by Equation [S10:](#page-2-1)

<span id="page-3-0"></span>
$$
\frac{dC}{dt} = \Psi \Delta p CO_2(t)
$$
  
=  $\Psi \Delta p CO_2^{cen}(0) \exp(-t\tau^{-1})$   
=  $\Delta C(0) \Psi \frac{dp CO_2^{ocn}}{dC^{ocn}} \exp(-t\tau^{-1})$  (S17)

The cumulative transfer of  $CO_2$  is then expressed by the integral of the above rate. Subtracting this integral from the initial perturbation  $\Delta C(0)$ , the time evolution of the ocean inventory perturbation in the ocean is obtained:

$$
\Delta C(t) = \Delta C(0) - \int_{0}^{t} \frac{\mathrm{d}C}{\mathrm{d}t} \mathrm{d}t \tag{S18}
$$

Evaluating the integral

$$
\Delta C(t) = \Delta C(0) - \Delta C(0) \Psi \frac{\mathrm{d}pCO_2^{ocn}}{\mathrm{d}C^{ocn}} \tau \left[1 - \exp(-t\tau^{-1})\right]
$$
  
= 
$$
\Delta C(0) - \Delta C(0) \frac{\frac{\mathrm{d}pCO_2^{ocn}}{\mathrm{d}C^{ocn}}}{\frac{\mathrm{d}pCO_2^{atm}}{\mathrm{d}C^{acm}} + \frac{\mathrm{d}pCO_2^{ocn}}{\mathrm{d}C^{ocn}}} \left[1 - \exp(-t\tau^{-1})\right]
$$
(S19)

<span id="page-4-1"></span>where  $\Psi$  conveniently cancels. Simplifying yields:

$$
\Delta C(t) = \Delta C(0) \left( \frac{\gamma}{1 + \gamma} + \frac{1}{1 + \gamma} \exp(-t\tau^{-1}) \right)
$$
\n(S20)

where  $\gamma$  expresses the ratio of the capacities of the two interacting reservoirs.

$$
\gamma = \frac{\mathrm{d}pCO_2^{atm}}{\mathrm{d}C^{atm}} \left/ \frac{\mathrm{d}pCO_2^{ocn}}{\mathrm{d}C^{ocn}} = \frac{V_m p \alpha V_{ml}}{V_{atm}} \frac{\partial [DIC]}{\partial [CO_2]} \right. \tag{S21}
$$

As expected, under finite, responsive atmosphere conditions, some fraction of the initial pulse will always remain in the reservoir it was induced in, even once the  $pCO<sub>2</sub>$  values have reached equilibrium. The fraction that will move to the other reservoir is  $\frac{1}{1+\gamma}$ , while the fraction that will remain in the ocean is  $\frac{\gamma}{1+\gamma}$ .

# ${\bf S1.3}$   $\;$  Equilibration of  ${\bf CO_2}$  inventories under prescribed atmospheres

The non-reactive atmosphere conditions, where the atmosphere is modelled with a constant or prescribed  $pCO_2^{atm}$ , correspond to the assumption that  $V_{atm}=\infty.$  In that case,  $\gamma\to 0$  and  $(1+\gamma)^{-1}\to 1.$  Under these circumstances, the evolution of a  $\Delta C(0)$  pulse is a simple exponential decay with the simpler e-folding time  $\tau_c$  given by Equation ?? and that will decay all the way to zero.

<span id="page-4-0"></span>
$$
\Delta pCO_2(t) = \Delta pCO_2(0) \exp(-t\tau_c^{-1})
$$
  
\n
$$
\Delta C(t) = \Delta C(0) \exp(-t\tau_c^{-1})
$$
  
\n
$$
\frac{\Delta dC(t)}{dt} = -\Delta C(0)\tau_c^{-1} \exp(-t\tau_c^{-1})
$$
\n(S22)

This is basis of the  $\eta(t)$  curve obtained in OAE efficiency simulations[\(Tyka et al., 2022;](#page-8-1) [He and Tyka, 2023;](#page-8-2) [Zhou et al., 2024;](#page-8-3) [Suselj et al., 2024\)](#page-8-4).

$$
\eta(t) = 1 - \frac{\Delta C(t)}{\Delta C(0)} = 1 - \exp(-t\tau_c^{-1})
$$
\n(S23)

#### S1.4 Equivalence

We have seen that the function  $\eta(t)$ , determined under constant  $pCO_2^{atm}$  conditions, does not properly describe the actual movement of  $CO_2$  from the atmosphere into the ocean under realistic conditions (where feedback of the atmosphere is taken into account). This raises the question, what does  $\eta(t)$  represent, if anything ? To illuminate this question, consider the following scenario. How would the total inventory of  $\mathrm{CO}_2$  in the atmosphere evolve, under realistic reactive-atmosphere conditions, if  $CO<sub>2</sub>$  was directly removed from the atmosphere at a rate proportional to  $d\eta(t)/dt$ , using a terrestrial technology like DAC, instead of indirectly by applying an alkalinity pulse to the ocean?

<span id="page-5-1"></span>Specifically, the rate of carbon removal out of the atmosphere would be

$$
\frac{\mathrm{d}\Delta C}{\mathrm{d}t} = \Delta C(0) \frac{\mathrm{d}\eta(t)}{\mathrm{d}t} = \Delta C(0)\tau_c^{-1} \exp(-t\tau_c^{-1})
$$
\n(S24)

<span id="page-5-0"></span>where  $\Delta C(0)$  is a scaling factor that sets the total, eventual amount of CO<sub>2</sub> removed. This steady removal of  $CO<sub>2</sub>$  from the atmosphere lowers the  $pCO<sub>2</sub>$  of the atmosphere at a rate of

$$
\frac{\mathrm{d}pCO_2^{atm}}{\mathrm{d}t} = \frac{dpCO_2^{atm}}{dC} \Delta C(0) \tau_c^{-1} \exp(-t\tau_c^{-1})
$$
\n
$$
(S25)
$$

Since the atmosphere now develops a  $\Delta pCO_2$  deficit, relative to the ocean, the ocean would react and degas into to the atmosphere (remember all ∆s are differences between a reference and a perturbed simulation). The rate of  $\Delta pCO_2$  change therefore would be given by the regular gas exchange ODE (Equation [S9\)](#page-2-2) which describes the ongoing ocean-atmosphere equilibration plus the bias induced by the  $CO_2$  removal process (Equation [S25\)](#page-5-0).

$$
\frac{\mathrm{d}\Delta pCO_2}{\mathrm{d}t} = -\tau^{-1}\Delta pCO_2 + \frac{dpCO_2^{atm}}{dC}\Delta C(0)\tau_c^{-1}\exp(-t\tau_c^{-1})\tag{S26}
$$

This ODE is of the form  $y'(t) = gy(t) - mh \exp(th)$  with the solution  $y(t) = c_1 \exp(gt) + \frac{hm}{g-h} \exp(ht)$ . The initial condition for this linear ODE is  $\Delta pCO_2(0) = 0$  since at the moment the removal starts, the reference and perturbed states are equal. This implies in the generalized form above  $y(t) = 0$  and therefore the constant  $c_1 = -\frac{hm}{g-h}$ . Hence  $y(t) = -\frac{hm}{g-h} \exp(gt) + \frac{hm}{g-h} \exp(ht) = \frac{hm}{g-h} (\exp(ht) - \exp(gt))$ 

<span id="page-5-2"></span>Substituting the placeholders  $g$ ,  $h$  and  $m$ :

$$
\Delta p CO_2^{atm}(t) = \Delta C(0) \left( \frac{dp CO_2^{atm}}{dC^{atm}} \right) \frac{\tau_c^{-1}}{\tau^{-1} - \tau_c^{-1}} [\exp(-t\tau_c^{-1}) - \exp(-t\tau^{-1})]
$$
  
=  $\Delta C(0) \frac{\tau_c^{-1}}{\Psi} [\exp(-t\tau_c^{-1}) - \exp(-t\tau^{-1})]$  (S27)

Given this trajectory of  $\Delta pCO_2$ , what is the combined effect on total atmospheric CO<sub>2</sub> inventory, i.e. the sum of direct removal and ocean reactance ? The total rate of change is the sum of the direct  $CO_2$  removal rate (Equation [S24\)](#page-5-1) and the rate due to gas exchange between ocean and atmosphere (due to the imbalance of  $\Delta pCO_2$ :

$$
\frac{\mathrm{d}\Delta C^{atm}}{\mathrm{d}t} = \Delta C(0)\tau_c^{-1}\exp(-t\tau_c^{-1}) - \Psi\Delta p C O_2(t)
$$
\n(S28)

Substituting the expression for  $\Delta p CO_2^{atm}(t)$  from Equation [S27:](#page-5-2)

$$
\frac{d\Delta C^{atm}}{dt} = \Delta C(0)\tau_c^{-1} \exp(-t\tau_c^{-1}) - \Delta C(0)\tau_c^{-1} [\exp(-t\tau_c^{-1}) - \exp(-t\tau^{-1})]
$$
  
=  $\Delta C(0)\tau_c^{-1} \exp(-t\tau^{-1})$   
=  $\Delta C(0)\Psi \frac{dpCO_2^{ocn}}{dC^{ocn}} \exp(-t\tau^{-1})$  (S29)

This net rate of change in atmospheric carbon inventory equals precisely that obtained earlier when consider-ing a pulsed DIC deficit in the ocean, Equation [\(S17\)](#page-3-0). This result clarifies the exact meaning of the  $\eta(t)$  OAE efficiency curve: The OAE curve of an *instantaneously* induced ocean-based DIC deficit (obtained under artificial, non-reactive conditions), gives the progression of the equivalent *gradual* removal of  $CO<sub>2</sub>$  directly from the atmosphere (under real, responsive atmosphere conditions), which would result in the same reduction in net atmospheric carbon inventory.

### S1.5 Calculating  $\eta(t)$  from a coupled simulation

When simulating OAE under responsive atmospheres or earth system models, it is still useful to calculate  $\eta(t)$ , especially or purposes of comparing different negative emissions technologies and calculating negative emission credits. The directly measured change in the ocean inventory unfortunately conflates the effect of the OAE intervention and the subsequent feedback. Therefore, a correction is necessary which can be used to factor out the effect of reservoir feedback. [Schwinger et al.](#page-8-5) [\(2024\)](#page-8-5) proposed a simple simulation setup to calculate this correction, which allows the conversion between the two carbon flows under the two different simulation conditions: First one simulates the fully coupled system with the OAE intervention. Then one conducts an additional reference simulation in which the atmosphere  $pCO_2$  is prescribed at the exact trajectory obtained in the fully-coupled simulation with the OAE intervention. However, in the new reference simulation no OAE is performed - only the atmospheric  $pCO_2$  is prescribed.

We can show below, using the two-box model, that the difference in ocean inventory between these two simulations indeed equals the  $\eta(t)$  value, had the OAE been simulated under a prescribed atmosphere in the first place.

The rate of  $CO_2$  transfer due to an OAE pulse under responsive atmosphere is Equation [S17](#page-3-0)

$$
\frac{\mathrm{d}C^{ocn}}{\mathrm{d}t} = \Delta C(0)\Psi \frac{\mathrm{d}pCO_2^{ocn}}{\mathrm{d}C^{ocn}} \exp(-t\tau^{-1}),\tag{S30}
$$

where  $\Delta C(0)$  gives the initial DIC deficit. But under prescribed conditions the rate is given by Equation [S22](#page-4-0)

$$
\frac{\mathrm{d}\Delta C^{ocn}}{\mathrm{d}t} = -\Delta C(0)\tau_c^{-1} \exp(-t\tau_c^{-1})\tag{S31}
$$

The correction required is equal to the difference of the two rates, i.e. the purely reactionary part of the carbon flux caused by the atmosphere feedback equals:

<span id="page-6-1"></span>
$$
\frac{\mathrm{d}C^{\text{fb}}}{\mathrm{d}t} = \Delta C(0)\Psi \frac{\mathrm{d}pCO_2^{ocn}}{\mathrm{d}C^{ocn}} \left[ \exp(-t\tau^{-1}) - \exp(-t\tau_c^{-1}) \right] \tag{S32}
$$

<span id="page-6-0"></span>Now consider the reference simulation with the  $pCO_2$  prescribed at the trajectory from the OAE-perturbed coupled-simulation. In this new reference simulation, the difference in atmospheric  $pCO_2$  compared to what it would have been without the prescription is equal to the change in  $pCO<sub>2</sub>$  caused by OAE in the fullycoupled simulation. That change is equal to the rate of  $CO_2$  transfer caused by OAE (from Equation [S20\)](#page-4-1) times  $\mathrm{d}pCO_{2}^{atm}/\mathrm{d}C^{atm},$  i.e.

$$
\frac{d\Delta p CO_2^{atm}}{dt} = -\frac{dp CO_2^{atm}}{dC^{atm}} \frac{dC^{ocn}}{dt} = \Delta C(0)\Psi \frac{dp CO_2^{atm}}{dC^{atm}} \frac{dp CO_2^{ocn}}{dC^{ocn}} \exp(-t\tau^{-1})
$$
\n
$$
(S33)
$$

In this new reference simulation, there will be offgassing of  $CO<sub>2</sub>$  from the ocean to the atmosphere (relative to a completely unperturbed simulation), because the prescribed atmospheric  $pCO_2$  is lower than it would have been otherwise. The total air-sea difference  $\Delta pCO_2$  will then evolve according to the difference between rates of change of the atmosphere and the ocean:

$$
\frac{\mathrm{d}\Delta pCO_2}{\mathrm{d}t} = -\tau_c^{-1} \Delta p CO_2 - \Delta C(0) \Psi \frac{\mathrm{d}p CO_2^{atm}}{\mathrm{d}C^{atm}} \frac{\mathrm{d}p CO_2^{ocn}}{\mathrm{d}C^{ocn}} \exp(-t\tau^{-1})
$$
\n(S34)

Where the first term is the regular ocean gas-exchange law under prescribed conditions (Equation [S13\)](#page-3-1) and the second term is the additional atmospheric  $pCO_2$  deviation due to the prescribed  $pCO_2$  trajectory (Equation [S33\)](#page-6-0). This, once again, is a linear ODE of the form  $y'(t) = gy(t) - m \exp(th)$  with the solution  $y(t) = c_1 \exp(gt) +$  $\frac{m}{g-h}$  exp(ht). Again, the initial condition for this linear ODE is  $\Delta pCO_2(0) = 0$  since up to the point where the atmospheric  $pCO_2$  is prescribed, the new reference state is identical to the original one.

The solution is (after simplification):

$$
\Delta p CO_2(t) = \Delta C(0) \frac{\mathrm{d}p CO_2^{ocn}}{\mathrm{d}C^{ocn}} \left[ \exp(-t\tau^{-1}) - \exp(-t\tau_c^{-1}) \right]
$$
(S35)

Which describes the time evolution of the  $pCO<sub>2</sub>$  difference across the air-sea interface in this new reference simulation. The rate of CO<sub>2</sub> transfer caused by this  $\Delta pCO_2(t)$  is then obtained from the gas transfer law (Equation [S2\)](#page-1-0), i.e.

$$
\frac{\mathrm{d}C^{ocn}}{\mathrm{d}t} = \Delta C(0)\Psi \frac{\mathrm{d}pCO_2^{ocn}}{\mathrm{d}C^{ocn}} \left[ \exp(-t\tau^{-1}) - \exp(-t\tau_c^{-1}) \right] \tag{S36}
$$

This matches exactly the sought after  $\frac{dC^{fb}}{dt}$  $\frac{C^{10}}{dt}$ , given by Equation [S32.](#page-6-1) In other words, as suggested by [Schwinger](#page-8-5) [et al.](#page-8-5) [\(2024\)](#page-8-5), the difference in  $CO_2$  ocean inventory between a simulation with OAE under a responsive atmosphere and one where the  $pCO_2^{atm}$  trajectory is prescribed to that of the former simulation but with no OAE applied yields the same metric (after normalization by  $\Delta A l k$ ) as given by  $\eta(t)$ .

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