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Supplement of

Substantial inter-model variation in OAE efficiency between the CESM2/MARBL and ECCO-Darwin ocean biogeochemistry models

Michael Dominik Tyka et al.

Correspondence to: Michael Dominik Tyka (mike.tyka@gmail.com)

S1 Alkalinity injection locations used in this study

Name	Average Latitude	Average Longitude	Polygon index
Alaska	58.2	213.2	180
Brazil	1.9	311.8	29
East Coast USA	41.1	290.6	17
Gulf of Mexico	27.1	268.7	87
Hawai‘i	18.4	206.8	157
Iceland	61.7	340.7	32
Japan	32.3	136.5	158
Kerguelen	-49.3	72.0	363
Norway	65.3	5.8	99
Oman	24.2	59.5	437
West Coast USA	30.6	241.3	214
Western Sahara	24.1	341.9	27
East Coast USA (Offshore)	35.2	294.0	134
Oman (Offshore)	14.8	64.1	592
West Coast USA (Offshore)	27.3	235.2	345
Western Sahara (Offshore)	24.6	330.6	111

Table S1. Summary of the twelve main alkalinity injection locations and the four additional offshore locations used in this study together with the geographical location of their center. The polygon index refers to the numbering scheme used in Zhou et al. (2025) and the exact area definitions can be found therein. See also Fig. 1 in main manuscript.

Name	Latitude	Longitude	Polygon index
East Coast USA	42.5	294.6	72
North Hawai‘i	27.7	203.4	157
North Pacific	55.9	185.6	305
Equatorial Pacific	1.5	162.4	334

Table S2. Alkalinity injection locations used in the comparison of interannual variability together with the geographical location of their center. The polygon index refers to the numbering scheme used in Yankovsky et al. (2025). See also Fig. 1 in main manuscript.

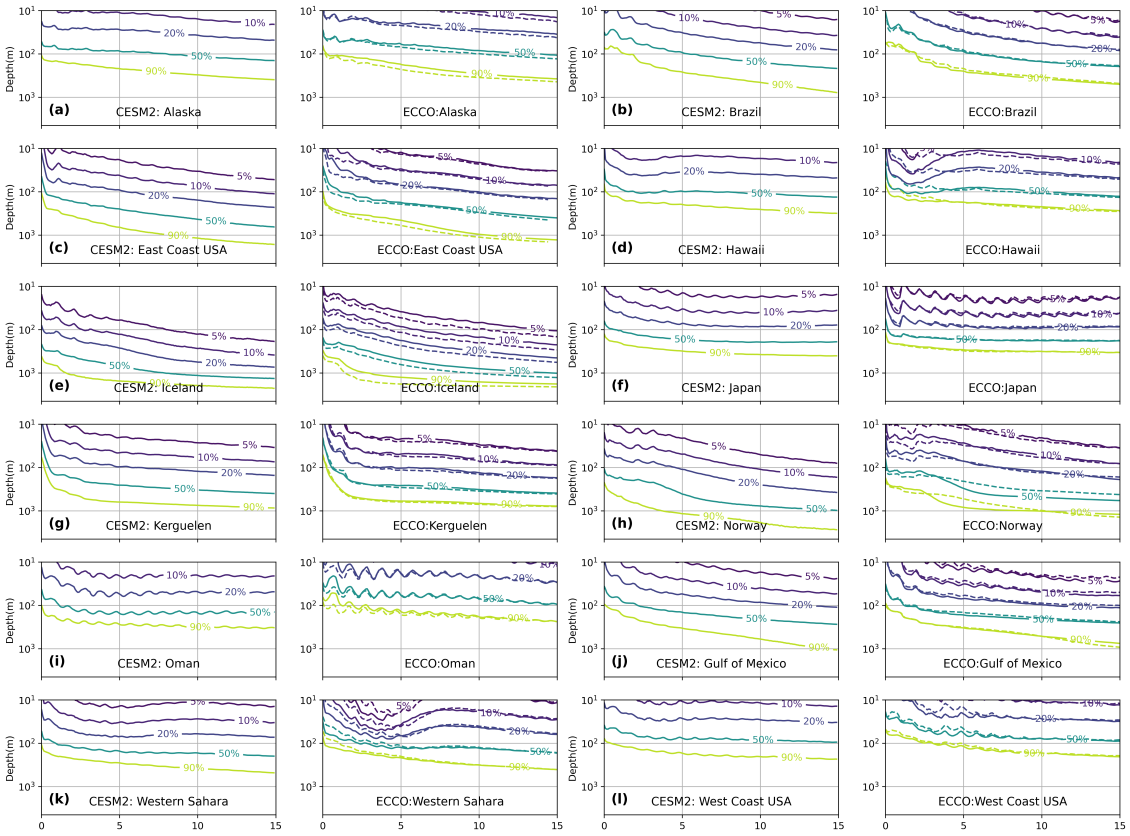

Figure S1. Distribution of excess alkalinity after injection over time, for CESM2/MARBL and ECCO-Darwin. Solid lines are 1999 simulations, dashed lines are 1992 simulations. The percentages indicate the amount of excess alkalinity situated above a particular depth. The rebound of alkalinity depth can be seen very clearly in the injection locations "Western Sahara" and "Hawaii", though it occurs to a different extent in the two models.

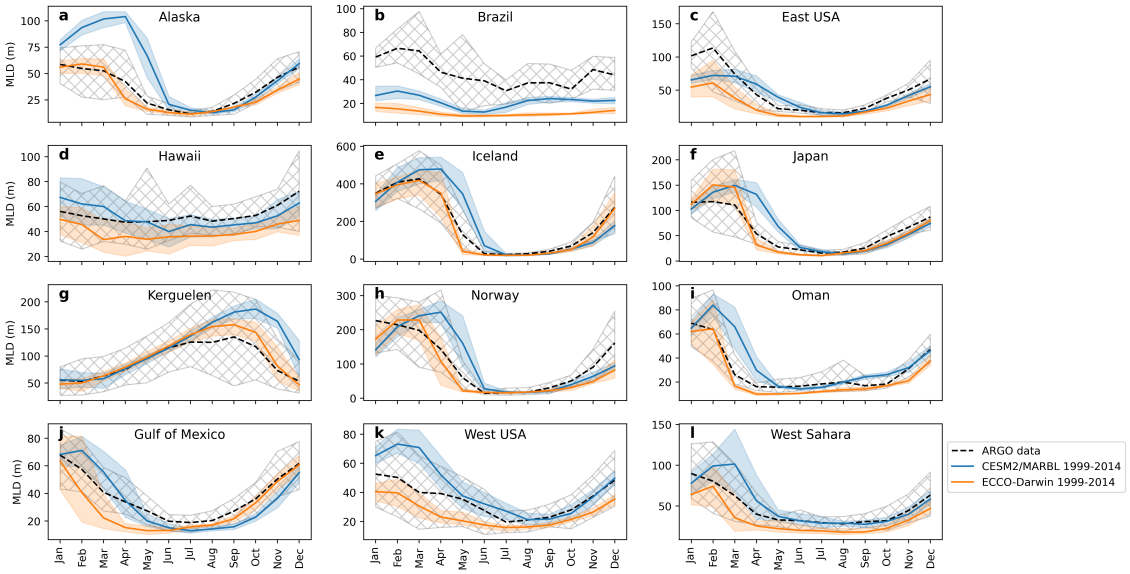

Figure S2. Mixed layer depths (MLD) (de Boyer Montégut et al., 2004), for each month, averaged over years 1999–2014 for each of the two models examined. In addition, MLDs estimated from Argo data (Holte et al., 2017) are overlaid in black (with hashed area indicating 5th and 95th percentiles within each alkalinity release patch). Note that the overlap of the alkalinity release patches with grid points where Argo climatological MLD data was available was only 28% for the locations "Brazil" and "East USA", which may account for the poorer agreement with the models. In particular, most of the Brazil release patch is located right at the mouth of the Amazon River in very shallow waters, where the depth is less than 40 m and whereas the Argo data that overlaps the patch represents locations just beyond the continental shelf.

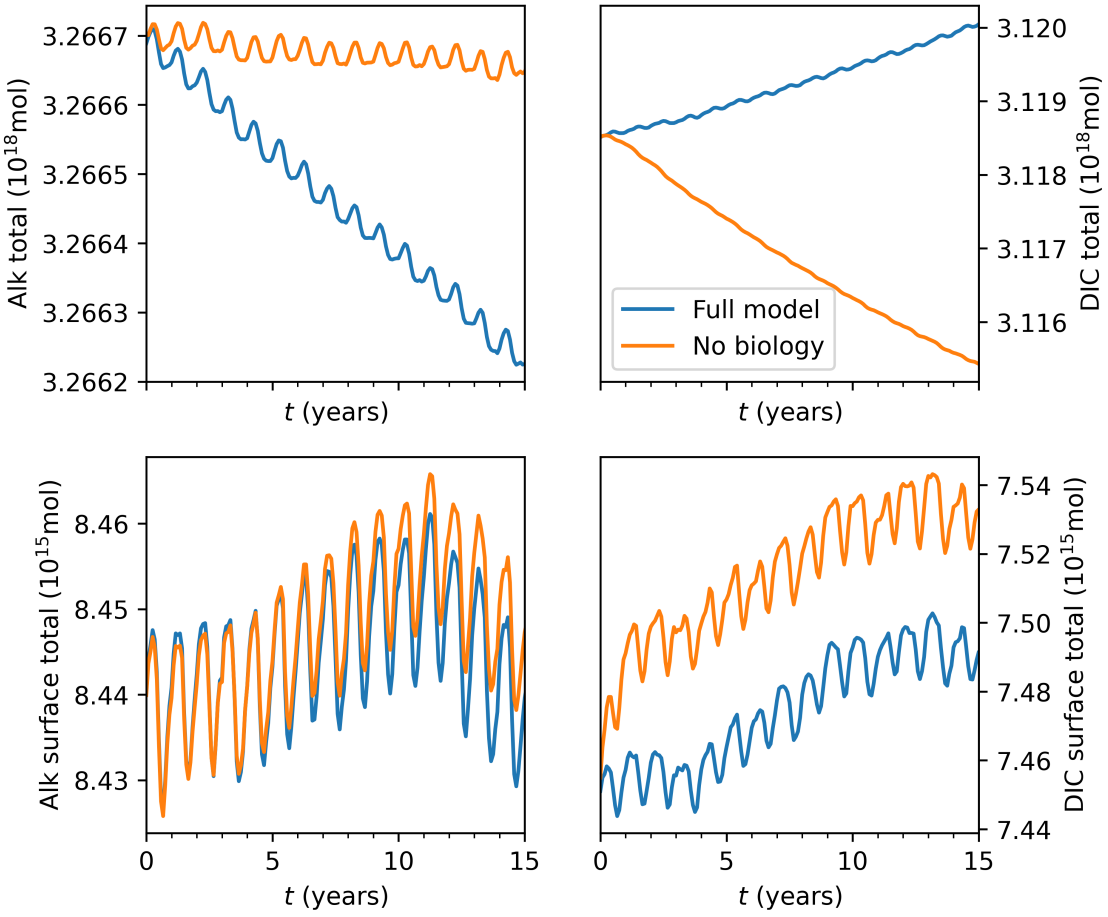

Figure S3. Absolute total amounts of DIC and Alkalinity in the regular ECCO-Darwin model and the ablated "no-biology" version, which only modelled gas exchange, with no alkalinity perturbation. As expected, the total amounts deviate as soon as the biological model is turned off at t=0. However, the changes are quite small over the 15 years of simulation time, reaching a magnitude on the order of 0.01–0.12% for the total amounts and 0.1–0.25% for surface quantities.

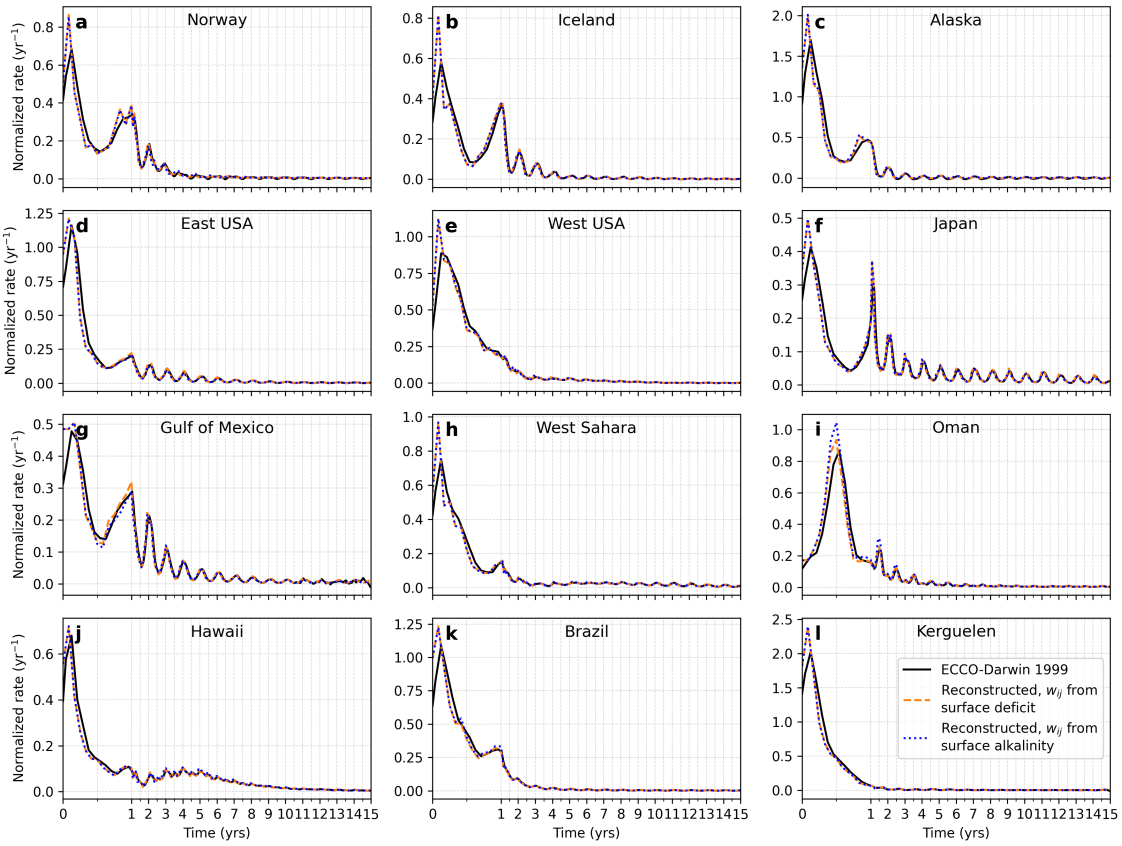

Figure S4. Black: Empirical equilibration rate from the ECCO-1999 runs (mol yr^{-1}), normalized by the amount of alkalinity added (mol) for each simulation, calculated as the time derivative of the simulated CO_2 uptake. Dashed orange: Equilibration rate "reconstructed" from the surface deficit (calculated using Equation 12, main text) and the local gas exchange parameters. Dotted blue: Equilibration rate "reconstructed" from the surface alkalinity distribution (calculated using Equation 12, main text), multiplied by the total surface deficit and the local gas exchange parameters.

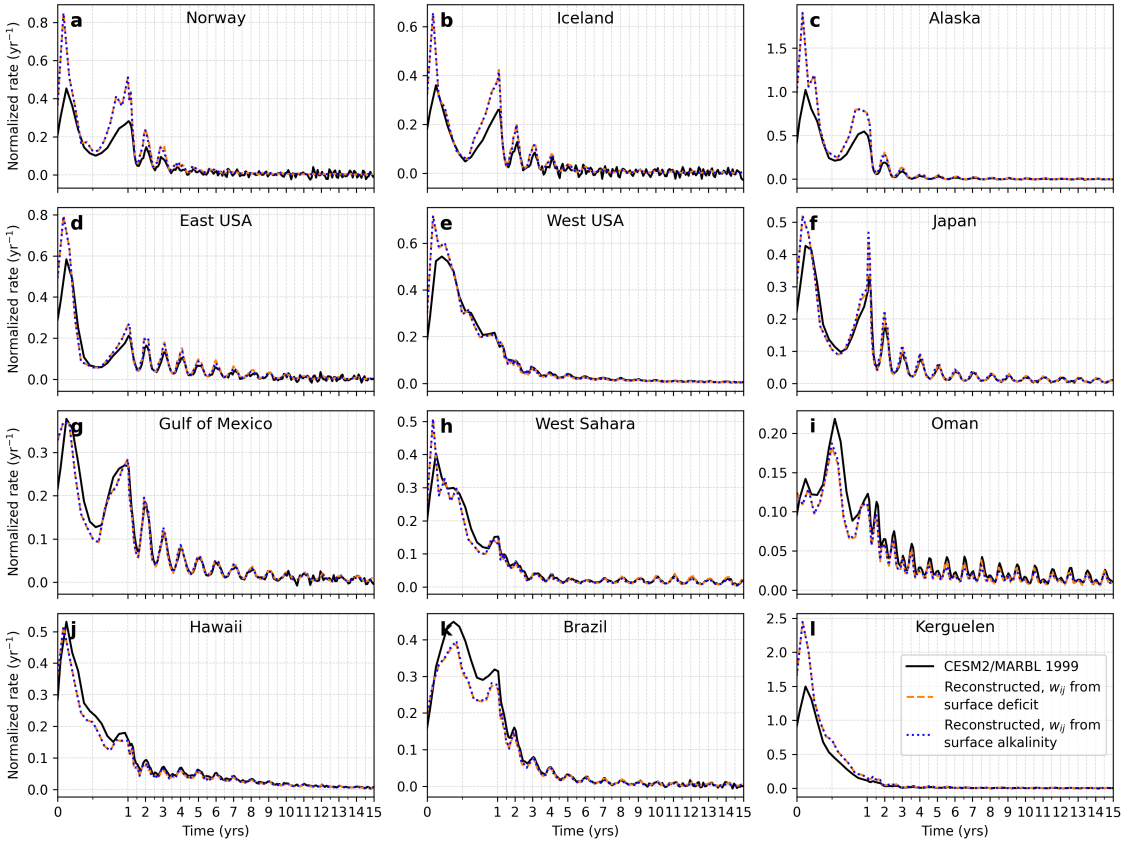

Figure S5. Black: Empirical equilibration rate from the CESM runs (mol yr^{-1}), normalized by the amount of alkalinity added (mol) for each simulation, calculated as the time derivative of the simulated CO_2 uptake. Dashed orange: Equilibration rate "reconstructed" from the surface deficit (calculated using Equation 12, main text) and the local gas exchange parameters. Dotted blue: Equilibration rate "reconstructed" from the surface alkalinity distribution (calculated using Equation 12, main text), multiplied by the total surface deficit and the local gas exchange parameters.

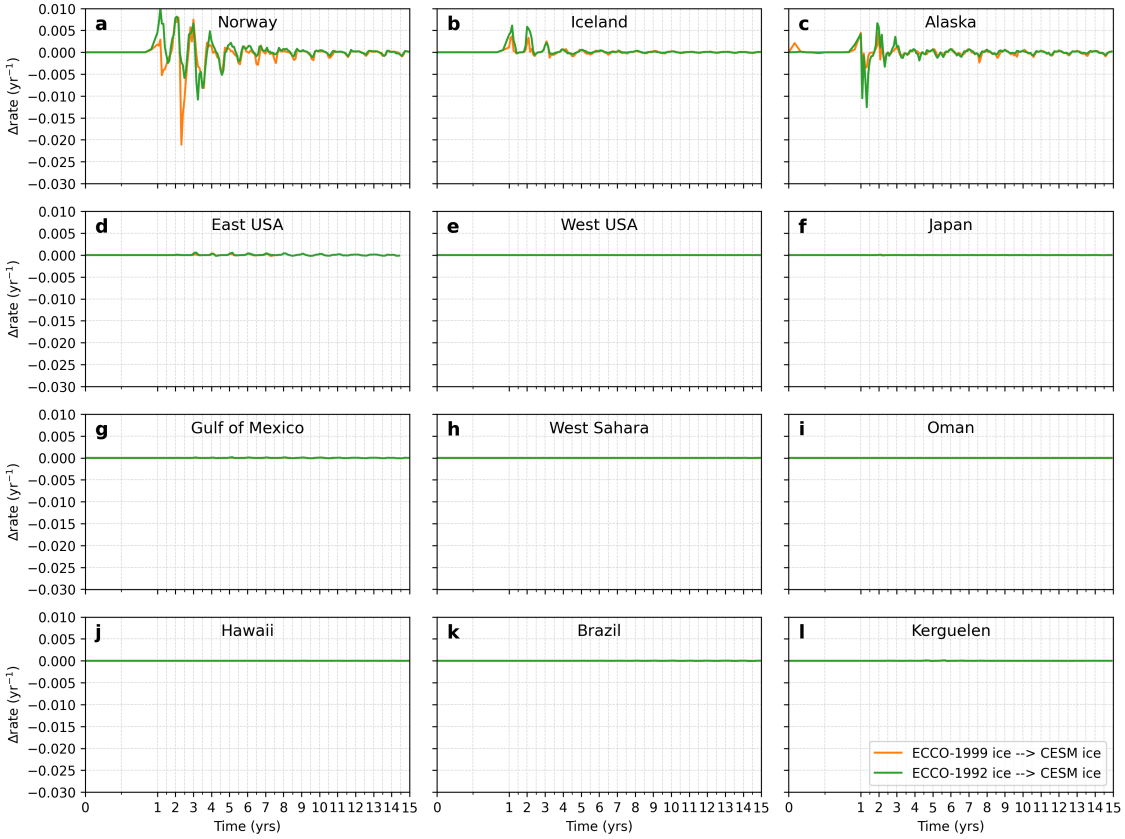

Figure S6. Change in the normalized equilibration rate in the two ECCO-Darwin runs when changing only the sea-ice parameters (α_{ice}) to those from CESM2/MARBL.

S2 Approximations and parameter dependence for $\partial[DIC]/\partial[Alk]$

The quantity $\eta_{max} = \frac{\partial[\mathrm{DIC}]}{\partial[Alk]}$ determines the maximal amount of additional CO_2 that can be taken up by a water parcel following the addition of a small quantity of alkalinity and therefore effectively determines the maximal efficiency of OAE at full equilibration. It is the reciprocal of the "isocapnic quotient" introduced by Humphreys et al. (2018). In principle, it is dependent on all the carbonate system parameters and can be calculated exactly using a carbonate system solver such as PyCO2SYS (Humphreys et al., 2020). An exact analytical formula has also been given by Humphreys et al. (2018):

$$1/\eta_{max} = \frac{(K_1 hs + 4K_1K_2s + K_w h + h^3)(K_B + h)^2 + K_B B_T h^3}{K_1 s(h + 2K_2)(K_B + h)^2}, \tag{S1}$$

where h is the $[H^+]$ concentration, s is the [CO_2] concentration, B_T is the total borate concentration and K_1,K_2,K_w and K_B are the equilibrium constants of the carbonate and borate equilibria. An approximation was also given as

$$1/\eta_{max} = 1 + \frac{2K_2[DIC]}{K_1[CO_2]} \tag{S2}$$

For more detail on the derivation and the approximation, see Humphreys et al. (2018). By dividing denominator and numerator by the factor $h^2(K_B + h)^2$ and replacing the terms with the equivalent concentration terms, this expression can be expressed exactly in terms of the concentrations of species contributing to the carbonate equilibrium. This can be useful for practical purposes.

$$1/\eta_{max} = \frac{[HCO_3^-] + 4[CO_3^{2-}] + [OH^-] + [H^+] + [B(OH)_4^-][B(OH)_3]/([B(OH)_4^-] + [B(OH)_3])}{[HCO_3^-] + 2[CO_3^{2-}]} \tag{S3}$$

All other parameters (salinity, etc) were also set to typical ocean values (default PyCO2SYS values were used).

When plotted against $[DIC]/[Alk]$, as shown in Figure S7a, all the points lie almost perfectly along a monotonically increasing line (Fig. S7). In other words, η_{max} appears to be almost purely a function of $[DIC]/[Alk]$. Temperature does not appear to strongly affect the value for most of the $[DIC]/[Alk]$ ratio, except around $[DIC]/[Alk] \approx 1.0$, where a small temperature dependence is observed. This suggests that there should be simple approximations for η_{max} as a function of $[DIC]/[Alk]$ for convenient calculation of this quantity without the need to compute the entire carbonate system. A very simple approximation was given by Tyka et al. (2022):

$$1/\eta_{max} \approx 3 - 2[DIC]/[Alk]. \tag{S4}$$

This approximation is shown as a dashed line in Figure S7 and is surprisingly good between $[DIC]/[Alk]$=0.45 and $[DIC]/[Alk]$=0.95, but fails to account for more alkaline and more acidic extremes. Its advantage is that it is easy and fast to calculate because it is not necessary to solve the carbonate system (i.e. there's no need to determine the value of pH, $[CO_2]$ or $[HCO_3^-]$, etc which are needed for the exact expression Eq. S3).

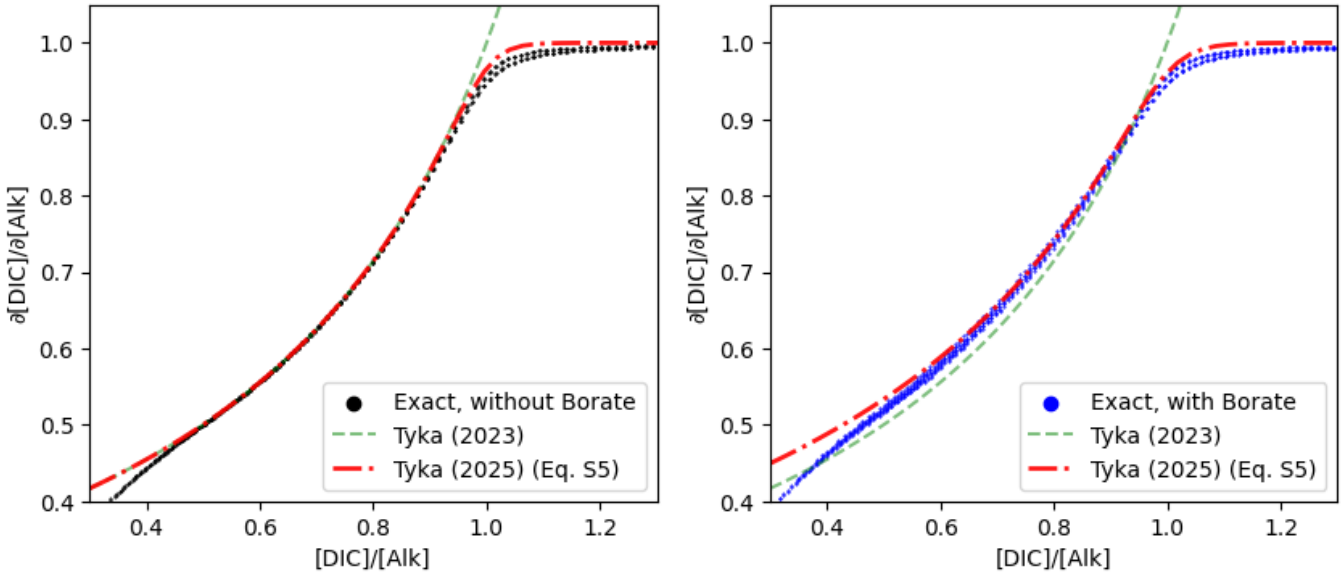

Figure S7. η_{max} as a function of [DIC]/[Alk] for carbonate systems with and without borate present. The approximation (Tyka et al., 2022) is shown as a dashed line. The superior approximation from Equation S5 is shown as a dash-dot line.

At values of $[DIC]/[Alk]$ greater than 0.9 the approximation begins to strongly diverge from the real value of η_{max}, which cannot exceed 1.0. The value of the approximation, however, continues beyond 1.0 because the term $3-2[DIC]/[Alk]$ continues to decrease below 1 (and eventually below 0), yielding nonsensical values. Here, we introduce an improvement on Eq. S4, which can be achieved by clipping $[DIC]/[Alk]$ above 1.0. If a simple truncation $\min(1,[DIC]/[Alk])$ were used, the result would be a sharp kink at $[DIC]/[Alk]=1.0$ and a poor fit in its vicinity. Instead, a much better result is obtained if one uses a softplus function, such as $\ln(\exp(\gamma x)+1)\gamma^{-1}$, where γ is an empirical sharpness parameter.

This yields an easy to compute, yet excellent approximation for almost all practically relevant values of $[DIC]/[Alk]$:

$$1/\eta_{max} = 1+\theta\gamma^{-1}\ln\{1+\exp(\gamma-\gamma[DIC]/[Alk])\}, \tag{S5}$$

with $\theta=2.0$ and $\gamma\approx 40$, which leads to about the right curvature near $[DIC]/[Alk]=1.0$. This approximation is shown in Figure S7a as a dash-dot line.

S2.1 Borate

When the effect of typical ocean borate concentrations is included in the calculation, η_{max} is somewhat increased and a small temperature dependence is now evident across the entire $[DIC]/[Alk]$ range (Fig. S7b). The earlier approximation is now a little too low (by around 0.03) compared to the accurate value of η_{max} due to the additional buffering provided by the borate

system. This can be empirically corrected by adjusting Equation S5 slightly, setting $\theta = 1.75$ and $\gamma \approx 30$. The comparison to the exact values calculated with PyCO2SYS is shown in Figure S7b.

S2.2 Other approximations

Other, less empirical, approximations can be constructed starting from Equation S3 by making some increasingly simplifying assumptions, which we note here for completeness. Some of them are extremely accurate, but generally require knowing the value of $[CO_2]$ or $[HCO_3^-]$.

Assuming the concentration of free $[H^+]$ is negligible and $B_T = 0$, Eq. S3 simplifies to:

$$1/\eta_{max} \approx \frac{[HCO_3^-] + 4[CO_3^{2-}] + [OH^-]}{[HCO_3^-] + 2[CO_3^{2-}]} \tag{S6}$$

The assumptions above also imply that $[Alk] = [HCO_3^-] + 2[CO_3^{2-}] + [OH^-]$ as the contributions to $[Alk]$ from the minor ions fall away. One can thus substitute the numerator and denominator as:

$$1/\eta_{max} \approx \frac{[Alk] + 2[CO_3^{2-}]}{[Alk] - [OH^-]} = \frac{[Alk] + ([Alk] - [OH^-] - [HCO_3^-])}{[Alk] - [OH^-]} = 1 + \frac{[Alk] - [HCO_3^-]}{[Alk] - [OH^-]} \tag{S7}$$

If one further assumes that $[OH^-]$ is also relatively small compared to $[Alk]$ in the denominator, another approximation arises (keeping in mind that $[DIC] = [CO_2] + [HCO_3^-] + [CO_3^{2-}]$):

$$\begin{aligned} 1/\eta_{max} &\approx \frac{[Alk] + 2[CO_3^{2-}]}{[Alk]} \\ &= \frac{[Alk] + 2([Alk] - [DIC] + [CO_2])}{[Alk]} \\ &= 3 - 2\frac{([DIC] - [CO_2])}{[Alk]} \end{aligned} \tag{S8}$$

Finally, this approximation simplifies further at typical ocean pH of 8 (where the concentration of free CO_2 is very small) and the earlier approximation from Tyka et al. (2022) is recovered ($1/\eta_{max} = 3 - 2[DIC]/[Alk]$).

Remarkably, the last approximation from Equation S7 and its slightly simpler relative

$$1/\eta_{max} \approx 1 + \frac{[Alk] - [HCO_3^-]}{[Alk]} \tag{S9}$$

work extremely well even when borate is present, even though the approximations do not explicitly include the concentrations of borate (unlike the exact expression).

Figure S8 shows some of these approximations and their respective errors against the exactly calculated values of η_{max} for both the borate-free carbonate system and the more realistic carbonate system with borate, respectively. Here, unlike Figure S7, the values for Alk and DIC were taken from OceanSODA (Gregor and Gruber, 2021) and thus the approximations are shown in the context of realistic ocean surface carbonate states. Note that large applications of OAE could push the system into very low

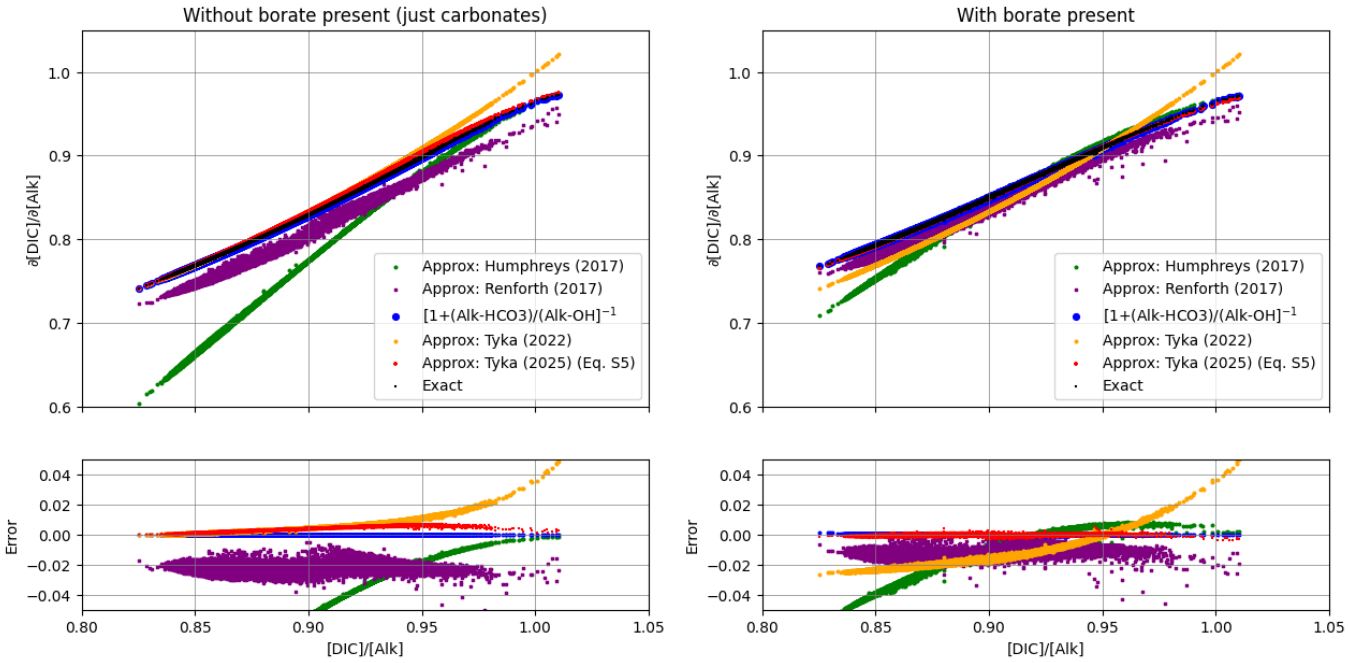

Figure S8. Various approximations to η_{max} discussed in the text. The approximations from Humphreys et al. (2018), Renforth and Henderson (2017) and from Tyka et al. (2022) are also shown for comparison.

DIC/Alk, below what is shown here, but only very transiently and local to the injection site. Since η_{max} is primarily concerned with the ultimate, long-term equilibration point, inaccuracies in transient low DIC/Alk are not particularly important.

S3 Analytical expression for $\partial[CO_2]/\partial[DIC]$

An analytic expression for $\partial[CO_2]/\partial[DIC]$ may be obtained using a similar approach to that used to derive $\partial[DIC]/\partial[Alk]$ by Humphreys et al. (2018). Beginning with their general expressions for alkalinity (A_T) and DIC (C_T)

$$\begin{aligned} A_T &= [HCO_3^-] + 2[CO_3^{2-}] + [B(OH)_4^-] + [OH^-] - [H^+] \\ &= K_1 s/h + 2K_1K_2 s/h^2 + K_B B_T/(K_B + h) + K_w/h - h \end{aligned} \tag{S10}$$

$$\begin{aligned} C_T &= [CO_2] + [HCO_3^-] + [CO_3^{2-}] \\ &= s + K_1 s/h + K_1K_2 s/h^2 \end{aligned} \tag{S11}$$

we calculate the two derivatives $\partial C/\partial h$ and $\partial s/\partial h$, both at constant alkalinity.

First expressing $s = [CO_2]$ in terms of A_T and h,

$$s = \frac{A_T - \frac{K_B B_T}{K_B+h} - \frac{K_W}{h} + h}{K_1/h + 2K_1K_2/h^2} \tag{S12}$$

one can take its derivative with respect to h:

$$\frac{\partial s}{\partial h} = h\frac{(h+4K_2)(A_T - \frac{K_B B_T}{K_B+h} - \frac{K_W}{h} + h) + h(h+2K_2)(\frac{B_T K_B}{(h+K_B)^2} + \frac{K_W}{h^2} + 1)}{K_1(h+2K_2)^2} \tag{S13}$$

Similarly, expressing C_T in terms of A_T and h,

$$C_T = \frac{(A_T - K_B B_T/(K_B+h) - K_W/h + h)(1 + K_1/h + K_1K_2/h^2)}{K_1/h + 2K_1K_2/h^2} \tag{S14}$$

and differentiating with respect to h yields:

$$\frac{\partial C_T}{\partial h} = \frac{C_T}{hs}\left[\left(1 - \frac{K_1/h + 4K_1K_2/h^2}{(K_1/h + 2K_1K_2/h^2)^2}\right)\left(\frac{K_B B_T}{K_B+h} - A_T - h + \frac{K_W}{h}\right) + \frac{\frac{h}{h+K_B}\frac{K_B B_T}{K_B+h} + h + K_W/h}{K_1/h + 2K_1K_2/h^2}\right] \tag{S15}$$

The sought-after derivative is then given by the quotient of these two derivatives:

$$\left.\frac{\partial C_T}{\partial s}\right|_{A_T} = \frac{\partial C_T}{\partial h} \Big/ \frac{\partial s}{\partial h} \tag{S16}$$

After some simplification one obtains:

$$\begin{aligned} \left.\frac{\partial C_T}{\partial s}\right|_{A_T} &= s^{-1}\left[C_T - \frac{(K_1 s/h + 2K_1K_2 s/h^2)^2}{K_1 s/h + 4K_1K_2 s/h^2 + h + K_w/h + h/(h+K_B)\frac{K_B B_T}{K_B+h}}\right] \\ &= s^{-1}\left[C_T - \frac{([HCO_3^-] + 2[CO_3^{2-}])^2}{[HCO_3^-] + 4[CO_3^{2-}] + [OH^-] + [H^+] + [B(OH)_4^-][B(OH)_3]/([B(OH)_4^-] + [B(OH)_3])}\right] \end{aligned} \tag{S17}$$

which can be further simplified by substituting η_{max} from Equation S3 to give

$$\left.\frac{\partial C_T}{\partial s}\right|_{A_T} = s^{-1}\left[C_T - \eta_{max}([HCO_3^-] + 2[CO_3^{2-}])\right] \tag{S18}$$

This equation is very convenient for calculating the β factor post-simulation when $[HCO_3^-]$ and $[CO_3^{2-}]$ have been saved as tracer values.

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