Supplementary material (part 2) of "Seasonal and inter-annual variability of plankton chlorophyll and primary production in the Mediterranean Sea: a modelling approach"

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1 Introduction

This document presents the list of equations solved by the OPATM-BFM model as referred in Lazzari et al. (this issue). The equation formulation follows the formalism and style proposed by Vichi et al. (2007).

2 Phytoplankton

The phytoplankton state variables are connected to 5 basic biogeochemical constituents (C, N, P, Si) and 1 chemical functional family (Chl) and thus for each group we have up to 5 equations:

$$
\begin{aligned}
\left.\frac{\partial P_c}{\partial t}\right|_{bio} &= \left.\frac{\partial P_c}{\partial t}\right|_{O^{(3)}}^{gpp} - \left.\frac{\partial P_c}{\partial t}\right|_{R_c^{(i)}}^{exu} - \left.\frac{\partial P_c}{\partial t}\right|_{O^{(3)}}^{rsp} - \left.\frac{\partial P_c}{\partial t}\right|_{R_c^{(i)}}^{lys} - \sum_j \left.\frac{\partial P_c}{\partial t}\right|_{Z_c^{(j)}}^{prd} && (1)\\
\left.\frac{\partial P_n}{\partial t}\right|_{bio} &= \left.\frac{\partial P_n}{\partial t}\right|_{N^{(3)},N^{(4)}}^{upt} - \left.\frac{\partial P_n}{\partial t}\right|_{R_n^{(i)}}^{lys} - \frac{P_n}{P_c}\sum_j \left.\frac{\partial P_c}{\partial t}\right|_{Z_c^{(j)}}^{prd} && (2)\\
\left.\frac{\partial P_p}{\partial t}\right|_{bio} &= \left.\frac{\partial P_p}{\partial t}\right|_{N^{(1)}}^{upt} - \left.\frac{\partial P_p}{\partial t}\right|_{R_p^{(i)}}^{lys} - \frac{P_p}{P_c}\sum_j \left.\frac{\partial P_c}{\partial t}\right|_{Z_c^{(j)}}^{prd} && (3)\\
\left.\frac{\partial P_s}{\partial t}\right|_{bio} &= \left.\frac{\partial P_s}{\partial t}\right|_{N^{(5)}}^{upt} - \left.\frac{\partial P_s}{\partial t}\right|_{R_s^{(6)}}^{lys} - \frac{P_s}{P_c}\sum_j \left.\frac{\partial P_c}{\partial t}\right|_{Z_c^{(j)}}^{prd} && (4)\\
&\text{if } P_s = P_s^{(1)},\ \text{otherwise } \left.\frac{\partial P_s}{\partial t}\right|_{bio} = 0 \\
\left.\frac{\partial P_l}{\partial t}\right|_{bio} &= \left.\frac{\partial P_l}{\partial t}\right|^{syn} - \frac{P_l}{P_c}\sum_j \left.\frac{\partial P_c}{\partial t}\right|_{Z_c^{(j)}}^{prd} && (5)
\end{aligned}
$$

2.1 Regulating factors

2.1.1 Temperature

$$f_P^T = Q_{10_P}^{\frac{T-10}{10}} \tag{6}$$

2.1.2 Nutrients

$$f_P^{n,p} = \min\left(\frac{P_n/P_c - n_P^{\min}}{n_P^{opt} - n_P^{\min}}, \frac{P_p/P_c - p_P^{\min}}{p_P^{opt} - p_P^{\min}}\right) \tag{7}$$

$$\begin{aligned} f_{P(1)}^s &= \frac{P_s/P_c - s_P^{\min}}{s_P^{opt} - s_P^{\min}} \\ f_{P(j)}^s &= 1, \quad j \neq 1 \end{aligned} \tag{8}$$

2.1.3 Light

$$E_{PAR} = E_{PAR}^k \exp\left(-\lambda^k \frac{\Delta z^k}{2}\right) \tag{9}$$

$$f_P^E = 1 - \exp\left(-\frac{\alpha_{chl}^0 E_{PAR} P_l}{f_P^T f_P^s r_P^0 P_c}\right). \tag{10}$$

derived from

$$f_P^E = 1 - \exp\left(-\frac{E_{PAR}}{E_K}\right)$$

$$\begin{aligned} E_k &= P_m^*/\alpha^* \\ P_m^* &= f_P^T f_P^s r_P^0 \frac{P_c}{P_l} \\ \alpha^* &= \alpha_{chl}^0 \end{aligned}$$

2.2 Photosynthesis

$$\left.\frac{\partial P_c}{\partial t}\right|_{O^{(3)}}^{gpp} = f_P^T f_P^E f_P^s r_P^0 P_c, \tag{11}$$

2.3 Exudation

$$\left.\frac{\partial P_c}{\partial t}\right|_{R_c^{(1)}}^{exu} = \beta_P \left.\frac{\partial P_c}{\partial t}\right|_{O^{(3)}}^{gpp} \tag{12}$$

2.4 Respiration

$$\left.\frac{\partial P_c}{\partial t}\right|_{O^{(3)}}^{rsp} = b_P f_P^T P_c + \gamma_P \left\{ \left.\frac{\partial P_c}{\partial t}\right|_{O^{(3)}}^{gpp} - \left.\frac{\partial P_c}{\partial t}\right|_{R_c^{(1)}}^{exu} \right\} \tag{13}$$

2.5 Lysis

$$\left.\frac{\partial P_c}{\partial t}\right|_{R_c^{(1)}}^{lys} = (1-\varepsilon_P^{n,p}) \frac{h_P^{p,n}}{f_P^{p,n} + h_P^{p,n}} d_P^{\circ} P_c \tag{14}$$

$$\left.\frac{\partial P_c}{\partial t}\right|_{R_c^{(6)}}^{lys} = \varepsilon_P^{n,p} \frac{h_P^{p,n}}{f_P^{p,n} + h_P^{p,n}} d_P^{\circ} P_c \tag{15}$$

$$\varepsilon_P^{n,p} = \min\left(1, \frac{p_P^{\min}}{P_p/P_c}, \frac{n_P^{\min}}{P_n/P_c}\right)$$

For $P^{(4)}$ there is an extra lysis rate ($d^{\circ}_{P^{(4)}}$):

$$\left.\frac{\partial P_c^{(4)}}{\partial t}\right|_{R_c^{(1)}}^{lys} = (1-\varepsilon_P^{n,p}) \frac{h_P^{p,n}}{f_P^{p,n} + h_P^{p,n}} d_P^{\circ} P_c^{(4)} + d^{\circ}_{P^{(4)}} \tag{16}$$

$$d^{\circ}_{P^{(4)}_c} = \frac{P_c^{(4)}}{P_c^{(4)} + 100} \tag{17}$$

2.6 Nutrient uptake

$$\left.\frac{\partial P_p}{\partial t}\right|_{N^{(1)}}^{upt} = \min\left(a_P^1 N^{(1)} P_c,\, G_P p_P^{\max} + f_P^T r_P^0 \left(p_P^{\max} - \frac{P_p}{P_c}\right) P_c\right) \tag{18}$$

$$\left.\frac{\partial P_n}{\partial t}\right|_{N^{(3)},N^{(4)}}^{upt} = \min\left(\left(a_P^3 N^{(3)} + a_P^4 N^{(4)}\right) P_c,\, G_P n_P^{\max} + f_P^T r_P^0 \left(n_P^{\max} - \frac{P_n}{P_c}\right) P_c\right) \tag{19}$$

$$a_P^3 = a_P \frac{l_{N4}}{l_{N4} + N^{(4)}} \tag{20}$$

$$a_P^4 = a_P \tag{21}$$

If the nitrogen uptake rate (19) is positive, then the partitioning between $N^{(3)}$and $N^{(4)}$uptake is done using the ratios $\frac{a_P^3 N^{(3)}}{a_P^3 N^{(3)} + a_P^4 N^{(4)}}$and $\frac{a_P^4 N^{(4)}}{a_P^3 N^{(3)} + a_P^4 N^{(4)}}$, respectively:

$$\left.\frac{\partial P_n}{\partial t}\right|_{N^{(3)}}^{upt} = \frac{a_P^3 N^{(3)}}{a_P^3 N^{(3)} + a_P^4 N^{(4)}} \left.\frac{\partial P_n}{\partial t}\right|_{N^{(3)},N^{(4)}}^{upt} \tag{22}$$

$$\left.\frac{\partial P_n}{\partial t}\right|_{N^{(4)}}^{upt} = \frac{a_P^4 N^{(4)}}{a_P^3 N^{(3)} + a_P^4 N^{(4)}} \left.\frac{\partial P_n}{\partial t}\right|_{N^{(3)},N^{(4)}}^{upt} \tag{23}$$

If it is negative, the whole flux is directed to the ammonia pool, phosphates and silicates

$$\left.\frac{\partial P_n}{\partial t}\right|_{N^{(4)}}^{rel} = \left.\frac{\partial P_n}{\partial t}\right|_{N^{(3)},N^{(4)}}^{upt} \tag{24}$$

$$\left.\frac{\partial P_p}{\partial t}\right|_{N^{(1)}}^{rel} = \left.\frac{\partial P_p}{\partial t}\right|_{N^{(1)}}^{upt} \tag{25}$$

$$\left.\frac{\partial P_s}{\partial t}\right|_{N^{(1)}}^{rel} = \left.\frac{\partial P_s}{\partial t}\right|_{N^{(5)}}^{upt} \tag{26}$$

$$\left.\frac{\partial P_s^{(1)}}{\partial t}\right|_{N^{(5)}}^{upt} = \max\left(0,\, s_{P^{(1)}}^{\max} G_{P^{(1)}}\right) - \max\left(0,\, \left(\frac{P_s^{(1)}}{P_c^{(1)}} - s_{P^{(1)}}^{opt}\right) P_c^{(1)}\right) \tag{27}$$

$$G_P = \left.\frac{\partial P_c}{\partial t}\right|_{O^{(3)}}^{gpp} - \left.\frac{\partial P_c}{\partial t}\right|_{R_c^{(i)}}^{exu} - \left.\frac{\partial P_c}{\partial t}\right|_{O^{(3)}}^{rsp} - \left.\frac{\partial P_c}{\partial t}\right|_{R_c^{(i)}}^{lys}$$

2.7 Nutrient release

$$\left.\frac{\partial P_p}{\partial t}\right|_{R_p^{(1)}}^{lys} = (1-\varepsilon_P^{n,p})\frac{h_P^{p,n}}{f_P^{p,n}+h_P^{p,n}} d_P^{\circ} P_p \tag{28}$$

$$\left.\frac{\partial P_p}{\partial t}\right|_{R_p^{(6)}}^{lys} = \varepsilon_P^{n,p}\frac{h_P^{p,n}}{f_P^{p,n}+h_P^{p,n}} d_P^{\circ} P_p \tag{29}$$

$$\left.\frac{\partial P_n}{\partial t}\right|_{R_n^{(1)}}^{lys} = (1-\varepsilon_P^{n,p})\frac{h_P^{p,n}}{f_P^{p,n}+h_P^{p,n}} d_P^{\circ} P_n \tag{30}$$

$$\left.\frac{\partial P_n}{\partial t}\right|_{R_n^{(6)}}^{lys} = \varepsilon_P^{n,p}\frac{h_P^{p,n}}{f_P^{p,n}+h_P^{p,n}} d_P^{\circ} P_n \tag{31}$$

$$\left.\frac{\partial P_s}{\partial t}\right|_{R_s^{(6)}}^{lys} = \frac{h_P^{p,n}}{f_P^{p,n}+h_P^{p,n}} d_P^{\circ} P_s \tag{32}$$

$$\varepsilon_P^{n,p} = \min\left(1, \frac{p_P^{\min}}{P_p/P_c}, \frac{n_P^{\min}}{P_n/P_c}\right)$$

2.8 Excrection of sugars, internal balance

$$\left.\frac{\partial P_c}{\partial t}\right|_{R_c^{(1)}}^{npp} = \left.\frac{\partial P_c}{\partial t}\right|_{O^{(3)}}^{gpp} - \left.\frac{\partial P_c}{\partial t}\right|_{R_c^{(1)}}^{exu} - \left.\frac{\partial P_c}{\partial t}\right|_{R_c^{(1)}}^{rsp} \tag{33}$$

$$\left.\frac{\partial P_c}{\partial t}\right|_{R_c^{(2)}}^{cor} = \left.\frac{\partial P_c}{\partial t}\right|_{R_c^{(1)}}^{npp} - \max\left(0, \min\left(\left.\frac{\partial P_c}{\partial t}\right|_{R_c^{(1)}}^{npp}, \frac{1}{p_P^{\min}}\left.\frac{\partial P_n}{\partial t}\right|_{N^{(3)},N^{(4)}}^{upt}, \frac{1}{n_P^{\min}}\left.\frac{\partial P_p}{\partial t}\right|_{N^{(1)}}^{upt}\right)\right) \tag{34}$$

2.9 Chlorophyll

$$\left.\frac{\partial P_l}{\partial t}\right|^{syn} = f_P^{p,n}\rho_{chl}\left(\left.\frac{\partial P_c}{\partial t}\right|^{gpp}_{O^{(3)}} - \left.\frac{\partial P_c}{\partial t}\right|^{exu}_{R_c^{(i)}} - \left.\frac{\partial P_c}{\partial t}\right|^{rsp}_{O^{(3)}} - \left.\frac{\partial P_c}{\partial t}\right|^{cor}_{R_c^{(i)}}\right) - \max\left(0, d^{\circ}_{P_l}(1-f_P^{p,n})\right)P_l + \\ + \min\left(0, \left.\frac{\partial P_c}{\partial t}\right|^{npp}_{R_c^{(1)}}\right) * \max\left(0, P_l - \theta^0_{chl}P_c\right) \tag{35}$$

$$\rho_{chl} = \theta^0_{chl}\frac{\left.\frac{\partial P_c}{\partial t}\right|^{gpp}_{O^{(3)}}}{\alpha^* E_{PAR}P_l} = \theta^0_{chl}\frac{f_P^T f_P^E f_P^s r_P^0 P_c}{\alpha^0_{chl}E_{PAR}P_l} \tag{36}$$

3 Bacterioplankton

The bacteria state variable is connected to 3 basic biogeochemical constituents, C, N and P, with 3 dynamical equations:

$$\left.\frac{\partial B_c}{\partial t}\right|_{bio} = \left.\frac{\partial B_c}{\partial t}\right|^{upt}_{R_c^{(i)}} - \left.\frac{\partial B_c}{\partial t}\right|^{rsp}_{O^{(3)}} - \sum_j \left.\frac{\partial B_c}{\partial t}\right|^{prd}_{Z_c^{(j)}} \tag{37}$$

$$\left.\frac{\partial B_n}{\partial t}\right|_{bio} = \frac{R_n^{(i)}}{R_c^{(i)}}\left.\frac{\partial B_c}{\partial t}\right|^{bcd}_{R_c^{(i)}} + \left.\frac{\partial B_n}{\partial t}\right|^{upt,rel}_{N^{(k)}} - \frac{B_n}{B_c}\sum_j \left.\frac{\partial B_c}{\partial t}\right|^{prd}_{Z_c^{(j)}} \tag{38}$$

$$\left.\frac{\partial B_p}{\partial t}\right|_{bio} = \frac{R_p^{(i)}}{R_c^{(i)}}\left.\frac{\partial B_c}{\partial t}\right|^{bcd}_{R_p^{(i)}} + \left.\frac{\partial B_p}{\partial t}\right|^{upt,rel}_{N^{(1)}} - \frac{B_p}{B_c}\sum_j \left.\frac{\partial B_c}{\partial t}\right|^{prd}_{Z_c^{(j)}} \tag{39}$$

3.1 Carbon uptake

$$\left.\frac{\partial B_c}{\partial t}\right|^{upt}_{R_c^{(1)}} = \left.\frac{\partial B_c}{\partial t}\right|^{upt}_{R_c^{(1)},R_c^{(2)},R_c^{(6)}}\frac{\nu_{_{R^{(1)}}}R_c^{(1)}}{sub\_avail} \tag{40}$$

$$\left.\frac{\partial B_c}{\partial t}\right|^{upt}_{R_c^{(2)}} = \left.\frac{\partial B_c}{\partial t}\right|^{upt}_{R_c^{(1)},R_c^{(2)},R_c^{(6)}}\frac{\nu_{_{R^{(2)}}}R_c^{(2)}}{sub\_avail} \tag{41}$$

$$\left.\frac{\partial B_c}{\partial t}\right|_{R_c^{(6)}}^{upt} = \left.\frac{\partial B_c}{\partial t}\right|_{R_c^{(1)},R_c^{(2)},R_c^{(6)}}^{upt} \frac{\nu_{_{R^{(6)}}} f_{_{R^{(6)}}}^{n,p} R_c^{(6)}}{sub\_avail} \tag{42}$$

where

$$\left.\frac{\partial B_c}{\partial t}\right|_{R_c^{(1)},R_c^{(2)},R_c^{(6)}}^{upt} = \min\left(f_{_B}^{n,p} Q_{10_B}^{\frac{T-10}{10}} r_{0_B} B_c, sub\_avail \right) \tag{43}$$

and *sub_avail* is the sub-strate availability:

$$sub\_avail = \nu_{_{R^{(1)}}} R_c^{(1)} + \nu_{_{R^{(2)}}} R_c^{(2)} + \nu_{_{R^{(6)}}} f_{_{R^{(6)}}}^{n,p} R_c^{(6)} \tag{44}$$

$$f_{_B}^{n,p} = \min\left(1, \frac{B_p/B_c}{p^{opt}}, \frac{B_n/B_c}{n^{opt}}\right); \quad f_{_{R^{(6)}}}^{n,p} = \min\left(1, \frac{R_p^{(6)}/R_c^{(6)}}{p^{opt}}, \frac{R_p^{(6)}/R_c^{(6)}}{n^{opt}}\right) \tag{45}$$

3.2 Respiration

$$\left.\frac{\partial B_c}{\partial t}\right|_{O^{(3)}}^{rsp} = b_{_B} f_B^T B_c + \left[1 - \eta_{_B} + \eta_{_B}^o (1 - f_B^o)\right] \left.\frac{\partial B_c}{\partial t}\right|_{R_c^{(1)},R_c^{(6)}}^{upt} \tag{46}$$

$$f_B^o = \frac{\left(O^{(2)}\right)^3}{\left(O^{(2)}\right)^3 + \left(h_{_B}^o\right)^3} \tag{47}$$

3.3 Mortality

$$\left.\frac{\partial B_c}{\partial t}\right|_{R_i^{(1)}}^{rel} = d_{_B}^0 f_B^T B_c \quad i = c, n, p \tag{48}$$

3.4 Nutrient uptake and release

Uptake from substrate (organic component)

$$\left.\frac{\partial B_p}{\partial t}\right|_{R_p^{(1)}}^{upt} = \frac{R_p^{(1)}}{R_c^{(1)}} \left.\frac{\partial B_c}{\partial t}\right|_{R_c^{(1)}}^{upt} \qquad \left.\frac{\partial B_p}{\partial t}\right|_{R_p^{(6)}}^{upt} = \frac{R_p^{(6)}}{R_c^{(6)}} \left.\frac{\partial B_c}{\partial t}\right|_{R_c^{(6)}}^{upt} \tag{49}$$

$$\left.\frac{\partial B_n}{\partial t}\right|_{R_n^{(1)}}^{upt} = \frac{R_n^{(1)}}{R_c^{(1)}} \left.\frac{\partial B_c}{\partial t}\right|_{R_c^{(1)}}^{upt} \qquad \left.\frac{\partial B_n}{\partial t}\right|_{R_n^{(6)}}^{upt} = \frac{R_n^{(6)}}{R_c^{(6)}} \left.\frac{\partial B_c}{\partial t}\right|_{R_c^{(6)}}^{upt} \tag{50}$$

for the uptake/release from inorganic substrate the difference between uptake rate of substrate organic P and rate of uptake of substrate C multiplied by the optimal ratio $p_{_B}^{opt}$ is considered, if we have phosphorus uptake higher than optimal phosphorus uptake based on carbon uptake:

$$\left.\frac{\partial B_p}{\partial t}\right|_{R_p^{(1)}}^{upt} + \left.\frac{\partial B_p}{\partial t}\right|_{R_p^{(6)}}^{upt} - p_{_B}^{opt}\left(\left.\frac{\partial B_c}{\partial t}\right|_{R_c^{(1)},R_c^{(2)},R_c^{(6)}}^{upt} - \left.\frac{\partial B_c}{\partial t}\right|_{O^{(3)}}^{rsp}\right) > 0 \tag{51}$$

there is a release of inorganic phosphorus equivalent to the surplus uptake:

$$\left.\frac{\partial B_p}{\partial t}\right|_{N^{(1)}}^{upt,rel} = -\left(\left.\frac{\partial B_p}{\partial t}\right|_{R_p^{(1)}}^{upt} + \left.\frac{\partial B_p}{\partial t}\right|_{R_p^{(6)}}^{upt} - p_{_B}^{opt}\left(\left.\frac{\partial B_c}{\partial t}\right|_{R_c^{(1)},R_c^{(2)},R_c^{(6)}}^{upt} - \left.\frac{\partial B_c}{\partial t}\right|_{O^{(3)}}^{rsp}\right)\right) \tag{52}$$

if eq.(51) is lower than zero, bacteria uptake from phosphate

$$\left.\frac{\partial B_p}{\partial t}\right|_{N^{(1)}}^{upt,rel} = min\left(a_P^1 N^{(1)} B_c,\, p_{_B}^{opt}\left(\left.\frac{\partial B_c}{\partial t}\right|_{R_c^{(1)},R_c^{(2)},R_c^{(6)}}^{upt} - \left.\frac{\partial B_c}{\partial t}\right|_{O^{(3)}}^{rsp}\right) - \left(\left.\frac{\partial B_p}{\partial t}\right|_{R_p^{(1)}}^{upt} + \left.\frac{\partial B_p}{\partial t}\right|_{R_p^{(6)}}^{upt}\right)\right) \tag{53}$$

Nitrogen fluxes in bacteria has a similar structure to phosphorus: for the uptake/release from inorganic substrate the difference between uptake rate of substrate organic N and rate of uptake of substrate C multiplied by the optimal ratio $n_{_B}^{opt}$ is considered, if we have nitrogen uptake higher than optimal nitrogen uptake based on carbon uptake:

$$\left.\frac{\partial B_n}{\partial t}\right|_{R_n^{(1)}}^{upt} + \left.\frac{\partial B_n}{\partial t}\right|_{R_n^{(6)}}^{upt} - n_B^{opt}\left(\left.\frac{\partial B_c}{\partial t}\right|_{R_c^{(1)},R_c^{(2)},R_c^{(6)}}^{upt} - \left.\frac{\partial B_c}{\partial t}\right|_{O^{(3)}}^{rsp}\right) \;>\; 0 \tag{54}$$

there is a release of ammonia equivalent to the surplus uptake:

$$\left.\frac{\partial B_p}{\partial t}\right|_{N^{(4)}}^{upt,rel} = -\left(\left.\frac{\partial B_n}{\partial t}\right|_{R_n^{(1)}}^{upt} + \left.\frac{\partial B_n}{\partial t}\right|_{R_n^{(6)}}^{upt} - n_B^{opt}\left(\left.\frac{\partial B_c}{\partial t}\right|_{R_c^{(1)},R_c^{(2)},R_c^{(6)}}^{upt} - \left.\frac{\partial B_c}{\partial t}\right|_{O^{(3)}}^{rsp}\right)\right) \tag{55}$$

if eq.(59) is lower than zero, bacteria uptake is done from nitrates and ammonia:

$$\begin{aligned}
\left.\frac{\partial B_p}{\partial t}\right|_{N^{(3)}}^{upt} &= min\left(\left(a_B^3 N^{(3)} + a_B^4 N^{(4)}\right) B_c,\, p_B^{opt}\left(\left.\frac{\partial B_c}{\partial t}\right|_{R_c^{(1)},R_c^{(2)},R_c^{(6)}}^{upt} - \left.\frac{\partial B_c}{\partial t}\right|_{O^{(3)}}^{rsp}\right) - \left(\left.\frac{\partial B_p}{\partial t}\right|_{R_p^{(1)}}^{upt} + \left.\frac{\partial B_p}{\partial t}\right|_{R_p^{(6)}}^{upt}\right)\right) \frac{a_B^3 N^{(3)}}{a_B^3 N^{(3)} + a_B^4 N^{(4)}} \\
\left.\frac{\partial B_p}{\partial t}\right|_{N^{(4)}}^{upt,rel} &= min\left(\left(a_B^3 N^{(3)} + a_B^4 N^{(4)}\right) B_c,\, p_B^{opt}\left(\left.\frac{\partial B_c}{\partial t}\right|_{R_c^{(1)},R_c^{(2)},R_c^{(6)}}^{upt} - \left.\frac{\partial B_c}{\partial t}\right|_{O^{(3)}}^{rsp}\right) - \left(\left.\frac{\partial B_p}{\partial t}\right|_{R_p^{(1)}}^{upt} + \left.\frac{\partial B_p}{\partial t}\right|_{R_p^{(6)}}^{upt}\right)\right) \frac{a_B^4 N^{(4)}}{a_B^3 N^{(3)} + a_B^4 N^{(4)}}
\end{aligned} \tag{56}$$

$$a_B^3 = a_B \frac{l_{N4}}{l_{N4} + N^{(4)}} \tag{57}$$

$$a_B^4 = a_B \tag{58}$$

3.5 Carbon correction

$$\left.\frac{\partial B_c}{\partial t}\right|_{R_c^{(7)}}^{cor} = \left.\frac{\partial B_c}{\partial t}\right|_{R_c^{(1)},R_c^{(2)},R_c^{(6)}}^{upt} - \left.\frac{\partial B_c}{\partial t}\right|_{O^{(3)}}^{rsp} - \min\left(\left.\frac{\partial B_c}{\partial t}\right|_{R_c^{(1)},R_c^{(2)},R_c^{(6)}}^{upt} - \left.\frac{\partial B_c}{\partial t}\right|_{O^{(3)}}^{rsp},\, \frac{\left.\frac{\partial B_n}{\partial t}\right|_{R_n^{(1)}}^{upt} + \left.\frac{\partial B_n}{\partial t}\right|_{R_n^{(6)}}^{upt} + \left.\frac{\partial B_n}{\partial t}\right|_{N^{(3)},N^{(4)}}^{upt,rel}}{n_B^{min}},\, \frac{\left.\frac{\partial B_p}{\partial t}\right|_{R_p^{(1)}}^{upt} + \left.\frac{\partial B_p}{\partial t}\right|_{R_p^{(6)}}^{upt} + \left.\frac{\partial B_p}{\partial t}\right|_{N^{(1)}}^{upt,rel}}{p_B^{min}}\right) \tag{59}$$

4 Zooplankton

$$\left.\frac{\partial Z_c}{\partial t}\right|_{bio} = \sum_{X=P,Z} \left.\frac{\partial Z_c}{\partial t}\right|_{X_c}^{prd} - \sum_{j=1,6} \left.\frac{\partial Z_c}{\partial t}\right|_{R_c^{(j)}}^{rel} - \left.\frac{\partial Z_c}{\partial t}\right|_{O^{(3)}}^{rsp} - \sum_{k=4,5,6} \left.\frac{\partial Z_c}{\partial t}\right|_{Z_c^{(k)}}^{prd} \quad (60)$$

$$\left.\frac{\partial Z_n}{\partial t}\right|_{bio} = \frac{F_n}{F_c} \sum_{X=P,Z} \left.\frac{\partial Z_c}{\partial t}\right|_{X_c}^{prd} - \sum_{j=1,6} \left.\frac{\partial Z_n}{\partial t}\right|_{R_n^{(j)}}^{rel} - \left.\frac{\partial Z_n}{\partial t}\right|_{N^{(4)}}^{rel} - \frac{Z_n}{Z_c} \sum_{k=4,5,6} \left.\frac{\partial Z_c}{\partial t}\right|_{Z_c^{(k)}}^{prd} \quad (61)$$

$$\left.\frac{\partial Z_p}{\partial t}\right|_{bio} = \frac{F_p}{F_c} \sum_{X=P,Z} \left.\frac{\partial Z_c}{\partial t}\right|_{X_c}^{prd} - \sum_{j=1,6} \left.\frac{\partial Z_p}{\partial t}\right|_{R_p^{(j)}}^{rel} - \left.\frac{\partial Z_p}{\partial t}\right|_{N^{(1)}}^{rel} - \frac{Z_p}{Z_c} \sum_{k=4,5,6} \left.\frac{\partial Z_c}{\partial t}\right|_{Z_c^{(k)}}^{prd} \quad (62)$$

4.1 Food availability

$$F_i = \sum_X \delta_{Z,X} e_{Z,X} X_i; \quad X_i \in \left\{ P_i^{(j)}, B_i, Z_i^{(j)} \right\}$$

where $\delta_{Z,X}$ is the availability of predator Z for prey X_i and $e_{Z,X}$ is the capture efficiency. $e_{Z,X} = \frac{X_c}{X_c+\mu_Z}$, μ_Z ($\mu_Z = 0$ for mesozooplankton).

4.2 Ingestion

$$\left.\frac{\partial Z_c}{\partial t}\right|_{X_c}^{prd} = -\left.\frac{\partial X_c}{\partial t}\right|_{Z_c}^{prd} = f_Z^T \, r_Z^0 \frac{\delta_{Z,X} e_{Z,X} X_c}{F_c} \frac{F_c}{F_c + h_Z^F} Z_c \quad (63)$$

which is rewritten in terms of the specific search volume in the case of mesozooplankton ($h_Z^F = \frac{r_{0Z}}{v_Z}$), because this parameter is generally available in the literature. Total ingestion rate:

$$\Im_i = \sum_X \left.\frac{\partial Z_i}{\partial t}\right|_{X_i}^{prd} \quad j = c,n,p. \quad (64)$$

4.3 Excretion/egestion

Microzooplankton formulations:

$$\left.\frac{\partial Z_c}{\partial t}\right|_{R_c^{(1)}}^{rel} = \varepsilon_Z^c \left(\beta_Z (1 - \eta_Z) \Im_c + \left(d_{0Z} f_Z^T + f_Z^o \right) Z_c \right) \quad (65)$$

$$\left.\frac{\partial Z_c}{\partial t}\right|_{R_c^{(6)}}^{rel} = \left(1-\varepsilon_Z^c\right)\left(\beta_Z\left(1-\eta_Z\right)\Im_c + \left(d_{0Z}\, f_Z^T + f_Z^o\right) Z_c\right) \tag{66}$$

$$f_Z^o = \min\left(1, \left(1-h_Z^O\right)\frac{\frac{O}{O^{sat}}}{\frac{O}{O^{sat}} + h_Z^O}\right) \tag{67}$$

for mesozooplankton:

$$\left.\frac{\partial Z_c}{\partial t}\right|_{R_c^{(6)}}^{rel} = \left(\beta_Z \Im_c + d_{0Z}\, f_Z^T Z_c + d_Z^{dns} Z_c^{\gamma_Z} + Q_Z^c\right) \tag{68}$$

The balancing flow of C Q_Z^c for the mesozooplankton is computed from the actual elemental ratios of ingested material:

$$\Gamma_Z^i = \frac{(1-\beta_Z)\Im_i}{\eta_Z \Im_c}, \quad i = n, p \tag{69}$$

the most limiting element is elected considering the lowest between :

$$\frac{\Gamma_Z^n}{\frac{Z_n}{Z_c}} \text{ and } \frac{\Gamma_Z^p}{\frac{Z_p}{Z_c}} \tag{70}$$

If nitrogen is most limiting and $\left(\Gamma_Z^n < \frac{Z_n}{Z_c}\right)$ then

$$Q_Z^c = \eta_Z \Im_c - \frac{(1-\beta_Z)}{n_Z^{opt}} \Im_n \tag{71}$$

or if phosphorus is most limiting and $\left(\Gamma_Z^p < \frac{Z_p}{Z_c}\right)$, then

$$Q_Z^c = \eta_Z \Im_c - \frac{(1-\beta_Z)}{p_Z^{opt}} \Im_p, \tag{72}$$

otherways $Q_Z^c = 0$.

4.4 Respiration

for microzooplankton

$$\left.\frac{\partial Z_c}{\partial t}\right|_{O^{(3)}}^{rsp} = (1-\beta_Z)(1-\eta_Z)\Im_c + b_Z f_Z^T Z_c \tag{73}$$

mesozooplankton:

$$\left.\frac{\partial Z_c}{\partial t}\right|_{O^{(3)}}^{rsp} = (1-\beta_Z-\eta_Z)\Im_c + b_Z f_Z^T Z_c \tag{74}$$

4.5 Organic nutrient excretion/egestion

for microzooplankton

$$\left.\frac{\partial Z_i}{\partial t}\right|_{R_i^{(1)}}^{rel} = \frac{Z_i}{Z_c}\min\left(\left.\frac{\partial Z_c}{\partial t}\right|_{R_c^{(1)}}^{rel} + \left.\frac{\partial Z_c}{\partial t}\right|_{R_c^{(6)}}^{rel}, \left.\frac{\partial Z_c}{\partial t}\right|_{R_c^{(1)}}^{rel} * 1.2\right) \tag{75}$$

$$\left.\frac{\partial Z_i}{\partial t}\right|_{R_i^{(6)}}^{rel} = \frac{Z_i}{Z_c}\left.\frac{\partial Z_c}{\partial t}\right|_{R_c^{(1)}}^{rel} - \left.\frac{\partial Z_i}{\partial t}\right|_{R_i^{(1)}}^{rel} \tag{76}$$

mesozooplankton excretes only in the particulate compartment:

$$\left.\frac{\partial Z_i}{\partial t}\right|_{R_i^{(6)}}^{rel} = \left(\beta_Z \Im_i + d_{0Z} f_Z^T Z_i + d_Z^{dns} Z_i^{\gamma_Z}\right) \tag{77}$$

4.6 Inorganic nutrient release

for microzooplankton:

$$\left.\frac{\partial Z_p}{\partial t}\right|_{N^{(1)}}^{rel} = \nu_Z^p \max\left(0, \frac{Z_p}{Z_c} - p_Z^{opt}\right) Z_p \tag{78}$$

$$\left.\frac{\partial Z_n}{\partial t}\right|_{N^{(4)}}^{rel} = \nu_Z^n \max\left(0, \frac{Z_n}{Z_c} - n_Z^{opt}\right) Z_n \tag{79}$$

for mesozooplankton

$$\left.\frac{\partial Z_p}{\partial t}\right|_{N^{(1)}}^{rel} = \left(d_{0_Z} f_Z^T Z_p + Q_Z^p\right) \tag{80}$$

The balancing flow of P Q_Z^p is computed from the actual elemental ratios of ingested material:

If we are in the condtion 1, limitation by carbon then

$$Q_Z^p = (1-\beta_Z)\Im_p - p_Z^{opt}\eta_Z\Im_c, \tag{81}$$

in case 2, limitiation by phosphorus, $Q_Z^p = 0$, and in case 3, limitation by nitrogen

$$Q_Z^p = (1-\beta_Z)\Im_p - p_Z^{opt}\eta_Z\left(\Im_c - Q_Z^c\right), \tag{82}$$

in the latter the correction on carbon grazing (net nitrogen limitation Q_Z^c case 3) is used as reference to calculate the balance for phosphorus internal quota.

Same arguments are valid for nitrogen internal quota:

$$\left.\frac{\partial Z_n}{\partial t}\right|_{N^{(4)}}^{rel} = \left(d_{0_Z} f_Z^T Z_n + Q_Z^n\right) \tag{83}$$

The balancing flow of P Q_Z^n is computed from the actual elemental ratios of ingested material:

If we are in the condtion 1, limitation by carbon then

$$Q_Z^n = (1-\beta_Z)\Im_n - n_Z^{opt}\eta_Z\Im_c \tag{84}$$

in case 2, limitiation by phosphorus:

$$Q_Z^n = (1-\beta_Z)\Im_n - n_Z^{opt}\eta_Z\left(\Im_c - Q_Z^c\right) \tag{85}$$

in the latter the correction on carbon grazing (net on phosphorus limitation Q_Z^c case 2) is used as reference to calculate the balance for nitrogen internal quota.

in the case 3, nitrogen limitation, $Q_Z^p = 0$

Symbol	$P^{(1)}$	$P^{(2)}$	$P^{(3)}$	$P^{(4)}$	**Description**
r_{0_P}	2.50	3.00	3.50	1.50	Maximum specific photosynthetic rate (d^{-1})
Q_{10_P}	2.00	2.00	2.00	2.00	Characteristic Q10 coefficient
$h^{s}_{P^{(1)}}$	1.00	-	-	-	Half saturation value for Si-limitation (mmolSi m^{-3})
b_P	0.10	0.05	0.10	0.10	Basal specific respiration rate (d^{-1})
γ_P	0.10	0.10	0.20	0.10	Activity respiration fraction (-)
β_P	0.05	0.20	0.20	0.20	Excreted fraction of primary production (-)
$h_P^{p,n,s}$	0.00	0.00	0.00	0.00	Nutrient stress threshold (-)
d_{0_P}	0.00	0.00	0.00	0.00	Maximum specific lysis rate (d^{-1})
a_1	$2.50\ 10^{-3}$	$2.50\ 10^{-3}$	$2.50\ 10^{-3}$	$2.50\ 10^{-3}$	Specific affinity constant for P (m^{-3} mg $C^{-1} d^{-1}$)
a_3	$2.50\ 10^{-2}$	$2.50\ 10^{-2}$	$2.50\ 10^{-2}$	$2.50\ 10^{-2}$	Specific affinity constant for N-NO_3 (m^{-3}mg C^{-1} d^{-1})
l_{N4}	1.00	0.50	0.10	1.00	Nitrate uptake limitation
$s^{opt}_{P^{(1)}}$	0.01	-	-	-	Standard Si:C ratio in diatoms (mmolSi mg C^{-1})
$\omega^{sink}_{P^{(1)}}$	5.00	-	-	2.50	Maximum sedimentation rate (m d^{-1})
$l^{sink}_{P^{(1)}}$	0.70	0.75	0.75	0.75	Nutrient stress threshold for sedimentation (-)
n_P^{min}, n_P^{opt}, n_P^{max}	$1.26\ 10^{-2}\times$ (0.55, 1, 2)	$1.26\ 10^{-2}\times$ (0.55, 1, 2)	$1.26\ 10^{-2}\times$ (0.55, 1, 2)	$1.26\ 10^{-2}\times$ (0.55, 1, 2)	Minimum, optimal and maximum nitrogen quota (mmolN mgC^{-1})
p_P^{min}, p_P^{opt}, p_P^{max}	$7.86\ 10^{-4}\times$(0.55, 1, 2)	$7.86\ 10^{-4}\times$(0.55, 1, 2)	$7.86\ 10^{-4}\times$(0.55, 1, 2)	$7.86\ 10^{-4}\times$(0.55, 1, 2)	Minimum, optimal and maximum phosphorus quota (mmolP mgC^{-1})
s_P^{min}, s_P^{opt}, s_P^{max}	$10^{-2}\times$(0.7, 1, 1.5)	-	-	-	Minimum, optimal and maximum Si quota (mmolP mgC^{-1})
α^0_{chl}	$1.38\ 10^{-5}$	$0.46\ 10^{-5}$	$1.52\ 10^{-5}$	$6.8\ 10^{-6}$	Maximum light utilization coefficient (mgC (mg chl)$^{-1}$ μE^{-1} m^2)
θ^0_{chl}	0.02	0.02	0.02	0.02	Maximum chl:C quotum (mg chl mg C^{-1})
$d^{\circ}_{P_l}$	0.2	0.2	0.2	0.2	Chl-specific turnover rate (d^{-1})

Table 1: Symbols, standard values and description of the phytoplankton parameters. $P^{(1)}$= diatoms; $P^{(2)}$= nanoflagellates; $P^{(3)}$= picophytoplankton; $P^{(4)}$= dinoflagellates

References

[1] Vichi, M., Pinardi, N., and Masina, S.: A generalized model of pelagic biogeochemistry for the global ocean ecosystem, Part I: Theory, J. Marine Syst., 64, 89–109, 2007.

Symbol	$Z^{(3)}$	$Z^{(4)}$	$Z^{(5)}$	$Z^{(6)}$	**Description**
Q_{10_Z}	2.00	2.00	2.00	2.00	Q10 coefficient (-)
h_Z^F	$r_{0_Z} \setminus v_Z$	$r_{0_Z} \setminus v_Z$	30.0	100	Michaelis constant for total food ingestion (mg C m^{-3})
μ_Z	0.00	0.00	50.0	50.0	Feeding threshold (mg C m^{-3})
r_{0_Z}	2.00	2.00	2.00	5.0	Potential specific growth rate (d^{-1})
v_Z	0.008	0.02	-	-	Specific search volume (m^3 mg C $^{-1}$)
b_Z	0.01	0.02	0.02	0.02	Basal specific respiration rate (d^{-1})
η_Z	0.60	0.60	0.50	0.30	Assimilation efficency (-)
β_Z	0.3	0.35	0.50	0.50	Excreted fraction of uptake (-)
ε_Z^c	0.00	0.00	0.70	0.70	Partition between dissolved and particulate excretion of C (-)
ε_Z^n	0.00	0.00	check	check	Partition between dissolved and particulate excretion of N (-)
ε_Z^p	0.00	0.00	check	check	Partition between dissolved and particulate excretion of P (-)
n_Z^{opt}, p_Z^{opt}	0.015, 0.00167	0.015, 0.00167	0.0167, 0.00185	0.0167, 0.00185	Maximum nutrient quota (mmolN mgC^{-1}, mmolP mgC^{-1})
ν_Z	-	-	0.5	0.5	Specific rate of nutrients and carbon excretion (d^{-1})
d_{0_Z}	0.01	0.01	0.00	0.00	Specific mortality rate (d^{-1})
d_Z^{dns}	0.0004	0.0004	-	-	Density-dependent specific mortality rate (m^3 mgC^{-1} d^{-1})
γ_Z	2.00	2.00	-	-	Exponent for density dependent mortality (-)

Table 2: Symbols, standard values and description of the zooplankton parameters.$Z^{(3)}$= mesozooplankton carnivorous ; $Z^{(4)}$= mesozooplankton omnivorous ; $Z^{(5)}$=microzooplankton; $Z^{(6)}$= heterotrophic nanoflagellates.

Symbol	*Value*	**Description**
Q_{10_B}	2.95	Characteristic Q10 coefficient
h_B^o	30.0	Half saturation value for oxygen limitation ($mmolO_2$ m^{-3})
r_{0_B}	8.38	Potential specific growth rate (d^{-1})
b_B	0.01	Basal specific respiration rate (d^{-1})
η_B	0.40	Assimilation efficiency (-)
η_B^o	0.20	Decrease in assimilation efficiency under anoxic conditions(-)
d_{0_B}	0.00	Specific mortality rate (d^{-1})
ν_R^1	0.5	Specific potential $R^{(1)}$ uptake (d^{-1})
ν_R^2	0.25	Specific potential $R^{(2)}$ uptake (d^{-1})
ν_R^6	0.1	Specific potential $R^{(6)}$ uptake (d^{-1})
n_B^{opt}, p_B^{opt}	0.017, 0.0019	Optimal nutrient quota (mmolN mgC^{-1}, mmolP mgC^{-1})
n_B^{min}, p_B^{min}	0.0085, 0.00095	Minimum nutrient quota (mmolN mgC^{-1}, mmolP mgC^{-1})

Table 3: Symbols, standard values and description of the bacterioplankton parameters.