



Supplement of

How beech ecophysiology shapes temperate forest gross primary productivity – Part 1: A wavelet-based framework for extracting seasonal dynamics

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S1: Cone of influence

Physical signals are inherently finite in length. For data points located sufficiently close to the boundaries of a signal, the wavelet may extend beyond the length of the signal, producing unreliable coefficients. The larger the scale of the wavelet, or the larger the period analyzed, the more data points are affected. To tackle this issue, practitioners employ the concept of the cone of influence (COI). For each data point, the COI refers to the boundary of the time-frequency plane region that encompasses all wavelet coefficients affected by the value of that data point (Mallat, 2009). By representing the COI at the edges of a signal on a scalogram, regions deemed affected by border effects can be delineated. An example of this procedure is provided for the Morlet wavelet in Fig. 1d.

The COI allows some latitude in the definition of the affected region. Torrence & Compo (1998) outlined this region based on the attenuation of the wavelet's amplitude from its peak value. They identified affected areas as those where the value of the wavelet's envelope decays to a fraction $1/e^2$ of its peak value. In this study, we applied a slightly more stringent definition, excluding coefficients for which more than 2 % of the wavelet extends beyond the signal border. The exceeding part of the wavelet was defined as the portion of the area under the wavelet curve that lies outside the signal.

S2: Wavelet choice for period bands

Wavelet choice influences the structure of period-band peaks and should therefore be considered during coefficient selection. The DOG2 wavelet provides a robust starting point for ecological signals, as it reliably highlights trend discontinuities and seasonal peaks (Mallat, 2009). While the DOG1 wavelet is equally effective at detecting discontinuities, it is less effective for peaks or drops in the signal if they are not pronounced enough. For signals dominated by oscillatory behavior, the Morlet wavelet is appropriate, although its complex nature requires separating real and imaginary parts during WAI interpretation (Fig. S4). Higher-order derivatives of the Gaussian (e.g. DOG6) can serve as real-valued alternatives to the Morlet wavelet.

S3: Wavelet transform adjustment (third step)

Since many wavelets share common oscillatory behavior, similar patterns can often be detected by a variety of wavelets (Torrence & Compo, 1998). However, inadequately suited wavelets may introduce substantial undesired area contributions (e.g. the left and central lobe of the wavelet in Fig. 2) or conversely fail to encapsulate the target pattern (e.g. the right lobe of Fig. 2, if we are interested in the entire growing season). Both situations can introduce uncertainties in the interpretations given to wavelet coefficients. To address this issue, the WT can be adjusted to align coefficient values with their intended interpretation. Four types of adjustments are proposed and applied in this study:

1. **Modifying the wavelet used** provides a means for matching the wavelet filter to the pattern sought in the signal, as visualized using the WAI. Non-admissible functions, such as functions with non-zero mean (Grossmann & Morlet, 1984), can also be used, in which case, the operation becomes a filtering process rather than a wavelet transform. Nevertheless, the interpretation based on WAI remains unchanged. As a general guideline, the Gaussian function (equivalent to a Gabor transform) is suitable for evaluating peak magnitude and smooth variations, the DOG1 wavelet for detecting discontinuities and evaluating rates of change, and the DOG2 wavelet for identifying and quantifying drops in the signal.
2. **Adjusting the period of analysis** controls the temporal extent captured by the wavelet. Although adjacent periods typically produce similar coefficient values, selecting a period consistent with the duration of the target process is essential for robust interpretations. This can be achieved by matching the wavelet support (or period for sinusoidal wavelets) to the duration of the event.
3. **Normalizing signals** eliminates the influence of absolute signal magnitude, enabling comparison across years or periods. Common approaches include scaling by signal amplitude or by total sum for each period. Amplitude-based normalization is useful for describing rates of change at the start and end of a trend, provided that sufficiently low periods are used, while sum-based normalization expresses coefficients as proportions of total signal magnitude.
4. **Combining coefficients** through simple mathematical operations can extend the range of interpretable dynamics without increasing methodological complexity. Division, for instance, allows direct comparison between different parts of the signal and is particularly useful for quantifying relative changes such as growth or decline amplitudes.

S4: Candidate indicators

1. **Yearly values**

The 1-year peak of the Mexican Hat wavelet (or alternatively a Gaussian function) can be used as a proxy for an annual integral, applying a bell-shaped weighting to the signal. This approach may be appropriate when the signal exhibits an approximately Gaussian seasonal pattern, as is often the case for GPP. However, its use remains limited for quantifying annual fluxes and is more suitable as a normalization factor based on a smooth weighting window.

2. **Steepness of decrease**

The indicator follows the same approach as IRise but is applied at the end of the GS. It was not retained due to (i) weaker signal detectability, (ii) higher uncertainty and (iii) limited support in the literature regarding ecophysiological processes governing senescence.

3. **End of season values**

The 6-month November peak (Fig. 5c) provides an estimate of late-season GPP levels. The Haar wavelet (Fig. S2) can be used to avoid Gaussian-type weighting, and similar wavelet adjustments as those applied for IPeak could be implemented. However, this indicator was not retained because IDrop captures mid-season dynamics, providing a more integrated description of seasonal decline than metrics focused solely on second-half values.

4. **Proportion of values at the onset of the GS**

This indicator is derived using the same procedure as IPeak applied to a normalized signal (each year's GPP divided by its annual sum). Alternatively, it can be defined as the ratio of the 6-month April peak (Fig. 5a) to annual GPP (either as a yearly coefficient or total sum). This metric is strongly correlated with IDrop ($r = 0.74$, $p\text{-value} < 0.001$), reflecting its sensitivity to mid-season decline. To avoid redundancy, only IDrop was retained. Additionally, IDrop is more directly related to drought effects, making it more relevant for ecophysiological interpretation.

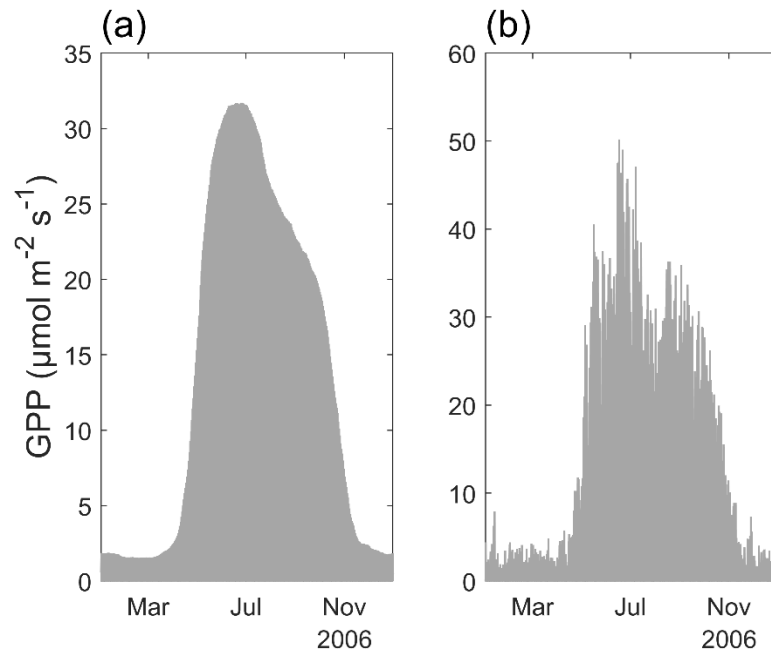


Figure S1: Example of the FR-Hes GPP signal for the year 2006: (a) representative GPP yearly cycle modelled from incoming radiation using light-response curves fitted across all years within 30-day sliding windows centered on day of year and applied to potential solar radiation, (b) GPP measured at the eddy-covariance tower (restricted to positive values only for readability). Beech is a deciduous species, producing new leaves in spring that last all summer and fall in autumn. As such, its photosynthetic activity (translated by the GPP signal) follows seasonal trends, reaching near-zero values during winter and maximal values at the beginning of summer.

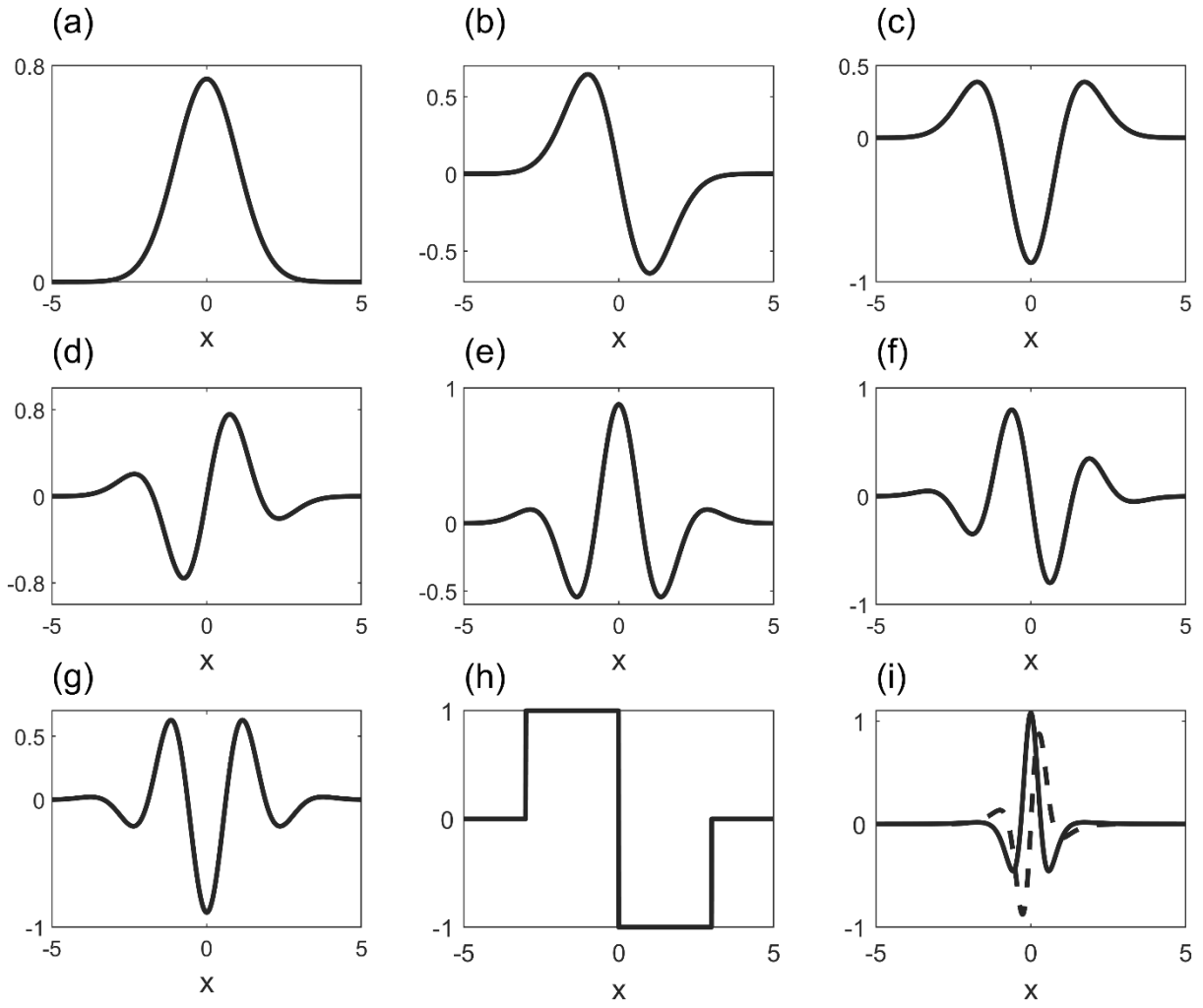


Figure S2: Illustration of useful analyzing functions for continuous wavelet transforms. (a) Gaussian function, (b–g) successive derivatives of the Gaussian function, with (b) indicating the first derivative and each subsequent letter corresponding to an increase of one derivative order, (h) Haar wavelet, (i) Paul wavelet of order four.

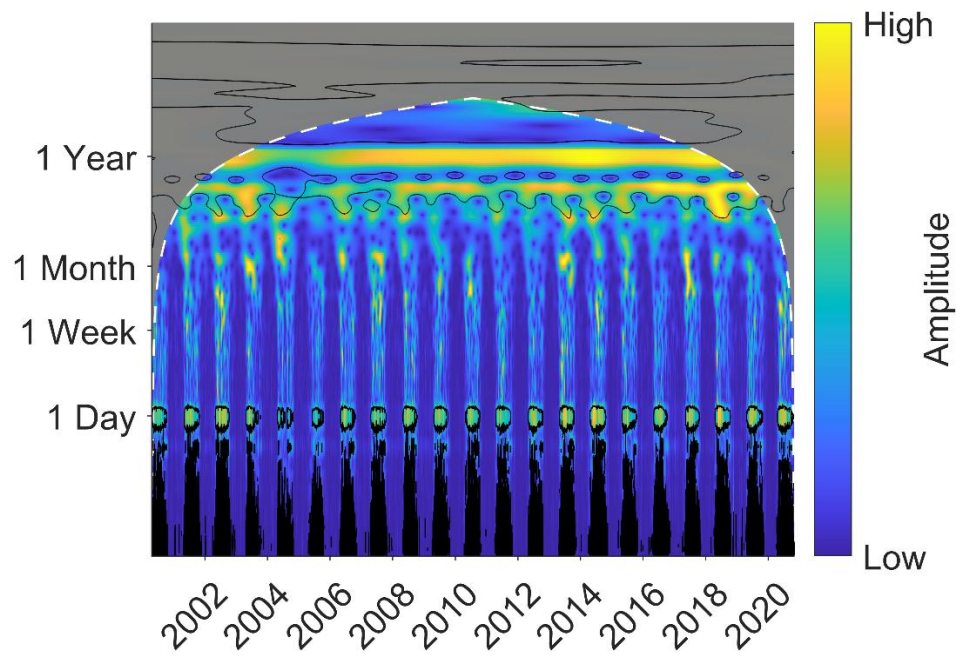


Figure S3: Scalogram depicted in Fig. 1d, for which coefficients are divided by their maximal value inside each period band.

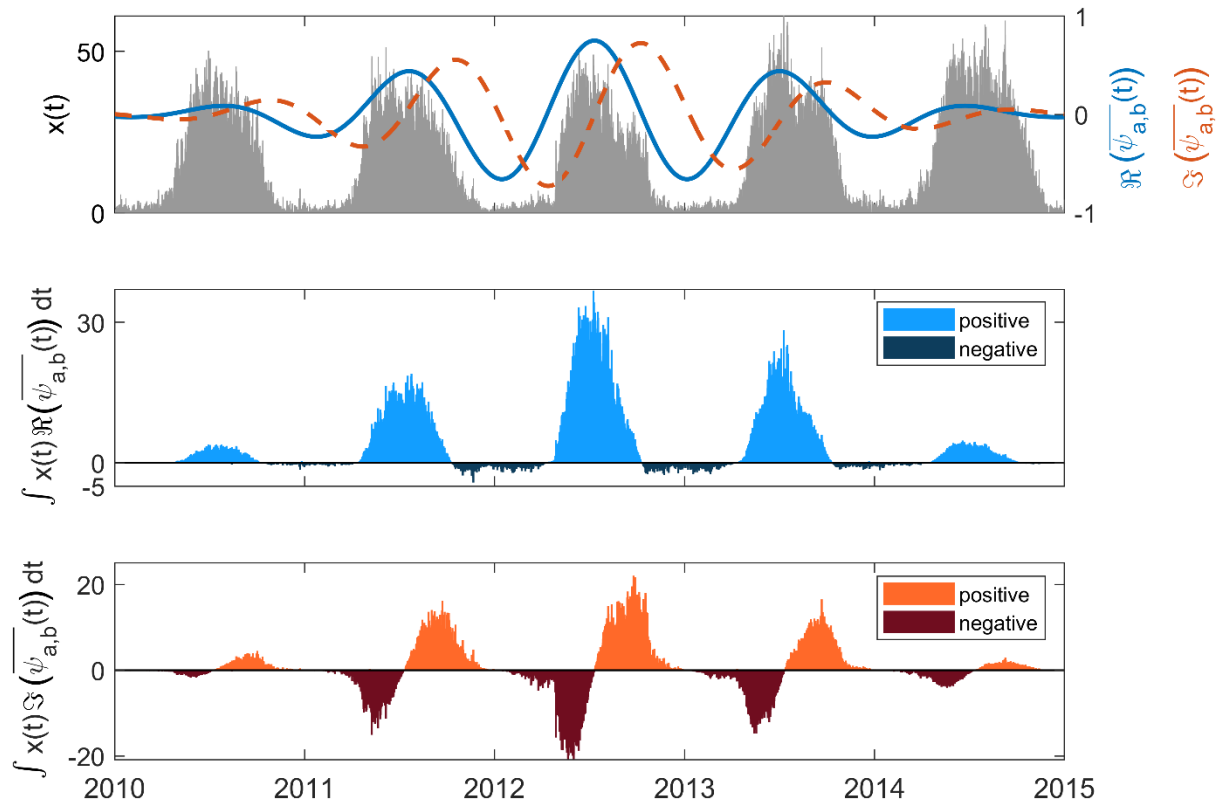


Figure S4: Detailed computation of a wavelet coefficient using the complex Morlet wavelet at a 1-year period. The blue color refers to the real part, while the red color denotes the imaginary part. In this example, the real part of the wavelet extracts the annual trend of the GPP signal. The imaginary part of the wavelet coefficient is close to zero. As the wavelet is translated, the relationship of information capture is inverted, yielding a consistently smoothed result on the 1-year period band throughout the translation process.

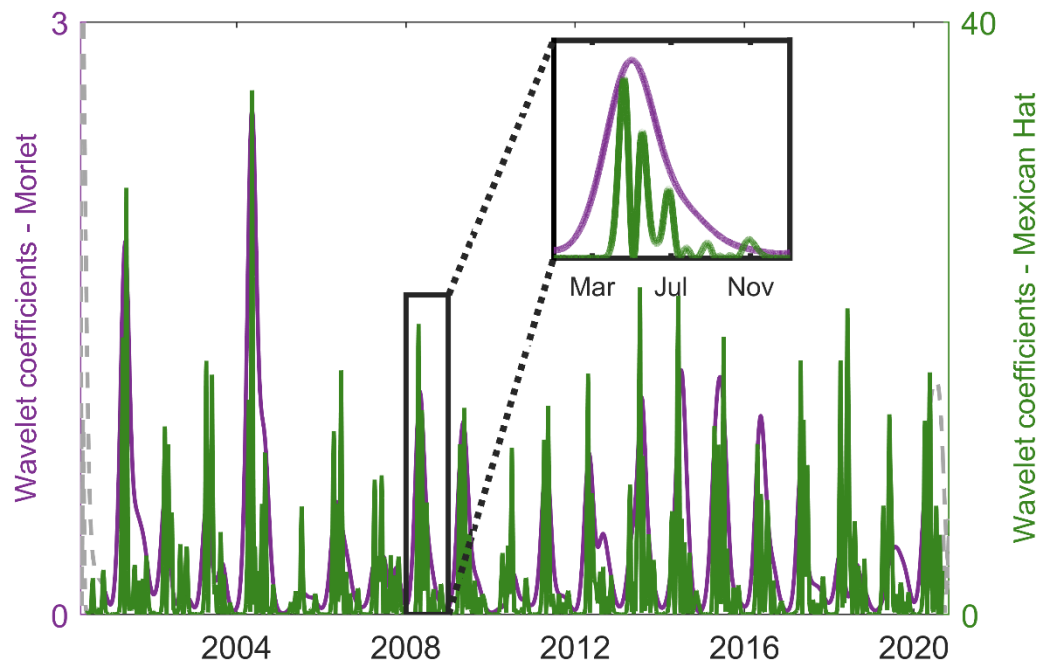


Figure S5: Amplitude of wavelet coefficients using the Morlet wavelet (purple) and the Mexican Hat wavelet (green) at the 2-month period band. Dashed lines represent coefficients affected by edge effects. The repeating peaks in March, June and November observed at the 6-month period are still discernible.

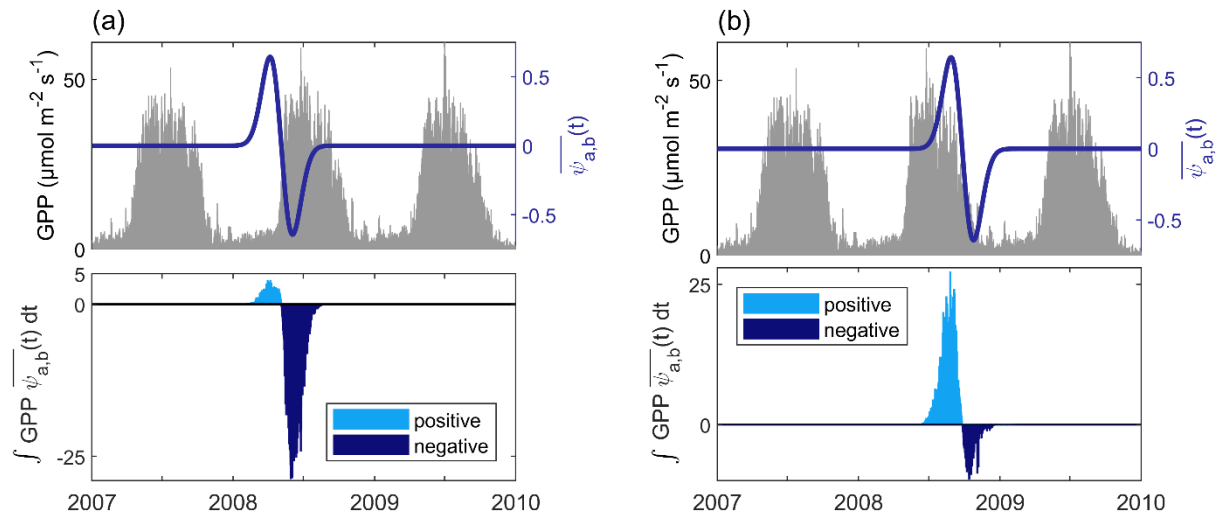


Figure S6: Computation steps for both wavelet coefficients involved in defining IDrop using the DOG1 wavelet. (a) April peak at a 6-month period, and (b) November peak at a 6-month period.

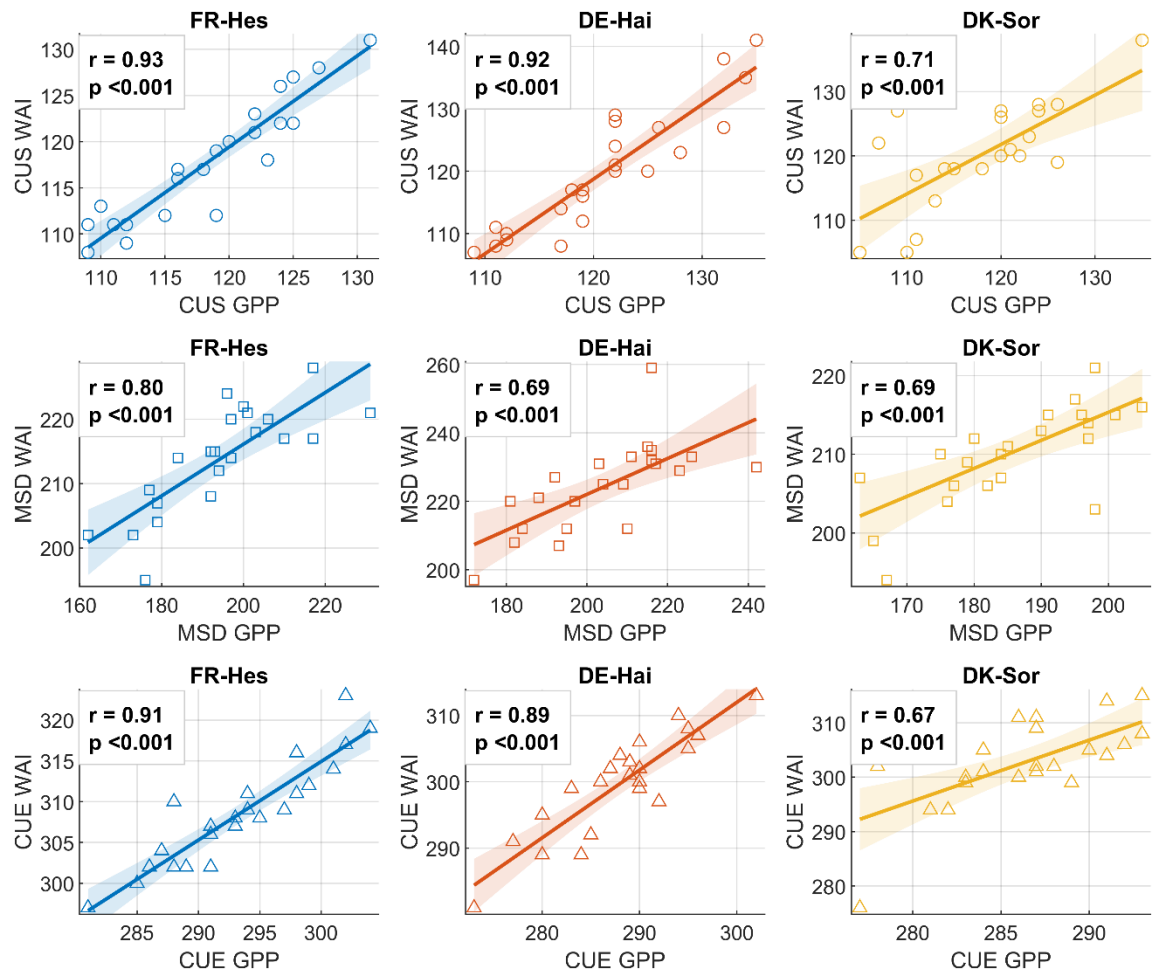


Figure S7: Benchmarking of three phenological metrics: dates of carbon uptake start (CUS), mid-season drop (MSD) and carbon uptake end (CUE). WAI-derived metrics (y-axis) are compared to dates estimated using thresholds on smoothed GPP series (x-axis) for three forest sites (FR-Hes, DE-Hai and DK-Sor). Units are expressed as day of year. See Methods for details.