

## SUPPLEMENTARY MATERIAL

### Model Development

#### *General*

*Ecosys* is an hourly time-step model with multiple canopy and soil layers that provide a framework for different plant and microbial populations to acquire, transform and exchange resources (energy, water, C, N and P). The model is constructed from algorithms representing basic physical, chemical and biological processes that determine process rates in plant and microbial populations interacting within complex biomes. These algorithms interact to simulate complex ecosystem behaviour across a wide range of spatial and biological scales. The model is designed to represent terrestrial ecosystems under range of natural and anthropogenic disturbances and environmental changes at patch (spatially homogenous one-dimensional) and landscape (spatially variable two- or three-dimensional) scales. A comprehensive description of *ecosys* with a detailed listing of inputs, outputs, governing equations, parameters, results and references can be found in Grant (2001). A more detailed description of model algorithms and parameters most relevant to simulating temperature, water and nutrient effects on *NEP* is given below, with reference to equations and variable definitions in Appendices A, B, C and D below.

### **Appendix A: Soil C, N and P Transformations**

#### *Decomposition*

Organic transformations in *ecosys* occur in five organic matter–microbe complexes (coarse woody litter, fine non-woody litter, animal manure, particulate organic matter (POM), and humus) in each soil layer. Each complex consists of five organic states: solid organic matter *S*, dissolved organic matter *Q*, sorbed organic matter *A*, microbial biomass *M*, and microbial residues *Z*, among which C, N, and P are transformed. Organic matter in litter and manure complexes are partitioned from proximate analysis results into carbohydrate, protein, cellulose, and lignin components of differing vulnerability to hydrolysis. Organic matter in POM, humus, microbial biomass and microbial residues in all complexes are also partitioned into components of differing vulnerability to hydrolysis.

The rate at which each component of each organic state in each complex is hydrolyzed during decomposition is a first-order function of the decomposer biomass *M* of all heterotrophic microbial populations [A1]. Decomposer biomasses are redistributed among complexes from active biomasses according to biomass – substrate concentration differences (priming) [A3]. The rate at which each component is hydrolyzed is also a Monod function of substrate concentration [A3,A5], calculated from the fraction of substrate mass colonized by *M* [A4]. Hydrolysis rates are controlled by  $T_s$  through an Arrhenius function [A6] and by soil water content ( $\theta$ ) through its effect on aqueous microbial concentrations [*M*] [A3,A5] in surface litter and in a spatially resolved soil profile.  $T_s$  and  $\theta$  are calculated from surface energy balances and from heat and water transfer schemes through canopy–snow–residue–soil profiles as described in *Energy*

*Exchange* above. Release of N and P from hydrolysis of each component in each complex is determined by its N and P concentrations [A7] which are determined from those of the originating litterfall as described in *Autotrophic Respiration and Growth* above. Most non-lignin hydrolysis products are released as dissolved organic C, N and P (DOC, DON, and DOP) which are adsorbed or desorbed according to a power function of their soluble concentrations [A8 – A10].

### *Microbial Growth*

The DOC decomposition product is the substrate for heterotrophic respiration ( $R_h$ ) by all  $M$  in each substrate-microbe complex [A13]. Total  $R_h$  for all soil layers [A11] drives CO<sub>2</sub> emission from the soil surface through volatilization and diffusion.  $R_h$  may be constrained by microbial N or P concentrations,  $T_s$ , DOC and O<sub>2</sub> [A12 - A14]. O<sub>2</sub> uptake by  $M$  is driven by  $R_h$  [A16] and constrained by O<sub>2</sub> diffusivity to microbial surfaces [A17], as described for roots in *Autotrophic Respiration and Growth* above. Thus  $R_h$  is coupled to O<sub>2</sub> reduction by all aerobic  $M$  according to O<sub>2</sub> availability.  $R_h$  not coupled with O<sub>2</sub> reduction is coupled with the sequential reduction of NO<sub>3</sub><sup>-</sup>, NO<sub>2</sub><sup>-</sup>, and N<sub>2</sub>O by heterotrophic denitrifiers, and with the reduction of organic C by fermenters and acetotrophic methanogens. In addition, autotrophic nitrifiers conduct NH<sub>4</sub><sup>+</sup> and NO<sub>2</sub><sup>-</sup> oxidation, and NO<sub>2</sub><sup>-</sup> reduction, and autotrophic methanogens and methanotrophs conduct CH<sub>4</sub> production and oxidation.

All microbial populations undergo maintenance respiration  $R_m$  [A18,A19], depending on microbial N and  $T_s$  as described earlier for plants.  $R_h$  in excess of  $R_m$  is used in growth respiration  $R_g$  [A20], the energy yield  $\Delta G$  of which drives growth in biomass  $M$  from DOC uptake according to the energy requirements of biosynthesis [A21, A22].  $R_m$  in excess of  $R_h$  causes microbial decay.  $M$  also undergoes first-order decay  $D_m$  [A23]. Internal retention and recycling of microbial N and P during decay [A24] is modelled whenever these nutrients constrain  $R_h$  [A12]. Changes in  $M$  arise from differences between gains from DOC uptake and losses from  $R_m + R_g + D_m$  [A25].

### *Microbial Nutrient Exchange*

During these changes, all microbial populations seek to maintain set minimum ratios of C:N or C:P in  $M$  by mineralizing or immobilizing NH<sub>4</sub><sup>+</sup>, NO<sub>3</sub><sup>-</sup>, and H<sub>2</sub>PO<sub>4</sub><sup>-</sup> [A26], thereby controlling solution [NH<sub>4</sub><sup>+</sup>], [NO<sub>3</sub><sup>-</sup>] and [H<sub>2</sub>PO<sub>4</sub><sup>-</sup>] that determine root and mycorrhizal uptake in *Nutrient Uptake and Translocation* above. If immobilization is inadequate to maintain these minimum ratios, then biomass C:N or C:P may rise, but  $R_h$  is constrained by N or P present in the lowest concentration with respect to that at the minimum ratio [A12]. Non-symbiotic heterotrophic diazotrophs can also fix aqueous N<sub>2</sub> [A27] to the extent that immobilization is inadequate to maintain their set minimum C:N, but at an additional respiration cost [A28]. Changes in microbial N and P arise from DON and DOP uptake plus NH<sub>4</sub><sup>+</sup>, NO<sub>3</sub><sup>-</sup>, and H<sub>2</sub>PO<sub>4</sub><sup>-</sup> immobilization and N<sub>2</sub> fixation, less NH<sub>4</sub><sup>+</sup>, NO<sub>3</sub><sup>-</sup>, and H<sub>2</sub>PO<sub>4</sub><sup>-</sup> mineralization and microbial N and P decomposition [A29].

### *Humification*

C, N and P decomposition products in each organic matter–microbe complex are gradually stabilized into more recalcitrant organic forms with lower C:N and C:P ratios. Products from lignin hydrolysis [A1,A7] combine with some of the products from protein and carbohydrate hydrolysis in the litterfall and manure complexes and are transferred to the POM complex [A31–A34]. Microbial decomposition products [A23, A24] from all complexes are partitioned between the humus complex and microbial residues in the originating complex according to soil clay content [A35, A36].

## **Appendix B: Soil-Plant Water Relations**

### *Canopy Transpiration*

Canopy energy exchange in *ecosys* is calculated from an hourly two-stage convergence solution for the transfer of water and heat through a multi-layered multi-population soil-root-canopy system. The first stage of this solution requires convergence to a value of canopy temperature  $T_c$  for each plant population at which the first-order closure of the canopy energy balance (net radiation  $R_n$ , latent heat flux  $LE$  [B1a,b,c], sensible heat flux  $H$  [B1d], and change in heat storage  $G$ ) is achieved. These fluxes are controlled by aerodynamic ( $r_a$ ) [B3] and canopy stomatal ( $r_c$ ) [B2] resistances. Two controlling mechanisms are postulated for  $r_c$  which are solved in two successive steps:

- (1) At the leaf level, leaf resistance  $r_l$  [C4] controls gaseous  $CO_2$  diffusion through each leaf surface when calculating  $CO_2$  fixation [C1] from concurrent solutions for diffusion  $V_g$  [C2] and carboxylation  $V_c$  [C3]. The value of  $r_l$  is calculated from a minimum leaf resistance  $r_{lmin}$  [C5] for each leaf surface that allows a set ratio for intercellular to canopy  $CO_2$  concentration  $C_i':C_b$  to be maintained at  $V_c$  under ambient irradiance, air temperature  $T_a$ ,  $C_a$  and zero canopy water potential ( $\psi_c$ ) ( $V_c'$ ). This ratio will be allowed to vary diurnally as described in *Gross Primary Productivity* below when  $\psi_c$  is solved in the second stage of the convergence solution, described under *Water Relations* below. Values of  $r_{lmin}$  are aggregated by leaf surface area to a canopy value  $r_{cmin}$  for use in the energy balance convergence scheme [B2a].
- (2) At the canopy level,  $r_c$  rises from  $r_{cmin}$  at zero  $\psi_c$  from step (1) above through an exponential function of canopy turgor potential  $\psi_t$  [B2b] calculated from  $\psi_c$  and osmotic water potential  $\psi_\pi$  [B4] during convergence for transpiration vs. water uptake.

### *Root and Mycorrhizal Water Uptake*

Root and mycorrhizal water uptake  $U$  [B5] is calculated from the difference between canopy water potential  $\psi_c$  and soil water potential  $\psi_s$  across soil and root hydraulic resistances  $\Omega_s$  [B9] and  $\Omega_r$  [B10 – B12] in each rooted soil layer [B6]. Root resistances are calculated from root radial [B10] and from primary [B11] secondary [B12] axial resistivities using root lengths and surface areas from a root system submodel [B13] driven by exchange of nonstructural C, N and P along concentration gradients generated by uptake vs. consumption of C, N and P in shoots and roots (Grant, 1998).

### *Canopy Water Potential*

After convergence for  $T_c$  is achieved, the difference between canopy transpiration  $E_c$  from the energy balance [B1] and total root water uptake  $U_c$  [B5] from all rooted layers in the soil is tested against the difference between canopy water content from the previous hour and that from the current hour [B14]. This difference is minimized in each iteration by adjusting  $\psi_c$  which in turn determines each of the three terms in [B14]. Because  $r_c$  and  $T_c$  both drive  $E_c$ , the canopy energy balance described under *Canopy Transpiration* above is recalculated for each adjusted value of  $\psi_c$  during convergence.

## **Appendix C: Gross Primary Productivity, Autotrophic Respiration, Growth and Litterfall**

### *C<sub>3</sub> Gross Primary Productivity*

After successful convergence for  $T_c$  and  $\psi_c$  (described in *Plant Water Relations* above),  $V_c$  is recalculated from that under zero  $\psi_c$  ( $V_c'$ ) to that under ambient  $\psi_c$ . This recalculation is driven by stomatal effects on  $V_g$  [C2] from the increase in  $r_{\min}$  at zero  $\psi_c$  [C5] to  $r_c$  at ambient  $\psi_c$  [C4], and by non-stomatal effects  $f_{\psi}$  [C9] on CO<sub>2</sub>- and light-limited carboxylation  $V_b$  [C6] and  $V_j$  [C7] (Grant and Flanagan, 2007). The recalculation of  $V_c$  is accomplished through a convergence solution for  $C_i$  and its aqueous counterpart  $C_c$  at which  $V_g$  [C2] equals  $V_c$  [C3] (Grant and Flanagan, 2007). The CO<sub>2</sub> fixation rate of each leaf surface at convergence is added to arrive at a value for gross primary productivity (*GPP*) by each plant population in the model [C1]. The CO<sub>2</sub> fixation product is stored in nonstructural C pools  $\sigma_C$  in each branch.

GPP is strongly controlled by nutrient uptake  $U_{\text{NH}_4}$ ,  $U_{\text{NO}_3}$  and  $U_{\text{PO}_4}$  [C23], products of which are added to nonstructural N ( $\sigma_N$ ) and P ( $\sigma_P$ ) in root and mycorrhizal layers where they are coupled with  $\sigma_C$  to drive growth of branches, roots and mycorrhizae as described in *Growth and Senescence* below. Low  $\sigma_N:\sigma_C$  or  $\sigma_P:\sigma_C$  in branches indicate excess CO<sub>2</sub> fixation with respect to N or P uptake for phytomass growth. Such ratios in the model have two effects on GPP:

- (1) They reduce activities of rubisco [C6a] and chlorophyll [C7a] through product inhibition [C11], thereby simulating the suppression of CO<sub>2</sub> fixation by leaf  $\sigma_C$  accumulation widely reported in the literature.
- (2) They reduce the structural N:C and P:C ratios at which leaves are formed because  $\sigma_C$ ,  $\sigma_N$  and  $\sigma_P$  are the substrates for leaf growth. Lower structural ratios cause a proportional reduction in areal concentrations of rubisco [C6b] and chlorophyll [C7b], reducing leaf CO<sub>2</sub> fixation.

### *Autotrophic Respiration*

The temperature-dependent oxidation of these nonstructural pools ( $R_c$ ) [C14], plus the energy costs of nutrient uptake [C23], drive autotrophic respiration ( $R_a$ ) [C13] by all branches, roots and mycorrhizae.  $R_c$  by roots and mycorrhizae is constrained by O<sub>2</sub> uptake  $U_{\text{O}_2}$  [C14b] calculated by solving for aqueous O<sub>2</sub> concentrations at root and mycorrhizal surfaces [ $\text{O}_{2r}$ ] at which convection + radial diffusion through the soil aqueous phase plus radial diffusion through the root aqueous phase [C14d] equals active

uptake driven by  $O_2$  demand from  $R_c$  [C14c] (Grant, 2004). These diffusive fluxes are in turn coupled to volatilization – dissolution between aqueous and gaseous phases in soil and root [D14]. The diffusion processes are driven by aqueous  $O_2$  concentrations sustained by transport and dissolution of gaseous  $O_2$  through soil and roots (Grant 2004), and are governed by lengths and surface areas of roots and mycorrhizae (Grant, 1998). Thus  $R_c$  is coupled to  $O_2$  reduction by all root and mycorrhizal populations according to  $O_2$  availability.  $R_c$  is first used to meet maintenance respiration requirements ( $R_m$ ), calculated independently of  $R_c$  from the N content in each organ, and a function of  $T_c$  or  $T_s$  [C16]. Any excess of  $R_c$  over  $R_m$  is expended as growth respiration  $R_g$ , constrained by branch, root or mycorrhizal  $\psi_t$  [C17]. When  $R_m$  exceeds  $R_c$ , the shortfall is met by the respiration of remobilizable C ( $R_s$ ) in leaves and twigs or roots and mycorrhizae [C15].

### *Growth and Litterfall*

$R_g$  drives the conversion of branch  $\sigma_C$  into foliage, twigs, branches, boles and reproductive material according to organ growth yields  $Y_g$  and phenology-dependent partitioning coefficients [C20], and the conversion of root and mycorrhizal  $\sigma_C$  into primary and secondary axes according to root and mycorrhizal growth yields. Growth also requires organ-specific ratios of nonstructural N ( $\sigma_N$ ) and P ( $\sigma_P$ ) from  $U_{NH_4}$ ,  $U_{NO_3}$  and  $U_{PO_4}$  [C23] which are coupled with  $\sigma_C$  to drive growth of branches, roots and mycorrhizae.

The translocation of  $\sigma_C$ ,  $\sigma_N$  and  $\sigma_P$  among branches and root and mycorrhizal layers is driven by concentration gradients generated by production of  $\sigma_C$  from branch GPP and of  $\sigma_N$  and  $\sigma_P$  from root and mycorrhizal uptake vs. consumption of  $\sigma_C$ ,  $\sigma_N$  and  $\sigma_P$  from  $R_c$ ,  $R_g$  and phytomass growth (Grant 1998). Low  $\sigma_N:\sigma_C$  or  $\sigma_P:\sigma_C$  in mycorrhizae and roots indicates inadequate N or P uptake with respect to  $CO_2$  fixation. These ratios affect translocation of  $\sigma_C$ ,  $\sigma_N$  and  $\sigma_P$  by lowering mycorrhizal – root – branch concentration gradients of  $\sigma_N$  and  $\sigma_P$  while raising branch – root – mycorrhizal concentration gradients of  $\sigma_C$ . These changes slow transfer of  $\sigma_N$  and  $\sigma_P$  from root to branch and hasten transfer of  $\sigma_C$  from branch to root, increasing root and mycorrhizal growth at the expense of branch growth, and thereby raising N and P uptake [C23] with respect to  $CO_2$  fixation. Conversely, high  $\sigma_N:\sigma_C$  or  $\sigma_P:\sigma_C$  in roots and mycorrhizae indicate excess N or P uptake with respect to  $CO_2$  fixation. Such ratios reduce specific activities of root and mycorrhizal surfaces for N or P uptake through a product inhibition function as has been observed experimentally. These changes hasten transfer of  $\sigma_N$  and  $\sigma_P$  from root to branch and slow transfer of  $\sigma_C$  from branch to root, increasing branch growth at the expense of root and mycorrhizal growth, and thereby slowing N and P uptake. Thus the modelled plant translocates  $\sigma_C$ ,  $\sigma_N$  and  $\sigma_P$  among branches, roots and mycorrhizae to maintain a functional equilibrium between acquisition and use of C, N and P by different parts of the plant.

$R_g$  is limited by  $\psi_t$  [C17], and because branch  $\psi_t$  declines relatively more with soil drying than does root  $\psi_t$ , branch  $R_g$  also declines relatively more with soil drying than does root  $R_g$ , slowing oxidation of  $\sigma_C$  in branches and allowing more translocation of  $\sigma_C$  from branches to roots. This change in allocation of  $\sigma_C$  enables more root growth to

reduce  $\Omega_s$ ,  $\Omega_t$  and  $\Omega_a$ , and hence increase  $U$  [B6], thereby offsetting the effects of soil drying on  $\psi_t$ . Thus the modelled plant translocates  $\sigma_C$ ,  $\sigma_N$  and  $\sigma_P$  among branches, roots and mycorrhizae to maintain a functional equilibrium between acquisition and use of water.

$R_s$  [C15] drives the withdrawal of remobilizable C, N and P (mostly nonstructural protein) from leaves and twigs or roots and mycorrhizae into  $\sigma_N$  and  $\sigma_P$ , and the loss of associated non-remobilizable C, N and P (mostly structural) as litterfall [C18, C19a,b]. Provision is also made to withdraw remobilizable N or P from leaves and twigs or roots and mycorrhizae when ratios of  $\sigma_N:\sigma_C$  or  $\sigma_P:\sigma_C$  become smaller than those required for growth of new phytomass [C19c,d]. This withdrawal drives the withdrawal of associated remobilizable C, and the loss of associated non-remobilizable C, N and P as litterfall. Environmental constraints such as water, heat, nutrient or  $O_2$  stress that reduce  $\sigma_C$  and hence  $R_c$  with respect to  $R_m$  therefore hasten litterfall. In addition, concentrations of  $\sigma_C$ ,  $\sigma_N$  and  $\sigma_P$  in roots and mycorrhizae drive exudation of nonstructural C, N and P to DOC, DON and DOP in soil [C19e-i].

$R_a$  of each branch or root and mycorrhizal layer is the total of  $R_c$  and  $R_s$ , and net primary productivity (NPP) is the difference between canopy GPP [C1] and total  $R_a$  of all branches and root and mycorrhizal layers [C13]. Phytomass net growth is the difference between gains driven by  $R_g$  and  $Y_g$ , and losses driven by  $R_s$  and litterfall [C20]. These gains are allocated to leaves, twigs, wood and reproductive material at successive branch nodes, and to roots and mycorrhizae at successive primary and secondary axes, driving leaf expansion [C21a] and root extension [C21b]. Losses from remobilization and litterfall in shoots start at the lowest node of each branch at which leaves or twigs are present, and proceed upwards when leaves or twigs are lost. Losses in roots and mycorrhizae start with secondary axes and proceeds to primary axes when secondary axes are lost.

#### *Root and Mycorrhizal Nutrient Uptake*

Root and mycorrhizal uptake of N and P  $U_{NH_4}$ ,  $U_{NO_3}$  and  $U_{PO_4}$  is calculated by solving for solution  $[NH_4^+]$ ,  $[NO_3^-]$  and  $[H_2PO_4^-]$  at root and mycorrhizal surfaces at which radial transport by mass flow and diffusion from the soil solution to these surfaces [C23a,c,e] equals active uptake by the surfaces [C23b,d,f]. Path lengths and surface areas for  $U_{NH_4}$ ,  $U_{NO_3}$  and  $U_{PO_4}$  are calculated from a root and mycorrhizal growth submodel driven by exchange of nonstructural C, N and P along concentration gradients generated by uptake vs. consumption of C, N and P in shoots and roots (Grant, 1998). A product inhibition function is included to avoid uptake in excess of nutrient requirements [C23g].

#### *C<sub>4</sub> Gross Primary Productivity*

##### *C<sub>4</sub> Mesophyll*

In  $C_4$  plants, the mesophyll carboxylation rate is the lesser of  $CO_2$ - and light-limited reaction rates [C26] (Berry and Farquhar, 1978). The  $CO_2$ -limited rate is a Michaelis-Menten function of PEP carboxylase (PEPc) activity and aqueous  $CO_2$  concentration in the mesophyll [C29] parameterized from Berry and Farquhar (1978) and from Edwards and Walker (1983). The light-limited rate [C30] is a hyperbolic function of

absorbed irradiance and mesophyll chlorophyll activity [C31] with a quantum requirement based on 2 ATP from Berry and Farquhar (1978). PEPc [C32] and chlorophyll [C33] activities are calculated from specific activities multiplied by set fractions of leaf surface N density, and from functions of C<sub>4</sub> product inhibition (Jiao and Chollet, 1988; Lawlor, 1993) [C34],  $\psi_c$  ([C35] as described in Grant and Flanagan, 2007) and  $T_c$  [C36]. Leaf surface N density is controlled by leaf structural N:C and P:C ratios calculated during leaf growth from leaf non-structural N:C and P:C ratios arising from root N and P uptake (Grant, 1998) vs. CO<sub>2</sub> fixation.

#### *C<sub>4</sub> Mesophyll-Bundle Sheath Exchange*

Differences in the mesophyll and bundle sheath concentrations of the C<sub>4</sub> carboxylation product drive mesophyll-bundle sheath transfer (Leegood, 2000) [C37]. The bundle sheath concentration of the C<sub>4</sub> product drives a product-inhibited decarboxylation reaction (Laisk and Edwards, 2000) [C38], the CO<sub>2</sub> product of which generates a concentration gradient that drives leakage of CO<sub>2</sub> from the bundle sheath to the mesophyll [C39]. CO<sub>2</sub> in the bundle sheath is maintained in 1:50 equilibrium with HCO<sub>3</sub><sup>-</sup> (Laisk and Edwards, 2000). At this stage of model development, the return of a C<sub>3</sub> decarboxylation product from the bundle sheath to the mesophyll is not simulated. Parameters used in Eqs. [C37 – C39] allowed mesophyll and bundle sheath concentrations of C<sub>4</sub> carboxylation products from [C40 – C41] to be maintained at values consistent with those in Leegood (2000), bundle sheath concentrations of CO<sub>2</sub> (from Eq. [C42]) to be maintained at values similar to those reported by Furbank and Hatch (1987), and bundle sheath CO<sub>2</sub> leakiness [C39]), expressed as a fraction of PEP carboxylation, to be maintained at values similar to those in Williams et al. (2001), in sorghum as described in Grant et al. (2004).

#### *C<sub>4</sub> Bundle Sheath*

A C<sub>3</sub> model in which carboxylation is the lesser of CO<sub>2</sub>- and light-limited reaction rates (Farquhar et al., 1980) has been parameterized for the bundle sheath of C<sub>4</sub> plants [C43] from Seeman et al. (1984). The CO<sub>2</sub>-limited rate [C44] is a Michaelis-Menten function of RuBP carboxylase (RuBPc) activity and bundle sheath CO<sub>2</sub> concentration [C42]. The light-limited rate [C45a] is a hyperbolic function of absorbed irradiance and activity of chlorophyll associated with the bundle sheath with a quantum yield based on 3 ATP [C46]. The provision of reductant from the mesophyll to the bundle sheath in NADP-ME species is not explicitly simulated. RuBPc [C47] and chlorophyll [C48] activities are the products of specific activities and concentrations multiplied by set fractions of leaf surface N density, and from functions of C<sub>3</sub> product inhibition (Bowes, 1991; Stitt, 1991) [C49],  $\psi_c$  (Eq. A12 from Grant and Flanagan, 2007) and  $T_c$  [C36].

Rates of C<sub>3</sub> product removal are controlled by phytomass biosynthesis rates driven by concentrations of nonstructural products from leaf CO<sub>2</sub> fixation and from root N and P uptake. If biosynthesis rates are limited by nutrient uptake, consequent depletion of nonstructural N or P and accumulation of nonstructural C will constrain specific activities of RuBP and chlorophyll [C47 – C49], and thereby slow C<sub>3</sub> carboxylation [C43], raise bundle sheath CO<sub>2</sub> concentration [C42], accelerate CO<sub>2</sub> leakage [C39], slow C<sub>4</sub> decarboxylation [C38], raise C<sub>4</sub> product concentration in the bundle sheath [C41],

slow  $C_4$  product transfer from the mesophyll [C37], raise  $C_4$  product concentration in the mesophyll [C40], and slow mesophyll  $CO_2$  fixation [C32 – C35]. This reaction sequence simulates the progressive inhibition of  $C_3$  and  $C_4$  carboxylation hypothesized by Sawada et al. (2002) following partial removal of C sinks in  $C_4$  plants.

#### *Shoot – Root - Mycorrhizal C, N, P Transfer*

Shoot – root C transfers  $Z_{sC}$  are calculated such that concentrations of  $\sigma_C$  with respect to structural phytomass in each branch and root layer approach equilibrium according to conductances  $g_{sC}$  calculated from shoot – root distances and axis numbers in each root layer [C50] (Grant, 1998). Because  $\sigma_C$  is generated by  $CO_2$  fixation in branches [C1],  $g_{sC}$  cause shoot-to-root gradients of  $\sigma_C$  that drive  $Z_{sC}$ . Shoot – root N and P transfers  $Z_{sN,P}$  are calculated such that concentrations of  $\sigma_{N,P}$  with respect to  $\sigma_C$  in each branch and root layer approach equilibrium according to rate constants  $g_{sN,P}$  [C51]. Because  $\sigma_{N,P}$  are generated by uptake in roots [C23],  $g_{sN,P}$  cause root-to-shoot gradients of  $\sigma_{N,P}$  that drive  $Z_{sN,P}$ .

Similarly, root - mycorrhizal C transfers  $Z_{rC}$  are calculated such that concentrations of  $\sigma_C$  with respect to structural phytomass in each root and mycorrhizal layer approach equilibrium according to rate constants  $g_{rC}$  [C52] (Grant, 1998). Because  $\sigma_C$  is maintained by  $Z_{sC}$  [C50],  $g_{rC}$  cause root-to-mycorrhizal gradients of  $\sigma_C$  that drive  $Z_{rC}$ . Root - mycorrhizal N and P transfers  $Z_{rN,P}$  are calculated such that concentrations of  $\sigma_{N,P}$  with respect to  $\sigma_C$  in each root and mycorrhizal layer approach equilibrium according to rate constants  $g_{rN,P}$  [C53]. Because mycorrhizal  $\sigma_{N,P}$  are generated by uptake with greater surface area and length with respect to phytomass [C23],  $g_{rN,P}$  cause mycorrhizal-to-root gradients of  $\sigma_{N,P}$  that drive  $Z_{rN,P}$ .

## **Appendix D: Soil Water, Heat, Gas and Solute Fluxes**

### *Surface Water Flux*

Surface runoff is modelled using Manning's equation [D1] with surface water velocity  $v$  [D3] calculated from surface geometry [D5a] and slope [D5b], and with surface water depth  $d$  [D2] calculated from surface water balance [D4] using kinematic wave theory.

### *Subsurface Water Flux*

Subsurface water flow [D7] is calculated from Richard's equation using bulk soil water potentials  $\psi_s$  of both cells if both source and destination cells are unsaturated [D9a], or Green-Ampt equation using  $\psi_s$  beyond the wetting front of the unsaturated cell if either source or destination cell is saturated [D9b] (Grant et al., 2004). Subsurface water flow can also occur through macropores using Poiseuille-Hagen theory for laminar flow in tubes (Dimitrov et al., 2010), depending on inputs for macropore volume fraction.

### *Exchange with Water Table*



If a water table is present in the model, subsurface boundary water fluxes between saturated boundary grid cells and a fixed external water table are calculated from lateral hydraulic conductivities of the grid cells, and from elevation differences and lateral distances between the grid cells and the external water table [D10]. These terms are determined from set values for the depth  $d_t$  of, and lateral distance  $L_t$  to, an external water table.

#### *Surface Heat Flux*

Surface heat fluxes ( $G$ ) arising from closure of the energy balance at snowpack, surface litter and soil surfaces [D11] (Grant et al., 1999) drive conductive – convective fluxes among snowpack, surface litter and soil layers [D12]. These fluxes drive freezing – thawing ( $Q_f$ ) and changes temperatures ( $T$ ) in snowpack, surface litter and soil layers [D13].

#### *Gas Flux*

All gases undergo volatilization – dissolution between the gaseous and aqueous phases in the soil [D14a] and root [D14b], and between the atmosphere and the aqueous phase at the soil surface [D15a], driven by gaseous – aqueous concentration differences calculated from solubility coefficients and coupled to diffusive uptake by roots [C14] and microbes [A17]. Gases also undergo convective - conductive transfer among soil layers driven by gaseous concentration gradients and diffusivities [D16a,b,c] calculated from air-filled porosities [D17a,b,c], and from each rooted soil layer directly to the atmosphere through roots driven by gaseous concentration gradients and diffusivities [D16d] calculated from root porosities [D17d]. Gases may also bubble upwards from soil zones in which the total partial pressure of all aqueous gases exceeds atmospheric pressure [D18].

#### *Solute Flux*

All gaseous and non-gaseous solutes undergo convective - dispersive transfer among soil layers driven by aqueous concentration gradients and dispersivities [D19] calculated from water-filled porosity [D20] and water flow length [D21].

## **Appendix E: Solute Transformations**

### *Precipitation - Dissolution Equilibria*

Solution  $[\text{NH}_4^+]$ ,  $[\text{NO}_3^-]$  and  $[\text{H}_2\text{PO}_4^-]$  that drive  $U_{\text{NH}_4}$ ,  $U_{\text{NO}_3}$  and  $U_{\text{PO}_4}$  [C23] are controlled by precipitation, adsorption and ion pairing reactions (Grant et al., 2004; Grant and Heaney, 1997), including precipitation-dissolution of  $\text{Al}(\text{OH})_3$ ,  $\text{Fe}(\text{OH})_3$ ,  $\text{CaCO}_3$ ,  $\text{CaSO}_4$ ,  $\text{AlPO}_4$ ,  $\text{FePO}_4$ ,  $\text{Ca}(\text{H}_2\text{PO}_4)_2$ ,  $\text{CaHPO}_4$ , and  $\text{Ca}_5(\text{PO}_4)_3\text{OH}$  [E1 – E9], cation exchange between  $\text{Ca}^{2+}$ ,  $\text{NH}_4^+$  and other cations [E10 – E15], anion exchange between adsorbed and soluble  $\text{H}_2\text{PO}_4^-$ ,  $\text{HPO}_4^{2-}$  and  $\text{OH}^-$  [E16 – E20], and ion pairing [E22 – E55].

Key governing equations for simulating net ecosystem productivity in *ecosys*. Variables input to the model appear in bold with values given in the **Definition of Variables** below.

## **Appendix F: N<sub>2</sub> Fixation**

### *Rhizobial Growth*

Modelling the activity of symbiotic N<sub>2</sub> fixing bacteria in roots follows a protocol similar to that of non-symbiotic N<sub>2</sub> fixing bacteria in soil. Respiration demand is driven by specific activity, microbial biomass  $M_n$ , and nonstructural C concentration  $[\chi_n]$  in root nodules [F1], and is constrained by temperature [F2] and microbial N or P status [F3]. Nodule respiration  $R$  is constrained by the extent to which O<sub>2</sub> uptake meets O<sub>2</sub> demand [F4] imposed by respiration demand [F5]. O<sub>2</sub> uptake is in turn constrained by rhizosphere [O<sub>2r</sub>] [F6a] which is controlled by radial diffusion of O<sub>2</sub> through soil water to roots and nodules [F6b]. Soil water [O<sub>2</sub>] is maintained by dissolution of O<sub>2</sub> from soil air which is in turn maintained by soil-atmosphere gas exchange and vertical diffusion (Grant, 2004).  $R_h$  is first allocated to maintenance respiration  $R_m$  [F7 – F8] and the remainder if any is allocated to growth respiration  $R_g$  [F9]. If  $R_m$  exceeds  $R_h$ , the shortfall is made up from respiration of microbial protein C, forcing senescence and litterfall of associated non-protein C [F10 – F11].

### *N<sub>2</sub> Fixation*

N<sub>2</sub> fixation  $V_{N_2}$  is driven by  $R_g$  [F12], but is constrained by accumulation of nonstructural N  $\nu_n$  with respect to nonstructural C and P also required for microbial growth in the nodule [F13]. Nonstructural N  $\nu_{nd}$  is the product of  $V_{N_2}$ , so that [F12] simulates the inhibition of N<sub>2</sub> fixation by its product (Postgate, 1998). The value of  $V_{N_2}$  is also limited by the additional N needed to maintain bacterial N content  $[N_n']$  of  $M_n$  [F12], so that N<sub>2</sub> fixation is constrained by the need of nodule bacteria for N not met from other sources (Postgate, 1998). Respiration required for N<sub>2</sub> fixation  $R_{N_2}$  [F14] is subtracted from  $R_g$  [F15] when calculating microbial growth [F16 – F18]. Microbial senescence drives N and P litterfall [F19 – F20].

### *Nodule – Root Exchange*

Exchange of nonstructural C, N and P between roots and nodules is driven by concentration gradients [F21 – F23] created by generation, transfer and consumption of nonstructural C, N and P in shoots, roots, mycorrhizae and nodules. Nonstructural C is generated in shoots and transferred along concentration gradients to roots and thence to nodules [F21]. Nonstructural P is generated in roots and transferred along concentration gradients to shoots and nodules [F23]. Nonstructural N is generated in roots through mineral uptake and in nodules through gaseous fixation [F22]. Nonstructural C, N and P in nodules is determined by root-nodule exchange, by nodule respiration and fixation, and by remobilization from nodule litterfall [F24 – F26].

Root nonstructural N ( $\nu_r$ ) may rise if high mineral N concentrations in soil sustain rapid N uptake by roots. Large  $\nu_r$  suppresses or even reverses the transfer of  $\nu_n$  from nodule to root [F22], raising  $\nu_n$  [F25] and hence suppressing  $V_{N_2}$  [F12 – F13]. Large  $\nu_r$  also accelerates the consumption of  $\chi_r$ , slowing its transfer to nodules [F21], reducing

$\chi_n$  [F24] and hence slowing nodule growth [F1]. Conversely, slow root N uptake caused by low soil mineral N concentrations would lower  $v_{rt}$  and raise  $\chi_{rt}$ , hastening the transfer of  $v_n$  from nodule to root and of  $\chi_{rt}$  from root to nodule, lowering  $v_n$ , raising  $\chi_n$ , and accelerating  $V_{N_2}$ . However [F13] also allows  $V_{N_2}$  to be constrained by nonstructural C and P concentrations arising from CO<sub>2</sub> fixation and root P uptake.

## **Appendix G: CH<sub>4</sub> Production and Consumption**

### *Anaerobic Fermenters and H<sub>2</sub> Producing Acetogens*

The states  $S_{i,j,k}$ ,  $B_{i,k}$  and  $Z_{i,j,k}$  in *ecosys* are substrates for hydrolysis by all active ( $j = a$ ) heterotrophic biomass communities  $M_{i,n,a}$  (Eqs. [1 - 7] of Grant et al., 1993a), which include fermenters plus acetogens. Hydrolysis products are transferred to soluble organic matter  $DOC_{i,k}$  which is the substrate for respiration and uptake by microbial biomass  $M_{i,n,j}$  as described for aerobic heterotrophs in Eq. [11] of Grant et al., (1993a). Respiration  $R_{i,f}$  of  $DOC_{i,c}$  by fermenters plus acetogens ( $n = f$ ) is a Michaelis-Menten function of  $[DOC_{i,c}]$  inhibited by O<sub>2</sub> (Eq. [G1]). Respiration products are partitioned among  $A_{i,c}$ , CO<sub>2</sub> and H<sub>2</sub> according to Brock and Madigan (1991) (Eq. [G2]).  $R_{i,f}$  beyond that used for maintenance respiration drives the uptake of additional  $DOC_{i,c}$  (Eq. [G3]) for microbial growth according to the growth yield  $Y_f$  of fermentation (Eq. [G4]). The growth yield from fermentation is calculated by dividing the free energy change of fermentation, adjusted for H<sub>2</sub> product concentration (Eq. [G5]), by the energy required to transform soluble organic C into microbial C (Eq. [G4]). Change in  $M_{i,f,j}$  is thus the difference between uptake and respiration of  $DOC_{i,c}$ , less decomposition (Eq. [G6]). This change determines  $M_{i,f,a}$  used in the following calculation of  $R_{i,f}$  (Eq. [G1]). Ratios of  $M_{i,f,j,c}$  to  $M_{i,f,j,n}$  determine mineralization-immobilization of N (Eq. [23] in Grant et al., 1993a). Decomposition products  $D_{i,f,j,k}$  are partitioned to microbial residues  $Z_{i,j,k}$  and soil organic matter  $S_{i,j,k}$  (where  $i =$  passive soil organic matter) ( Eqs. [26-28] in Grant et al., 1993a) which undergo further hydrolysis.

### *Acetotrophic Methanogens*

The fermenter product  $A_{i,c}$  (Eq. [G2]) is the substrate for respiration  $R_{i,m}$  by acetotrophic methanogens ( $n = m$ ) (Eq. [G7]). Respiration products are partitioned between CH<sub>4</sub> and CO<sub>2</sub> according to Brock and Madigan (1991) (Eq. [G8]).  $R_{i,m}$  beyond that used for maintenance respiration drives the uptake of additional  $A_{i,c}$  (Eq. [G9]) for microbial growth according to the growth yield  $Y_m$  of acetotrophic methanogenesis (Eq. [G10]). This growth yield is calculated by dividing the free energy change of acetotrophic methanogenesis (Brock and Madigan, 1991) by the energy required to transform acetate into microbial C. Acetogenic methanogens in the model use acetate as their sole carbon and energy source (Smith and Mah, 1980). Change in  $M_{i,m,j}$  is thus the difference between uptake and respiration of  $A_{i,c}$ , less decomposition (Eq. [G11]). This change determines  $M_{i,m,a}$  used in the following calculation of  $R_{i,m}$  (Eq. [G7]). Mineralization and decomposition processes are the same as those for other microbial populations.

### *Hydrogenotrophic Methanogens*

The fermenter products  $\text{CO}_2$  and  $\text{H}_2$  (Eq. [G2]) are the substrates for  $\text{CO}_2$  reduction by hydrogenotrophic methanogens ( $n = h$ ) which are assumed to be autotrophic (Eq. [G12]). Respiration products are partitioned between  $\text{CH}_4$  and  $\text{H}_2\text{O}$  according to Brock and Madigan (1991) (Eq. [G13]).  $R_h$  beyond that used for maintenance respiration drives the uptake of additional  $\text{CO}_2$  (Eq. [G14]) for microbial growth according to the growth yield  $Y_h$  of hydrogenotrophic methanogenesis (Brock and Madigan, 1991) (Eq. [G15]). This growth yield is calculated by dividing the free energy change of hydrogenotrophic methanogenesis, adjusted for  $\text{H}_2$  substrate concentration (Eq. [G16]), by the energy required to transform  $\text{CO}_2$  into microbial C. Change in  $M_{h,j}$  is thus the difference between uptake and respiration of  $\text{CO}_2$ , less decomposition (Eq. [G17]). This change determines  $M_{h,a}$  used in the following calculation of  $R_h$  (Eq. [G12]). Mineralization and decomposition processes are the same as those for other microbial populations.

### *Autotrophic Methanotrophs*

Methane generated by acetotrophic and hydrogenotrophic methanogens is the substrate for  $\text{CH}_4$  oxidation by autotrophic methanotrophs ( $n = t$ ) (Eq. [G18]). The stoichiometry and energetics of the methanotrophic reactions (Eqs. [G22 – G24]) are based on those of  $\text{CH}_4$  to  $\text{CO}_2$  in Brock and Madigan (1991). The oxidation of  $\text{CH}_4$  to  $\text{CO}_2$  is coupled through an energy yield with the oxidation of  $\text{CH}_4$  to organic C used in microbial respiration (Eq. [G19]). The energy yield from  $\text{CH}_4$  oxidation is calculated by dividing the free energy change of  $\text{CH}_4$  oxidation by the energy required to transform  $\text{CH}_4$  into organic C (Eq. [G20]). Oxygen requirements to sustain  $\text{CH}_4$  oxidation rates are then calculated from the stoichiometries of  $\text{CH}_4$  oxidation (Eq. [G22 and G23]) and aerobic microbial respiration (Eq. [G24]). The  $\text{O}_2$  concentrations at methanotrophic microsites are then found at which active  $\text{O}_2$  uptake driven by requirements for  $\text{CH}_4$  oxidation equals spherical  $\text{O}_2$  diffusion to the microsites from the soil solution. These microsites are considered to be uniformly distributed on soil surfaces and are separated from the soil atmosphere (if present) by a water film of uniform thickness that depends upon soil water potential. The  $\text{O}_2$  uptake by each aerobic microbial population in the model competes with that by all other aerobic microbial populations (e.g. Grant, 1995; Grant and Rochette, 1994), and is constrained by  $\text{O}_2$  transfer rates through the gaseous and aqueous phases of the soil profile. The ratio of  $\text{O}_2$  uptake to  $\text{O}_2$  requirement  $f_{\text{O}_2,t}$  is then used to constrain  $\text{CH}_4$  oxidation rates (Eq. [G21]) so that  $\text{CH}_4$  oxidation is stoichiometrically coupled to  $\text{O}_2$  uptake. Growth respiration by methanotrophs is calculated as the difference between total respiration  $R_t$  from Eq. [G21b] and maintenance respiration  $R_{\text{mt}}$  from Eqs. [18-19] of Grant et al. (1993a). Growth respiration drives the uptake and transformation of additional  $\text{CH}_4$  into microbial biomass  $M_{t,c}$  (Eq. [G25]) according to the growth yield. This yield is calculated by dividing the free energy change of  $\text{CH}_4$  oxidation (Brock and Madigan, 1991) (Eq. [G18]) by the energy required to construct new microbial biomass from  $\text{CH}_4$  (Eq. [G26]). Net growth of the methanotrophic population  $M_{t,j,c}$  is calculated as the uptake of  $\text{CH}_4 - \text{C}$  minus respiration and decomposition of assimilated C (Eq. [G27]). This change determines  $M_{t,a}$  used in the following calculation of  $X'_t$  (Eq. [G18]). Mineralization and decomposition processes are the same as those for other microbial populations.

This submodel of autotrophic methanotrophy has been used to simulate methanotrophic growth yields, specific growth rates, CH<sub>4</sub> concentration profiles and the sensitivity of CH<sub>4</sub> uptake to temperature and water content in soil columns (Grant, 1999). The combined submodels of anaerobic fermentation, acetotrophic methanogenesis, hydrogenotrophic methanogenesis and autotrophic methanotrophy have been used to simulate methanogenic growth yields, specific growth rates, and the time course of CH<sub>4</sub> emissions from differently amended soil columns at different temperatures (Grant, 1998b). All input parameter values used in Eqs. [G1] to [G27] (Table 1) were derived from the microbiological literature and remain unchanged from those used in Grant (1998b) and in Grant (1999).

## Appendix H: Inorganic N Transformations

### *Mineralization and Immobilization of Ammonium by All Microbial Populations*

Each functional component  $j$  ( $j$  = labile or resistant) of each microbial population  $m$  ( $m$  = obligately aerobic bacteria, obligately aerobic fungi, facultatively anaerobic denitrifiers, anaerobic fermenters plus H<sub>2</sub>-producing acetogens, acetotrophic methanogens, hydrogenotrophic methanogens and methanotrophs, NH<sub>4</sub><sup>+</sup> and NO<sub>2</sub><sup>-</sup> oxidizers, and non-symbiotic diazotrophs) in each substrate-microbe complex  $i$  ( $i$  = animal manure, coarse woody plant residue, fine non-woody plant residue, particulate organic matter, or humus) seeks to maintain a set C:N ratio by mineralizing NH<sub>4</sub><sup>+</sup> ([H1a]) or by immobilizing NH<sub>4</sub><sup>+</sup> ([H1b]) or NO<sub>3</sub><sup>-</sup> ([H1c]). Provision is made for C:N ratios to rise above set values during immobilization, but at a cost to microbial function. These transformations control the exchange of N between organic and inorganic states.

### *Oxidation of DOC and Reduction of Oxygen by Heterotrophs*

Constraints on heterotrophic oxidation of DOC imposed by O<sub>2</sub> uptake are solved in four steps:

- 1) DOC oxidation under non-limiting O<sub>2</sub> is calculated from active biomass and DOC concentration ([H2]),
- 2) O<sub>2</sub> reduction under non-limiting O<sub>2</sub> is calculated from 1) using a set respiratory quotient ([H3]),
- 3) O<sub>2</sub> reduction under ambient O<sub>2</sub> is calculated from radial O<sub>2</sub> diffusion through water films of thickness determined by soil water potential ([H4a]) coupled with active uptake at heterotroph surfaces driven by 2) ([H4b]). O<sub>2</sub> diffusion and active uptake is population-specific, allowing the development of more anaerobic conditions at microbial surfaces associated with more biologically active substrates. O<sub>2</sub> uptake by heterotrophs also accounts for competition with O<sub>2</sub> uptake by nitrifiers, roots and mycorrhizae,
- 4) DOC oxidation under ambient O<sub>2</sub> is calculated from 2) and 3) ([H5]). The energy yield of DOC oxidation drives the uptake of additional DOC for construction of microbial biomass  $M_{i,h}$  according to construction energy costs of each heterotrophic population

(Eqs. [7] to [13] in Grant and Pattey, 2003). Energy costs of denitrifiers are slightly larger than those of obligate heterotrophs, placing denitrifiers at a competitive disadvantage for growth and hence DOC oxidation if electron acceptors other than  $O_2$  are not used.

#### *Oxidation of DOC and Reduction of Nitrate, Nitrite and Nitrous Oxide by Denitrifiers*

Constraints imposed by  $NO_3^-$  availability on DOC oxidation by denitrifiers are solved in five steps:

- 1)  $NO_3^-$  reduction under non-limiting  $NO_3^-$  is calculated from a fraction of electrons demanded by DOC oxidation but not accepted by  $O_2$  because of diffusion limitations ([H6]),
- 2)  $NO_3^-$  reduction under ambient  $NO_3^-$  is calculated from 1) ([H7]),
- 3)  $NO_2^-$  reduction under ambient  $NO_2^-$  is calculated from demand for electrons not met by  $NO_3^-$  in 2) ([H8]),
- 4)  $NO_2^-$  reduction under ambient  $NO_2^-$  is calculated from demand for electrons not met by  $NO_2^-$  in 3) ([H9]),
- 5) additional DOC oxidation enabled by  $NO_x$  reduction in 2), 3) and 4) is added to that enabled by  $O_2$  reduction from [H5], the energy yield of which drives additional DOC uptake for construction of  $M_{i,n}$ . This additional uptake offsets the disadvantage incurred by the larger construction energy costs of denitrifiers.

#### *Oxidation of Ammonia and Reduction of Oxygen by Nitrifiers*

Constraints on nitrifier oxidation of  $NH_3$  imposed by  $O_2$  uptake are solved in four steps:

- 1) substrate ( $NH_3$ ) oxidation under non-limiting  $O_2$  is calculated from active biomass and from  $NH_3$  and  $CO_2$  concentrations ([H11]),
- 2)  $O_2$  reduction under non-limiting  $O_2$  is calculated from 1) using set respiratory quotients ([H12]),
- 3)  $O_2$  reduction under ambient  $O_2$  is calculated from radial  $O_2$  diffusion through water films of thickness determined by soil water potential ([H13a]) coupled with active uptake at nitrifier surfaces driven by 2) ([H13b]).  $O_2$  uptake by nitrifiers also accounts for competition with  $O_2$  uptake by heterotrophic DOC oxidizers, roots and mycorrhizae,
- 4)  $NH_3$  oxidation under ambient  $O_2$  is calculated from 2) and 3) ([H14]). The energy yield of  $NH_3$  oxidation drives the fixation of  $CO_2$  for construction of microbial biomass  $M_{i,n}$  according to construction energy costs of each nitrifier population (Eqs. [32] to [34] in Grant and Pattey, 2003).

#### *Oxidation of Nitrite and Reduction of Oxygen by Nitrifiers*

Constraints on nitrifier oxidation of  $NO_2^-$  imposed by  $O_2$  uptake ([H15] to [H18]) are solved in the same way as are those of  $NH_3$  ([H11] to [H14]). The energy yield of  $NO_2^-$  oxidation drives the fixation of  $CO_2$  for construction of microbial biomass  $M_{i,o}$  according to construction energy costs of each nitrifier population.

#### *Oxidation of Ammonia and Reduction of Nitrite by Nitrifiers*

Constraints on nitrifier oxidation imposed by  $\text{NO}_2^-$  availability are solved in three steps:

- 1)  $\text{NO}_2^-$  reduction under non-limiting  $\text{NO}_2^-$  is calculated from a fraction of electrons demanded by  $\text{NH}_3$  oxidation but not accepted by  $\text{O}_2$  because of diffusion limitations ([H19]),
- 2)  $\text{NO}_2^-$  reduction under ambient  $\text{NO}_2^-$  and  $\text{CO}_2$  is calculated from step (1) ([H20]), competing for  $\text{NO}_2^-$  with [H18],
- 3) additional  $\text{NH}_3$  oxidation enabled by  $\text{NO}_2^-$  reduction in 2) [H21] is added to that enabled by  $\text{O}_2$  reduction from [H14]. The energy yield from this oxidation drives the fixation of additional  $\text{CO}_2$  for construction of  $M_{i,n}$ .

## Appendix A: Soil C, N and P Transformations

### *Decomposition*

$D_{Si,j,l,C} = D'_{Si,j,l,C} M_{i,d,l,C} f_{\text{igl}} (S_{i,l,C} / G_{i,l,C})$	decomposition of litter, POC, humus	[A1a]
$D_{Zi,j,l,C} = D'_{Zi,j,l,C} M_{i,d,l,C} f_{\text{igl}} (Z_{i,l,C} / G_{i,l,C})$	decomposition of microbial residues	[A1b]
$D_{Ai,l,C} = D'_{Ai,l,C} M_{i,d,l,C} f_{\text{igl}} (A_{i,l,C} / G_{i,l,C})$	decomposition of adsorbed SOC	[A1c]
$S_{i,l,C} = \sum_j S_{i,j,l,C}$	total C in all kinetic components of litter, POC, humus	[A2a]
$Z_{i,l,C} = \sum_j Z_{i,j,l,C}$	total C in all kinetic components of microbial residues	[A2b]
$G_{i,l,C} = S_{i,l,C} + Z_{i,l,C} + A_{i,l,C}$	total C in substrate-microbe complexes	[A2c]
$M_{i,d,l,C} = M_{i,a,l,C} + q_m (M_{i,a,l,C} G_{ix,l,C} - M_{ix,a,l,C} G_{i,l,C}) / (G_{ix,l,C} + G_{i,l,C})$	redistribution of active microbial biomass populations from each substrate-microbe complex $i$ to other substrate-microbe complexes $ix$ according to concentration differences (priming)	[A3a]
$M_{i,a,l,C} = \sum_n M_{i,n,a,l,C}$	substrate and water constraint on $D$ from colonized litter, POC and humus, microbial residues and adsorbed SOC	[A3b]
$D'_{Si,j,l,C} = \{D_{Sj,C} [S_{i,j,l,C}]\} / \{[S_{i,j,l,C}] + K_{mD} (1.0 + [\sum M_{i,d,l,C}] / K_{iD})\}$	substrate and water constraint on $D$ from colonized litter, POC and humus, microbial residues and adsorbed SOC	[A4a]
$D'_{Zi,j,l,C} = \{D_{Zj,C} [Z_{i,j,l,C}]\} / \{[Z_{i,j,l,C}] + K_{mD} (1.0 + [M_{i,d,l,C}] / K_{iD})\}$	substrate and water constraint on $D$ from colonized litter, POC and humus, microbial residues and adsorbed SOC	[A4b]
$D'_{Ai,l,C} = \{D_{A,C} [A_{i,l,C}]\} / \{[A_{i,l,C}] + K_{mD} (1.0 + [M_{i,d,l,C}] / K_{iD})\}$	substrate and water constraint on $D$ from colonized litter, POC and humus, microbial residues and adsorbed SOC	[A4c]
$\delta S_{i,j,k,l,C} / \delta t = \beta \sum_n (U_{i,n,l,C} - R_{hi,n,l}) (S'_{i,j,k,l,C} / S'_{i,j,l,C}) \{ (S'_{i,j,l,C} / S_{i,j,l,C}) / (S'_{i,j,l,C} / S_{i,j,l,C} + K_{iS}) \}$	colonized litter increases with microbial growth into uncolonized litter	[A5]
$f_{\text{igl}} = T_{sl} \{ e^{[B - H_a / (R T_{sl})]} \} / \{ 1 + e^{[(H_{dl} - ST_{sl}) / (R T_{sl})]} + e^{[(ST_{sl} - H_{dh}) / (R T_{sl})]} \}$	Arrhenius function for $D$ and $R_h$	[A6]



$D_{Si,j,l,N,P} = D_{Si,j,l,C} (S_{i,j,l,N,P} / S_{i,j,l,C})$	decomposition of N and P are driven by that of C in litter, POC, humus, microbial residues and adsorbed SOC	[A7a]
$D_{Zi,j,l,N,P} = D_{Zi,j,l,C} (Z_{i,j,l,N,P} / Z_{i,j,l,C})$		[A7b]
$D_{Ai,l,N,P} = D_{Ai,l,C} (A_{i,l,N,P} / A_{i,l,C})$		[A7c]
$Y_{i,l,C} = k_{ts} (G_{i,l,C} F_s [Q_{i,l,C}]^b - X_{i,l,C})$	Freundlich sorption of DOC	[A8]
$Y_{i,l,N,P} = Y_{i,l,C} (Q_{i,l,N,P} / Q_{i,l,C})$	$(Y_{i,l,C} > 0)$ adsorption of DON, DOP	[A9]
$Y_{i,l,N,P} = Y_{i,l,C} (X_{i,l,N,P} / X_{i,l,C})$	$(Y_{i,l,C} < 0)$ desorption of DON, DOP	[A10]

### Microbial Growth

$R_h = \sum_i \sum_n \sum_l R_{hi,n,l}$	total heterotrophic respiration	[A11]
$R_{hi,n,l} = R'_{hn} \min\{C_{Ni,n,l,a} / C_{Nj}, C_{Pi,n,l,a} / C_{Pj}\}$	$R_h$ constrained by microbial N, P	[A12]
$R'_{hi,n,l} = M_{i,n,a,l,C} \{R_{hi,n,l} [Q_{i,l,C}]\} / \{(K_{mqC} + [Q_{i,l,C}])\} f_{vgl} f_{\psi g}$	$R_h$ constrained by substrate DOC	[A13]
$R_{hi,n,l} = R'_{hi,n,l} (U_{O2i,n,l} / U'_{O2i,n,l})$	$R_h$ constrained by O <sub>2</sub>	[A14]
$f_{vgl} = 1.0 - 6.67 (1.0 - e^{(M \psi_s / (R T_{sl})})$	$\psi_s$ constraints on microbial growth	[A15]
$U'_{O2i,n,l} = 2.67 R'_{hi,n,l}$	O <sub>2</sub> demand driven by potential $R_h$	[A16]
$U_{O2i,n,l} = U'_{O2i,n,l} [O_{2mi,n,l}] / ([O_{2mi,n,l}] + K_{O2})$	active uptake coupled with radial diffusion of O <sub>2</sub>	[A17a]
$= 4 \pi n M_{i,n,a,l,C} D_{sO2l} [r_m r_{wl} / (r_{wl} - r_m)] ([O_{2sl}] - [O_{2mi,n,l}])$		[A17b]
$R_{mi,n,j,l} = R_m M_{i,n,j,l,N} f_{tml}$	maintenance respiration	[A18]
$f_{tml} = e^{[y (T_{sl} - 298.16)]}$	temperature sensitivity of $R_m$	[A19]
$R_{gi,n,l} = R_{hi,n,l} - \sum_j R_{mi,n,j,l}$	growth respiration	[A20]
$U_{i,n,l,C} = \min (R_{hi,n,l}, \sum_j R_{mi,n,j,l}) + R_{gi,n,l} (1 + \Delta G / E_m)$	DOC uptake driven by $R_g$	[A21]

$$U_{i,n,lN,P} = U_{i,n,l} Q_{i,lN,P} / Q_{i,lC}$$

$$D_{Mi,n,j,l,C} = D_{Mij} M_{i,n,j,C} f_{igl}$$

$$D_{Mi,n,j,N,P} = D_{Mij} M_{i,n,j,lN,P} f_{igl} f_{di,n,lN,P}$$

$$\delta M_{i,n,j,l,C} / \delta t = F_j U_{i,n,lC} - F_j R_{hi,n,l} - D_{Mi,n,j,l,C}$$

$$\delta M_{i,n,j,l,C} / \delta t = F_j U_{i,n,lC} - R_{mi,n,j,l} - D_{Mi,n,j,l,C}$$

DON, DOP uptake driven by  $U_{i,n,lC}$  [A22]

first-order decay of microbial C, [A23]

partial release of microbial N, P [A24]

$[R_{hi,n,l} > R_{mi,n,j,l}]$  growth [A25a]

$[R_{hi,n,l} < R_{mi,n,j,l}]$  senescence [A25b]

### Microbial Nutrient Exchange

$$U_{NH_4i,n,j,l} = (M_{i,n,j,l,C} C_{Nj} - M_{i,n,j,l,N})$$

$U_{NH_4} < 0$  mineralization [A26a]

$$U_{NH_4i,n,j,l} = \min \{ (M_{i,n,j,l,C} C_{Nj} - M_{i,n,j,l,N}), \\ U'_{NH_4} a_{i,n,j,l} ([NH_4^+_{i,n,j,l}] - [NH_4^+_{mn}]) / ([NH_4^+_{i,n,j,l}] - [NH_4^+_{mn}] + K_{NH_4}) \}$$

$U_{NH_4} > 0$  immobilization [A26b]

$$U_{NO_3i,n,j,l} = \min \{ (M_{i,n,j,l,C} C_{Nj} - (M_{i,n,j,l,N} + U_{NH_4i,n,j,l})), \\ U'_{NO_3} a_{i,n,j,l} ([NO_3^-_{i,n,j,l}] - [NO_3^-_{mn}]) / ([NO_3^-_{i,n,j,l}] - [NO_3^-_{mn}] + K_{NO_3}) \}$$

$U_{NO_3} > 0$  immobilization [A26c]

$$U_{PO_4i,n,j,l} = (M_{i,n,j,l,C} C_{Pj} - M_{i,n,j,l,P})$$

$U_{PO_4} < 0$  mineralization [A26d]

$$U_{PO_4i,n,j,l} = \min \{ (M_{i,n,j,l,C} C_{Pj} - M_{i,n,j,l,P}), \\ U'_{PO_4} A_{i,n,j,l} ([H_2PO_4^-_{i,n,j,l}] - [H_2PO_4^-_{mn}]) / ([H_2PO_4^-_{i,n,j,l}] - [H_2PO_4^-_{mn}] + K_{PO_4}) \}$$

$U_{PO_4} > 0$  immobilization [A26e]

$$\Phi_{i,n=fj,l} = \max \{ 0, M_{i,n=fj,l,C} C_{Nj} - M_{i,n=fj,l,N} - \max \{ 0, U_{i,n=fj,l,N} \} \}$$

$N_2$  fixation driven by N deficit of [A27]

$$R_{\phi i,n=fj,l} = E_{\phi} \Phi_{i,n=fj,l}$$

diazotrophic population [A27]

respiration needed to drive  $N_2$  [A28]

$$\delta M_{i,n,j,l,N} / \delta t = F_j U_{i,n,lN} + U_{NH_4i,n,j,l} + U_{NO_3i,n,j,l} + \Phi_{i,n=fj,l} - D_{Mi,n,j,l,N}$$

fixation [A28]

growth vs. losses of microbial N, P [A29a]

$$\delta M_{i,n,j,l,P} / \delta t = F_j U_{i,n,lP} + U_{PO_4i,n,j,l} - D_{Mi,n,j,l,P}$$

[A29b]

$$M_{i,n,a,l,C} = M_{i,n,j=labile,l,C} + M_{i,n,j=resistant,l,C} F_r / F_l$$

active microbial biomass [A30a]  
calculated from labile fraction

### Humification

$$H_{Si,j=\text{lignin},l,C} = D_{Si,j=\text{lignin},l,C}$$

decomposition products of litter [A31]

$$H_{Si,j=\text{lignin},l,N,P} = D_{Si,j=\text{lignin},l,N,P}$$

substrate added to POC depending  
on lignin [A32]

$$H_{Si,j\neq\text{lignin},l,C} = H_{Si,j=\text{lignin},l,C} L_{hj}$$

[A33]

$$H_{Si,j\neq\text{lignin},l,N,P} = H_{Si,j\neq\text{lignin},l,C} S_{i,l,N,P} / S_{i,l,C}$$

[A34]

$$H_{Mi,n,j,l,C} = D_{Mi,n,j,l,C} F_h$$

decay products of microbes added  
to humus [A35]

$$H_{Mi,n,j,l,N,P} = H_{Mi,n,j,l,C} M_{i,n,j,l,N,P} / M_{i,n,j,l,C}$$

[A36]

$$F_h = 0.167 + 0.167 F_{\text{clay}} + 0.167 \times 10^{-6} G_{i,l,C}$$

fraction of  $D_M$  added to humus  
depends on clay and SOC [A37]

## Definition of Variables in Appendix A

Variable	Definition	Unit	Equation	Value	Reference
<i>subscripts</i>					
<i>i</i>	substrate-microbe complex: coarse woody litter, fine non-woody litter, POC, humus				
<i>j</i>	kinetic component: labile <i>l</i> , resistant <i>r</i> , active <i>a</i>				
<i>l</i>	soil or litter layer				
<i>n</i>	microbial functional type: heterotrophic (bacteria, fungi), autotrophic (nitrifiers, methanotrophs), diazotrophic, obligate aerobe, facultative anaerobes (denitrifiers), obligate anaerobes (methanogens)				
<i>variables</i>					
$A_{i,l,C}$	mass of adsorbed SOC	$\text{g C m}^{-2}$	[A1c,A2c]		
$[A_{i,l,C}]$	concentration of adsorbed SOC in soil	$\text{g C Mg}^{-1}$	[A4c]		
<i>a</i>	microbial surface area	$\text{m}^2 \text{ m}^{-2}$	[A26]		
<b><i>B</i></b>	parameter such that $f_{\text{ig}} = 1.0$ at $T_l = 298.15 \text{ K}$		[A6]	26.230	
<b><i>b</i></b>	Freundlich exponent for sorption isotherm		[A8]	0.85	Grant et al. (1993a,b)
<b><math>\beta</math></b>	specific colonization rate of uncolonized substrate	-	[A5]	2.5	Grant et al. (2010)
$C_{N,Pi,n,a,l}$	ratio of $M_{i,n,a,N,P}$ to $M_{i,n,a,C}$	$\text{g N or P g C}^{-1}$	[A12]		

$C_{N,Pj}$	maximum ratio of $M_{i,n,j,N,P}$ to $M_{i,n,j,C}$ maintained by $M_{i,n,j,C}$	g N or P g C <sup>-1</sup>	[A12,A26,A27]	0.22 and 0.13 (N), 0.022 and 0.013 (P) for $j$ = labile and resistant, respectively	Grant et al. (1993a,b)
$D_{Mi,j}$	specific decomposition rate of $M_{i,n,j}$ at 30°C	g C g C <sup>-1</sup> h <sup>-1</sup>	[A23,A24]	0.0125 and 0.00035 for $j$ = labile and resistant, respectively	Grant et al. (1993a,b)
$D_{Mi,n,j,l,C}$	decomposition rate of $M_{i,n,j,l,C}$	g C m <sup>-2</sup> h <sup>-1</sup>	[A23,A25,A35]		
$D_{Mi,n,j,l,N,P}$	decomposition rate of $M_{i,n,j,l,N,P}$	g N or P m <sup>-2</sup> h <sup>-1</sup>	[A24,A29]		
$D_{sO_2l}$	aqueous dispersivity–diffusivity of O <sub>2</sub> during microbial uptake in soil	m <sup>2</sup> h <sup>-1</sup>	[A17]		
$D_{Ai,l,C}$	decomposition rate of $A_{i,l,C}$ by $M_{i,d,l,C}$ producing $Q$ in [A13]	g C m <sup>-2</sup> h <sup>-1</sup>	[A1c,A7c,A31c]		
$D_{A_f,C}$	specific decomposition rate of $A_{i,l,C}$ by $M_{i,d,l,C}$ at 25°C and saturating[ $A_{i,l,C}$ ]	g C g C <sup>-1</sup> h <sup>-1</sup>	[A4c]	0.025	Grant et al. (1993a,b)
$D_{A_{i,j,l,N,P}}$	decomposition rate of $A_{i,l,N,P}$ by $M_{i,d,l,C}$	g N or P m <sup>-2</sup> h <sup>-1</sup>	[A7c]		
$D'_{A_{i,j,l,C}}$	specific decomposition rate of $S_{i,j,l,C}$ by $\sum_n M_{i,n,a,l}$ at 25°C	g C g C <sup>-1</sup> h <sup>-1</sup>	[A1a,A4c]		
$D_{S_{i,j,l,C}}$	decomposition rate of $S_{i,j,l,C}$ by $\sum_n M_{i,n,a,l}$ producing $Q$ in [A13]	g C m <sup>-2</sup> h <sup>-1</sup>	[A1a,A7a,A31a]		
$D_{S_f,C}$	specific decomposition rate of $S_{i,j,l,C}$ by $\sum_n M_{i,n,a,l}$ at 25°C and saturating [ $S_{i,l,C}$ ]	g C g C <sup>-1</sup> h <sup>-1</sup>	[A4a]	1.0, 1.0, 0.15, and 0.025 for $j$ = protein, carbohydrate, cellulose, and lignin, 0.009 for POC, and 0.009 and 0.003 for active and passive humus.	Grant et al. (1993a,b)

$D_{S_{i,j,l,N,P}}$	decomposition rate of $S_{i,j,l,N,P}$ by $\sum_n M_{i,n,a,l}$	g N or P m <sup>-2</sup> h <sup>-1</sup>	[A7a, A32]		
$D'_{S_{i,j,l,C}}$	specific decomposition rate of $S_{i,j,l,C}$ by $\sum_n M_{i,n,a,l}$ at 25°C	g C g C <sup>-1</sup> h <sup>-1</sup>	[A1a,A4a]		
$D_{Z_{i,j,l,C}}$	decomposition rate of $Z_{i,j,l,C}$ by $\sum_n M_{i,n,a,l}$ producing $Q$ in [A13]	g C m <sup>-2</sup> h <sup>-1</sup>	[A1b,A7b]		
$D_{Z_{i,j,l,N,P}}$	decomposition rate of $Z_{i,j,l,N,P}$ by $\sum_n M_{i,n,a,l}$	g N or P m <sup>-2</sup> h <sup>-1</sup>	[A7b]		
$D_{Z_{i,C}}$	specific decomposition rate of $Z_{i,j,l,C}$ by $\sum_n M_{i,n,a,l}$ at 25°C and saturating[ $Z_{i,l,C}$ ]	g C g C <sup>-1</sup> h <sup>-1</sup>	[A4b]	0.25 and 0.05 for $j =$ labile and resistant biomass	Grant et al. (1993a,b)
$D'_{Z_{i,j,l,C}}$	specific decomposition rate of $Z_{i,j,l,C}$ by $\sum_n M_{i,n,a,l}$ at 25°C	g C g C <sup>-1</sup> h <sup>-1</sup>	[A1b,A4b]		
$\Delta G$	energy yield of C oxidation and O <sub>2</sub> reduction	kJ g C <sup>-1</sup>	[A21]	37.5	
$E_m$	energy requirement for growth of $M_{i,n,a,l}$	kJ g C <sup>-1</sup>	[A21]	25	
$E_\phi$	energy requirement for non-symbiotic N <sub>2</sub> fixation by heterotrophic diazotrophs ( $n = f$ )	g C g N <sup>-1</sup>	[A28]	5	Waring and Running (1998)
$F_{\text{clay}}$	fraction of mineral soil as clay	Mg Mg <sup>-1</sup>	[A37]		
$F_h$	fraction of products from microbial decomposition that are humified (function of clay content)		[A35, A37]		Sørensen (1981)
$F_l$	fraction of microbial growth allocated to labile component $M_{i,n,l}$		[A25,A29,A30]	0.55	Grant et al. (1993a,b)
$F_r$	fraction of microbial growth allocated to resistant component $M_{i,n,r}$		[A25,A29,A30]	0.45	Grant et al. (1993a,b)
$F_s$	equilibrium ratio between $Q_{i,l,C}$ and $H_{i,l,C}$		[A8]		
$f_{di,n,N,P}$	fraction of N or P released with $D_{M_{i,n,j,l,C}}$ during decomposition	dimensionless	[A24]	0.33 $U_{\text{NH}_4} > 0$ 1.00 $U_{\text{NH}_4} < 0$ 0.33 $U_{\text{PO}_4} > 0$ 1.00 $U_{\text{PO}_4} < 0$	
$f_{\text{igl}}$	temperature function for microbial growth respiration	dimensionless	[A1,A6,A13, A23,A24]		
$f_{\text{iml}}$	temperature function for maintenance respiration	dimensionless	[A18,A19]		

$f_{vgl}$	soil water potential function for microbial, root or mycorrhizal growth respiration	dimensionless	[A13,A15]		Pirt (1975)
$\Phi_{i,n=f,j,l}$	non-symbiotic N <sub>2</sub> fixation by heterotrophic diazotrophs ( $n = f$ )	g N m <sup>-2</sup> h <sup>-1</sup>	[A27,A28,A29]		
$G_{i,l,C}$	total C in substrate-microbe complex	g C Mg <sup>-1</sup>	[A1,A2c,A3a,A8,A37]		
[H <sub>2</sub> PO <sub>4</sub> <sup>-</sup> ]	concentration of H <sub>2</sub> PO <sub>4</sub> <sup>-</sup> in soil solution	g P m <sup>-3</sup>	[A26]		
$H_a$	energy of activation	J mol <sup>-1</sup>	[A6,C10]	65 x 10 <sup>3</sup>	Addiscott (1983)
$H_{dh}$	energy of high temperature deactivation	J mol <sup>-1</sup>	[A6,C10]	225 x 10 <sup>3</sup>	
$H_{dl}$	energy of low temperature deactivation	J mol <sup>-1</sup>	[A6,C10]	198 x 10 <sup>3</sup>	
$H_{Mi,n,j,l,C}$	transfer of microbial C decomposition products to humus	g C m m <sup>-2</sup> h <sup>-1</sup>	[A35,A36]		
$H_{Mi,n,j,l,N,P}$	transfer of microbial N or P decomposition products to humus	g N or P m <sup>-2</sup> h <sup>-1</sup>	[A36]		
$H_{Si,j,l,C}$	transfer of C hydrolysis products to particulate OM	g C m <sup>-2</sup> h <sup>-1</sup>	[A31,A32,A33,A34]		
$H_{Si,j,l,N,P}$	transfer of N or P hydrolysis products to particulate OM	g N or P m <sup>-2</sup> h <sup>-1</sup>	[A32,A34]		
$K_{iS}$	inhibition constant for microbial colonization of substrate	-	[A5]	0.5	Grant et al. (2010)
$K_{NH_4}$	M-M constant for NH <sub>4</sub> <sup>+</sup> uptake at microbial surfaces	g N m <sup>-3</sup>	[A26]	0.40	
$K_{NO_3}$	M-M constant for NO <sub>3</sub> <sup>-</sup> uptake at microbial surfaces	g N m <sup>-3</sup>	[A26]	0.35	
$K_{PO_4}$	M-M constant for H <sub>2</sub> PO <sub>4</sub> <sup>-</sup> uptake at microbial surfaces	g P m <sup>-3</sup>	[A26]	0.125	
$K_{iD}$	inhibition constant for [ $M_{i,n,a}$ ] on $S_{i,C}$ , $Z_{i,C}$	g C m <sup>-3</sup>	[A4]	25	Grant et al. (1993a,b); Lizama and Suzuki (1990)
$K_{mD}$	Michaelis–Menten constant for $D_{Si,j,C}$	g C Mg <sup>-1</sup>	[A4]	75	
$K_{mQC}$	Michaelis–Menten constant for $R'_{hi,n}$ on [ $Q_{i,C}$ ]	g C m <sup>-3</sup>	[A13]	36	
$K_{O_2}$	Michaelis–Menten constant for reduction of O <sub>2s</sub> by microbes, roots and mycorrhizae	g O <sub>2</sub> m <sup>-3</sup>	[A17]	0.064	Griffin (1972)

$k_{ts}$	equilibrium rate constant for sorption	$\text{h}^{-1}$	[A8]	0.01	Grant et al. (1993a,b)
$L_{bj}$	ratio of nonlignin to lignin components in humified hydrolysis products		[A33]	0.10, 0.05, and 0.05 for $j =$ protein, carbohydrate, and cellulose, respectively	Shulten and Schnitzer (1997)
$M$	molecular mass of water	$\text{g mol}^{-1}$	[A15]	18	
$M_{i,d,l,C}$	heterotrophic microbial C used for decomposition	$\text{g C m}^{-2}$	[A1,A3a,A4]		
$M_{i,n,j,l,C}$	microbial C	$\text{g C m}^{-2}$	[A13,A17A23,A25,A26, A30,A36]		
$M_{i,n,j,l,N}$	microbial N	$\text{g N m}^{-2}$	[A18,A27,A29]		
$M_{i,n,j,l,P}$	microbial P	$\text{g P m}^{-2}$	[A24,A29,A26, A36]		
$M_{i,n,a,l,C}$	active microbial C from heterotrophic population $n$ associated with $G_{i,l,C}$	$\text{g C m}^{-2}$	[A3,A13,A17, A30]		
$[M_{i,n,a,l,C}]$	concentration of $M_{i,n,a}$ in soil water = $M_{i,n,a,l,C} / \theta_l$	$\text{g C m}^{-3}$	[A3, A5]		
$[\text{NH}_4^+_{i,n,j,l}]$	concentration of $\text{NH}_4^+$ at microbial surfaces	$\text{g N m}^{-3}$	[A26]		
$[\text{NH}_4^+_{mn}]$	concentration of $\text{NH}_4^+$ at microbial surfaces below which $U_{\text{NH}_4} = 0$	$\text{g N m}^{-3}$	[A26]	0.0125	
$[\text{NO}_3^-_{i,n,j,l}]$	concentration of $\text{NO}_3^-$ at microbial surfaces	$\text{g N m}^{-3}$	[A26]		
$[\text{NO}_3^-_{mn}]$	concentration of $\text{NO}_3^-$ at microbial surfaces below which $U_{\text{NO}_3} = 0$	$\text{g N m}^{-3}$	[A26]	0.03	
$[\text{H}_2\text{PO}_4^-_{i,n,j,l}]$	concentration of $\text{H}_2\text{PO}_4^-$ at microbial surfaces	$\text{g N m}^{-3}$	[A26]		
$[\text{H}_2\text{PO}_4^-_{mn}]$	concentration of $\text{H}_2\text{PO}_4^-$ at microbial surfaces below which $U_{\text{PO}_4} = 0$	$\text{g N m}^{-3}$	[A26]	0.002	
$[\text{O}_{2mi,n,l}]$	$\text{O}_2$ concentration at heterotrophic microsities	$\text{g O}_2 \text{ m}^{-3}$	[A17]		



$[O_{2s,l}]$	$O_2$ concentration in soil solution	$g O_2 m^{-3}$	[A17]		
$Q_{i,l,C}$	DOC from products of $D_{Si,j,l,C}$ [A3] and $D_{Zi,j,l,C}$ [A5]	$g C m^{-2}$	[A8,A13,A22]		
$[Q_{i,l,C}]$	solution concentration of $Q_{i,l,C}$	$g C Mg^{-1}$	[A8,A13]		
$Q_{i,l,N,P}$	DON and DOP from products of $(D_{Si,j,l,N,P} + D_{Zi,j,l,N,P})$	$g N$ or $P m^{-2}$	[A9,A22]		
$q_m$	rate constant for reallocating $M_{i,a,l,C}$ to $M_{i,d,l,C}$	$h^{-1}$	[A3a]	0.5	
$R$	gas constant	$J mol^{-1} K^{-1}$	[A6,A15,C10]	8.3143	
$R_{\phi i,n=f,j,l}$	respiration for non-symbiotic $N_2$ fixation by heterotrophic diazotrophs ( $n = f$ )	$g C m^{-2} h^{-1}$	[A28]		
$R_{gi,n,l}$	growth respiration of $M_{i,n,a,l}$ on $Q_{i,l,C}$ under nonlimiting $O_2$ and nutrients	$g C g C^{-1} h^{-1}$	[A20]		
$R_h$	total heterotrophic respiration of all $M_{i,n,a,l}$ under ambient DOC, $O_2$ , nutrients, $\theta$ and temperature	$g C m^{-2} h^{-1}$	[A11]		
$R_{hi,n,l}$	heterotrophic respiration of $M_{i,n,a,l}$ under ambient DOC, $O_2$ , nutrients, $\theta$ and temperature	$g C m^{-2} h^{-1}$	[A5,A11,A14,A20, A21,A25]		
$R_{hi,n,l}$	specific heterotrophic respiration of $M_{i,n,a,l}$ under nonlimiting $O_2$ , DOC, $\theta$ and $25^\circ C$	$g C g C^{-1} h^{-1}$	[A12,A13]		
$R_h' n$	specific heterotrophic respiration of $M_{i,n,a,l}$ under nonlimiting DOC, $O_2$ , nutrients, $\theta$ and $25^\circ C$	$g C g C^{-1} h^{-1}$	[A12]	0.125	Shields et al. (1973)
$R_h' i,n,l$	heterotrophic respiration of $M_{i,n,a,l}$ under nonlimiting $O_2$ and ambient DOC, nutrients, $\theta$ and temperature	$g C m^{-2} h^{-1}$	[A13,A14,A16]		
$R_m$	specific maintenance respiration at $25^\circ C$	$g C g N^{-1} h^{-1}$	[A18]	0.0115	Barnes et al. (1998)
$R_{mi,n,j,l}$	maintenance respiration by $M_{i,n,j,l}$	$g C m^{-2} h^{-1}$	[A18,A20,A21,A25]		
$r_{wl}$	radius of $r_m$ + water film at current water content	m	[A17]		
$r_m$	radius of heterotrophic microsite	m	[A17]	$2.5 \times 10^{-6}$	
$r_{wl}$	thickness of water films	m	[A17]		

<b>S</b>	change in entropy	$\text{J mol}^{-1} \text{K}^{-1}$	[A6,C10]	710	Sharpe and DeMichelle (1977)
$[S_{i,j,l,C}]$	concentration of $S_{i,j,l,C}$ in soil	$\text{g C Mg}^{-1}$	[A4a]		
$S_{i,j,l,C}$	mass of colonized litter, POC or humus C	$\text{g C m}^{-2}$	[A2a,A5,A7a,A33]		
$S'_{i,j,l,C}$	mass of uncolonized litter, POC or humus C	$\text{g C m}^{-2}$	[A5]		
$S_{i,j,l,N,P}$	mass of litter, POC or humus N or P	$\text{g N or P m}^{-2}$	[A7a,A33]		
$T_{sl}$	soil temperature	K	[A6,A15.A19]		
$U_{i,n,C}$	uptake of $Q_{i,l,C}$ by $\Sigma_n M_{i,n,a,l}$ under limiting nutrient availability	$\text{g C m}^{-2} \text{h}^{-1}$	[A5,A21,A22,A25]		
$U_{i,n,N,P}$	uptake of $Q_{i,l,N,P}$ by $\Sigma_n M_{i,n,a,l}$ under limiting nutrient availability	$\text{g N or P m}^{-2} \text{h}^{-1}$	[A22,A29]		
$U_{\text{NH}_4^+,n,j,l}$	$\text{NH}_4^+$ uptake by microbes	$\text{g N m}^{-2} \text{h}^{-1}$	[A26, A27,A29]		
$U'_{\text{NH}_4}$	maximum $U_{\text{NH}_4}$ at 25 °C and non-limiting $\text{NH}_4^+$	$\text{g N m}^{-2} \text{h}^{-1}$	[A26]	$5.0 \times 10^{-3}$	
$U_{\text{NO}_3^-,n,j,l}$	$\text{NO}_3^-$ uptake by microbes	$\text{g N m}^{-2} \text{h}^{-1}$	[A26,A27,A29]		
$U'_{\text{NO}_3}$	maximum $U_{\text{NO}_3}$ at 25 °C and non-limiting $\text{NO}_3^-$	$\text{g N m}^{-2} \text{h}^{-1}$	[A26]	$5.0 \times 10^{-3}$	
$U_{\text{O}_2,n}$	$\text{O}_2$ uptake by $M_{i,n,a,l}$ under ambient $\text{O}_2$	$\text{g m}^{-2} \text{h}^{-1}$	[A14,A17]		
$U'_{\text{O}_2,n}$	$\text{O}_2$ uptake by $M_{i,n,a,l}$ under nonlimiting $\text{O}_2$	$\text{g m}^{-2} \text{h}^{-1}$	[A14,A16,A17]		
$U_{\text{PO}_4^-,n,j,l}$	$\text{H}_2\text{PO}_4^-$ uptake by microbes	$\text{g N m}^{-2} \text{h}^{-1}$	[A26,A27,A29]		
$U'_{\text{PO}_4}$	maximum $U_{\text{PO}_4}$ at 25 °C and non-limiting $\text{H}_2\text{PO}_4^-$	$\text{g N m}^{-2} \text{h}^{-1}$	[A26]	$5.0 \times 10^{-3}$	
$X_{i,l,C}$	adsorbed C hydrolysis products	$\text{g C Mg}^{-1}$	[A8,A10]		
$X_{i,l,N,P}$	adsorbed N or P hydrolysis products	$\text{g P Mg}^{-1}$	[A10]		
<b>y</b>	selected to give a $Q_{10}$ for $f_{\text{im}}$ of 2.25		[A19]	0.081	

$\psi_s$	soil or residue water potential	MPa	[A15]
$Y_{i,l,C}$	sorption of C hydrolysis products	$\text{g C m}^{-2} \text{ h}^{-1}$	[A8,A9,A10]
$Y_{i,l,N,P}$	sorption of N or P hydrolysis products	$\text{g P m}^{-2} \text{ h}^{-1}$	[A9,A10]
$[Z_{i,j,l,C}]$	concentration of $Z_{i,j,l,C}$ in soil	$\text{g C Mg}^{-1}$	[A4b]
$Z_{i,j,l,C}$	mass of microbial residue C in soil	$\text{g C m}^{-2}$	[A2b,A7b]
$Z_{i,j,l,N,P}$	mass of microbial residue N or P in soil	$\text{g P m}^{-2}$	[A7b]

## Appendix B: Soil-Plant Water Relations

### *Canopy Transpiration*

$Rn_{ci} + LE_{ci} + H_{ci} + G_{ci} = 0$	canopy energy balance	[B1a]
$LE_{ci} = L (e_a - e_{ci(T_{ci}, \psi_{ci})}) / r_{ai}$	$LE$ from canopy evaporation	[B1b]
$LE_{ci} = L (e_a - e_{ci(T_{ci}, \psi_{ci})}) / (r_{ai} + r_{ci}) - LE_{ci}$ from [B1b]	$LE$ from canopy transpiration	[B1c]
$H_{ci} = \rho C_p (T_a - T_{ci}) / r_{ai}$	$H$ from canopy energy balance	[B1d]
$r_{cmini} = 0.64 (C_b - C_i') / V_c'$	$r_c$ driven by rates of carboxylation	[B2a]
$r_{ci} = r_{cmini} + (r_{cmaxi} - r_{cmini}) e^{(-\beta \psi_{ci})}$	vs. diffusion	[B2b]
$r_{ai} = \{(\ln((z_u - z_{di}) / z_{ri})^2 / (K^2 u_a))\} / (1 - 10 Ri)$	$r_c$ constrained by water status	[B3a]
$Ri = \{g (z_u - z_{ri}) / (u_a^2 T_a)\} (T_a - T_c)$	$r_a$ driven by windspeed, surface	[B3b]
$\psi_{ti} = \psi_{ci} - \psi_{\pi i}$	$r_a$ adjusted for stability vs. buoyancy	[B4]

### *Root and Mycorrhizal Water Uptake*

$U_{wi} = \sum_l \sum_r U_{wi,r,l}$		[B5]
$U_{wi,r,l} = (\psi_{c'i} - \psi_{s'l}) / (\Omega_{si,r,l} + \Omega_{ri,r,l} + \sum_x \Omega_{ai,r,l,x})$	$U_w$ along hydraulic gradient	[B6]
$\psi_{c'i} = \psi_{ci} + 0.01 z_{bi}$		[B7]
$\psi_{s'l} = \psi_{sl} - 0.01 z_l$		[B8]
$\Omega_{si,r,l} = \ln\{(d_{i,r,l} / r_{i,r,l}) / (2\pi L_{i,r,l} \kappa_{ri,r,l})\} \theta_{wl} / \theta_{pl}$		[B9]
$\Omega_{ri,r,l} = \Omega_{ri,r} / L_{i,r,l}$		[B10]
$\Omega_{ai,r,l,x=1} = \Omega_{ai,r} z_l / \{n_{i,r,l,1} (r_{i,r,l,1} / r'_{i,r})^4\} + \gamma \Omega_{ai,r} z_{bi} / \{n_{i,r,l,1} (r_{bi} / r'_b)^4\} \sum_{i,r,l} (M_{i,r,l}) / M_{i,r,l}$		[B11]
$\Omega_{ai,r,l,x=2} = \Omega_{ai,r} (L_{i,r,l,2} / n_{i,r,l,2}) / \{n_{i,r,l,2} (r_{i,r,l,2} / r'_{i,r})^4\}$		[B12]

$$\delta L_{i,r,l} / \delta t = \delta M_{i,r,l} / \delta t v_r / \{ \rho_r (1 - \theta_{p_{i,r}}) (\pi r_{i,r,l}^2) \} \quad [\text{B13}]$$

*Canopy Water Potential*

$$(e_a - e_{i(T_{ci})}) / (r_{ai} + r_{ci}) [\text{B1}] = \sum_l \sum_r (\psi_{c'i} - \psi_{s'l}) / (\Omega_{si,r,l} + \Omega_{ri,r,l} + \sum_x \Omega_{ai,r,l,x}) + X_{ci} \delta \psi_{ci} / \delta t \quad [\text{B14}]$$

$\psi_c$  solved when transpiration from [B1-B4] (LHS) equals uptake from [B5-B13] + change in storage (RHS)

### Definition of Variables in Appendix B

Variable	Definition	Unit	Equation	Value	Reference
<i>subscripts</i>					
<i>i</i>	plant species or functional type: coniferous, deciduous, annual, perennial, C <sub>3</sub> , C <sub>4</sub> , monocot, dicot etc.				
<i>j</i>	branch or tiller				
<i>k</i>	node				
<i>l</i>	soil or canopy layer				
<i>m</i>	leaf azimuth				
<i>n</i>	leaf inclination				
<i>o</i>	leaf exposure (sunlit vs. shaded)				
<i>r</i>	root or mycorrhizae				
<i>variables</i>					
$\beta$	stomatal resistance shape parameter	MPa <sup>-1</sup>	[B2b,C4,C9]	-5.0	Grant and Flanagan (2007)

$C_b$	[CO <sub>2</sub> ] in canopy air	$\mu\text{mol mol}^{-1}$	[B2,C2,C5]		
$C_i'$	[CO <sub>2</sub> ] in canopy leaves at $\psi_{c_i} = 0$ MPa	$\mu\text{mol mol}^{-1}$	[B2]	$0.70 C_b$	Larcher (2001)
$d_{i,r,l}$	half distance between adjacent roots	m	[B9]		
$E_{ci}$	canopy transpiration	$\text{m}^3 \text{m}^{-2} \text{h}^{-1}$	[B1,B14]		
$e_a$	atmospheric vapor density at $T_a$ and ambient humidity	$\text{g m}^{-3}$	[B1]		
$e_{ci(T_{ci}, \psi_{ci})}$	canopy vapor density at $T_{ci}$ and $\psi_{ci}$	$\text{g m}^{-3}$	[B1]		
$G_{ci}$	canopy storage heat flux	$\text{W m}^{-2}$	[B1]		
$H_{ci}$	canopy sensible heat flux	$\text{W m}^{-2}$	[B1]		
$K$	von Karman's constant		[B3a]	0.41	
$\kappa_{i,r,l}$	hydraulic conductivity between soil and root surface	$\text{m}^2 \text{MPa}^{-1} \text{h}^{-1}$	[B9]		
$\gamma$	scaling factor for bole axial resistance from primary root axial resistance	-	[B11]	$1.6 \times 10^4$	Grant et al. (2007)
$L$	latent heat of evaporation	$\text{J g}^{-1}$	[B1]	2460	
$LE_{ci}$	latent heat flux between canopy and atmosphere	$\text{W m}^{-2}$	[B1]		
$L_{i,r,l}$	length of roots or mycorrhizae	$\text{m m}^{-2}$	[B9,B10,B12,B13]		
$M_{i,r,l}$	mass of roots or mycorrhizae	$\text{g m}^{-2}$	[B11,B13]		
$n_{i,r,l,x}$	number of primary ( $x = 1$ ) or secondary ( $x = 2$ ) axes	$\text{m}^{-2}$	[B11,B12]		
$\mathcal{R}_{ai,r}$	axial resistivity to water transport along root or mycorrhizal axes	$\text{MPa h m}^{-4}$	[B11,B12]	$4.0 \times 10^9$ deciduous $1.0 \times 10^{10}$ coniferous	Larcher (2001)
$\mathcal{R}_{ai,r,l,x}$	axial resistance to water transport along axes of primary ( $x = 1$ ) or secondary ( $x = 2$ ) roots or mycorrhizae	$\text{MPa h m}^{-1}$	[B6,B11,B12]		

$\Omega_{ri,r}$	radial resistivity to water transport from surface to axis of roots or mycorrhizae	MPa h m <sup>-2</sup>	[B10]	1.0 x 10 <sup>4</sup>	Doussan et al. (1998)
$\Omega_{ri,r,l}$	radial resistance to water transport from surface to axis of roots or mycorrhizae	MPa h m <sup>-1</sup>	[B6,B10]		
$\Omega_{si,r,l}$	radial resistance to water transport from soil to surface of roots or mycorrhizae	MPa h m <sup>-1</sup>	[B6,B9]		
$\theta_{wl}$	soil water content	m <sup>3</sup> m <sup>-3</sup>	[B9]		
$\theta_{pl}$	soil porosity	m <sup>3</sup> m <sup>-3</sup>	[B9]		
$\theta_{Pi,r}$	root porosity	m <sup>3</sup> m <sup>-3</sup>	[B13]		
$Ri$	Richarson number		[B3a,B3b]		van Bavel and Hillel (1976)
$Rn_{ci}$	canopy net radiation	W m <sup>-2</sup>	[B1]		
$r_{ai}$	aerodynamic resistance to vapor flux from canopy	s m <sup>-1</sup>	[B1,B3a]		
$r_{bi}$	radius of bole at ambient $\psi_{ci}$	m	[B11]		
$r_{b'i}$	radius of bole at $\psi_{ci} = 0$ MPa	m	[B11]		
$r_{ci}$	canopy stomatal resistance to vapor flux	s m <sup>-1</sup>	[B1,B2b]		
$r_{cmaxi}$	canopy cuticular resistance to vapor flux	s m <sup>-1</sup>	[B2b]	5.0 x 10 <sup>3</sup>	Larcher (2001)
$r_{cmini}$	minimum $r_{ci}$ at $\psi_{ci} = 0$ MPa	s m <sup>-1</sup>	[B2,B2b]		
$r_{i,r,l,x}$	radius of primary ( $x=1$ ) or secondary ( $x=2$ ) roots or mycorrhizae at ambient $\psi_{ri,l,z}$	m	[B9,B11,B12,B13]		
$r'_{i,r}$	radius of secondary roots or mycorrhizae at $\psi_{ri,l,z} = 0$ MPa	m	[B11,B12]	2.0 x 10 <sup>-4</sup> tree 1.0 x 10 <sup>-4</sup> bush 0.05 x 10 <sup>-4</sup> mycorrhizae	
$\rho_r$	root specific density	g C g FW <sup>-1</sup>	[B13]	0.05	Grant (1998)
$T_a$	air temperature	K	[B3b]		

$T_c$	canopy temperature	K	[B3b]		
$U_{wi}$	total water uptake from all rooted soil layers	$m^3 m^{-2} h^{-1}$	[B5,B14]		
$U_{wi,r,l}$	water uptake by root and mycorrhizal surfaces in each soil layer	$m^3 m^{-2} h^{-1}$	[B5,B6]		
$u_a$	wind speed measured at $z_u$	$m s^{-1}$	[B3a,B3b]		
$V_c'$	potential canopy CO <sub>2</sub> fixation rate at $\psi_{c_i} = 0$ MPa	$\mu mol m^{-2} s^{-1}$	[B2]		
$v_r$	root specific volume	$m^3 g FW^{-1}$	[B13]	$10^{-6}$	Grant (1998)
$X_{ci}$	canopy capacitance	$m^3 m^{-2} MPa^{-1}$	[B14]		
$\psi_{ci}$	canopy water potential	MPa	[B4,B7,B14]		
$\psi_{c'i}$	$\psi_{ci} +$ canopy gravitational potential	MPa	[B6,B7]		
$\psi_{\pi i}$	canopy osmotic potential	MPa	[B4]		
$\psi_{sl}$	soil water potential	MPa	[B8]		
$\psi_{s'l}$	$\psi_{sl} +$ soil gravitational potential	MPa	[B6,B8]		
$\psi_{ti}$	canopy turgor potential	MPa	[B2b,B4]	1.25 at $\psi_c = 0$	
$z_{bi}$	length of bole from soil surface to top of canopy	m	[B7,B11]		
$z_{di}$	canopy zero-plane displacement height	m	[B3a]		Perrier (1982)
$z_l$	depth of soil layer below surface	m	[B8,B11]		
$z_r$	canopy surface roughness	m	[B3a,B3b]		Perrier (1982)
$z_u$	height of wind speed measurement	m	[B3a,B3b]		



## Appendix C: Gross Primary Productivity, Autotrophic Respiration, Growth and Litterfall

### *C<sub>3</sub> Gross Primary Productivity*

$GPP = \sum_{i,j,k,l,m,n,o} (V_{ci,j,k,l,m,n,o} = V_{gi,j,k,l,m,n,o}) A_{i,j,k,l,m,n,o}$	solve for $C_{ii,j,k,l,m,n,o}$ at which	[C1]
$V_{gi,j,k,l,m,n,o} = (C_b - C_{ii,j,k,l,m,n,o}) / r_{li,j,k,l,m,n,o}$	$V_{ci,j,k,l,m,n,o} = V_{gi,j,k,l,m,n,o}$ diffusion	[C2]
$V_{ci,j,k,l,m,n,o} = \min\{V_{bi,j,k,l,m,n,o}, V_{ji,j,k,l,m,n,o}\}$	carboxylation	[C3]
$r_{li,j,k,l,m,n,o} = r_{lmini,j,k,l,m,n,o} + (r_{lmaxi} - r_{lmini,j,k,l,m,n,o}) e^{(-\beta \psi_{li})}$	$r_l$ is leaf-level equivalent of $r_c$	[C4]
$r_{lmini,j,k,l,m,n,o} = (C_b - C_i') / V_c'_{i,j,k,l,m,n,o}$	minimum $r_l$ is driven by carboxylation	[C5]
$V_{bi,j,k,l,m,n,o} = V_{bmaxi,j,k} (C_{ci,j,k,l,m,n,o} - \Gamma_{i,j,k}) / (C_{ci,j,k,l,m,n,o}) + K_{ci} f_{\psi_{i,j,k,l,m,n,o}}$	CO <sub>2</sub> and water $f_{\psi}$ constraints on $V_b$	[C6a]
$V_{bmaxi,j,k} = V_{b'i} F_{rubisco_i} M_{L_{Ri,j,k}} / A_{i,j,k} f_{tbi} f_{ci}$	temperature $f_{tb}$ and nutrient $f_{ic}$ constraints on $V_{bmax}$	[C6b]
$\Gamma_{i,j,k} = 0.5 O_c V_{omaxi,j,k} K_{ci} / (V_{bmaxi,j,k} K_{oi})$	CO <sub>2</sub> compensation point	[C6c]
$V_{omaxi,j,k} = V_{o'i} F_{rubisco_i} M_{L_{Ri,j,k}} / A_{i,j,k} f_{toi}$	oxygenation	[C6d]
$K_{ci} = K_{ci} f_{tkci} (1 + O_c / (K_{oi} f_{tkoi}))$	M-M constant for $V_b$	[C6e]
$V_{ji,j,k,l,m,n,o} = J_{i,j,k,l,m,n,o} Y_{i,j,k,l,m,n,o} f_{\psi_{i,j,k,l,m,n,o}}$	water constraints on $V_j$	[C7a]
$Y_{i,j,k,l,m,n,o} = (C_{ci,j,k,l,m,n,o} - \Gamma_{i,j,k}) / (4.5 C_{ci,j,k,l,m,n,o} + 10.5 \Gamma_{i,j,k})$	carboxylation efficiency of $V_j$	[C7b]
$J_{i,j,k,l,m,n,o} = (\epsilon I_{i,l,m,n,o} + J_{maxi,j,k} - ((\epsilon I_{i,l,m,n,o} + J_{maxi,j,k})^2 - 4\alpha\epsilon I_{i,l,m,n,o} J_{maxi,j,k})^{0.5}) / (2\alpha)$	irradiance constraints on $J$	[C8a]
$J_{maxi,j,k} = V_j' F_{chlorophyll_i} M_{i,j,k,prot} / A_{i,j,k} f_{tji} f_{ci}$	temperature and nutrient constraints on $J_{max}$	[C8b]
$f_{\psi_{i,j,k,l,m,n,o}} = (r_{lmini,j,k,l,m,n,o} / r_{li,j,k,l,m,n,o})^{0.5}$	non-stomatal effect related to stomatal effect	[C9]

$$f_{tbi} = \exp[\mathbf{B}_v - \mathbf{H}_{av} / (RT_{ci})] / \{1 + \exp[(\mathbf{H}_{dl} - ST_{ci}) / (RT_{ci})] + \exp[(ST_{ci} - \mathbf{H}_{dh}) / (RT_{ci})]\}$$

Arrhenius functions for [C10a]

$$f_{toi} = \exp[\mathbf{B}_o - \mathbf{H}_{ao} / (RT_{ci})] / \{1 + \exp[(\mathbf{H}_{dl} - ST_{ci}) / (RT_{ci})] + \exp[(ST_{ci} - \mathbf{H}_{dh}) / (RT_{ci})]\}$$

carboxylation, oxygenation and [C10b]

$$f_{tji} = \exp[\mathbf{B}_j - \mathbf{H}_{aj} / (RT_{ci})] / \{1 + \exp[(\mathbf{H}_{dl} - ST_{ci}) / (RT_{ci})] + \exp[(ST_{ci} - \mathbf{H}_{dh}) / (RT_{ci})]\}$$

electron transport [C10c]

$$f_{tkci} = \exp[\mathbf{B}_{kc} - \mathbf{H}_{akc} / (RT_{ci})]$$

temperature sensitivity of  $\mathbf{K}_{c_i}$ ,  $\mathbf{K}_{o_i}$  [C10d]

$$f_{tkoi} = \exp[\mathbf{B}_{ko} - \mathbf{H}_{ako} / (RT_{ci})]$$

[C10e]

$$f_{iCi} = \min\{\sigma_{Ni,j} / (\sigma_{Ni,j} + \sigma_{Ci,j} / \mathbf{K}_{iCN}), \sigma_{Pi,j} / (\sigma_{Pi,j} + \sigma_{Ci,j} / \mathbf{K}_{iCP})\}$$

control of  $\sigma_N$  and  $\sigma_P$  vs.  $\sigma_C$  in [C11]

shoots on  $V_b$ ,  $V_j$  through product inhibition and on leaf protein growth through leaf structural C:N:P ratios

$$\delta M_{LRi,j,k} / \delta t = \delta M_{Li,j,k} / \delta t \min\{[N'_{leaf} + (N_{leaf} - N'_{leaf}) f_{iCi}] / N_{prot}, [P'_{leaf} + (P_{leaf} - P'_{leaf}) f_{iCi}] / P_{prot}\}$$

growth of remobilizable leaf protein C [C12]

### Autotrophic Respiration

$$R_a = \sum_i \sum_j (R_{ci,j} + R_{si,j}) + \sum_i \sum_z (R_{ci,r,l} + R_{si,r,l}) + \mathbf{E}_{N,P} (U_{NH4i,r,l} + U_{NO3i,r,l} + U_{PO4i,r,l})$$

total autotrophic respiration [C13]

$$R_{ci,j} = \mathbf{R}_c' \sigma_{Ci,j} f_{tai}$$

$O_2$  constraint on root respiration [C14a]

$$R_{ci,r,l} = \mathbf{R}_c' \sigma_{Ci,r,l} f_{tai} (U_{O2i,r,l} / U'_{O2i,r,l})$$

from active uptake coupled with [C14b]

$$U_{O2i,r,l} = U'_{O2i,r,l} [O_{2ri,r,l}] / ([O_{2ri,r,l}] + \mathbf{K}_{O_2})$$

diffusion of  $O_2$  from soil as for [C14c]

heterotrophic respiration in [A17], [C14c]

and from active uptake coupled [C14c]

with diffusion of  $O_2$  from roots [C14d]

$$= U_{wi,r,l} [O_{2sl}] + 2\pi L_{i,r,l} D_{sO_2} ([O_{2sl}] - [O_{2ri,r,l}]) \ln\{(r_{sl} + r_{ri,r,l}) / r_{ri,r,l}\} + 2\pi L_{i,r,l} D_{rO_2} ([O_{2qi,r,l}] - [O_{2ri,r,l}]) \ln(r_{qi,r,l} / r_{ri,r,l})$$

[C14d]

$$U'_{O2i,r,l} = 2.67 R_{a'i,r,l}$$

[C14e]

$$R_{si,j} = -\min\{0.0, R_{ci,j} - R_{mi,j}\}$$

remobilization when  $R_m > R_c$  [C15]

$$R_{mi,j} = \sum_z (N_{i,j,z} \mathbf{R}_m' f_{umi})$$

maintenance respiration [C16]

$$R_{gi,j} = \max\{0.0, \min\{R_{ci,j} - R_{mi,j}\} \min\{1.0, \max\{0.0, \psi_{ti} - \psi_{ti}'\}\}\}$$

growth when  $R_m < R_c$  [C17]

## Growth and Litterfall

$l_{i,j,z,C} = R_{si,j} M_{L_N^{i,j}} / M_{L_R^{i,j}}$	senescence drives litterfall of non-remobilizable material	[C18]
$l_{i,j,z,N} = l_{i,j,z,C} \mathbf{N}_{\text{prot}} (1.0 - \mathbf{X}_{\text{mx}} f_{\lambda N i,j})$	litterfall of N and P is driven by that of C but reduced by	[C19a]
$l_{i,j,z,P} = l_{i,j,z,C} \mathbf{P}_{\text{prot}} (1.0 - \mathbf{X}_{\text{mx}} f_{\lambda P i,j})$	translocation to $\sigma_N$ and $\sigma_P$	[C19b]
$f_{\lambda N i,j} = \sigma_{C i,j} / (\sigma_{C i,j} + \sigma_{N i,j} / \mathbf{K}_{\lambda N})$	according to ratios of $\sigma_N$ and $\sigma_P$ with $\sigma_C$ , root and mycorrhizal	[C19c]
$f_{\lambda P i,j} = \sigma_{C i,j} / (\sigma_{C i,j} + \sigma_{P i,j} / \mathbf{K}_{\lambda P})$	litterfall ( $i, r, l$ ) calculated as for branch litterfall ( $i, j, z$ )	[C19d]
$x_{i,r,l,C} = r_x \sigma_{C i,r,l}$	root and mycorrhizal exudation driven by $\sigma_C$ , $\sigma_N$ and $\sigma_P$ , and by $\sigma_C: \sigma_N$ and $\sigma_C: \sigma_P$ .	[C19e]
$x_{i,r,l,N} = r_x \sigma_{N i,r,l} f_{x i,r,l,N}$		[C19f]
$x_{i,r,l,P} = r_x \sigma_{P i,r,l} f_{x i,r,l,P}$		[C19g]
$f_{x i,r,l,N} = \sigma_{N i,j} / (\sigma_{N i,j} + \sigma_{C i,j} / \mathbf{K}_{xN})$		[C19h]
$f_{x i,r,l,P} = \sigma_{P i,j} / (\sigma_{P i,j} + \sigma_{C i,j} / \mathbf{K}_{xP})$		[C19i]
$\delta M_{B i,j} / \delta t = \sum_z [R_{g i,j} (1 - \mathbf{Y}_{g i,z}) / \mathbf{Y}_{g i,z}] - R_{s i,j} - l_{i,j,C}$	branch growth driven by $R_g$	[C20a]
$\delta M_{R i,r,l} / \delta t = [R_{g i,r,l} (1 - \mathbf{Y}_{g i,r}) / \mathbf{Y}_{g i,r}] - R_{s i,r,l} - l_{i,r,l,C}$	root growth driven by $R_g$	[C20b]
$\delta A_{L i,j,k,l} / \delta t = \chi (M_{L i,j,k,l} / y_i)^{-0.33} \delta M_{L i,j,k,l} / \delta t \min\{1, \max\{0, \psi_i - \psi_i'\}\}$	leaf expansion driven by leaf mass growth	[C21a]
$\delta L_{i,r,l,1} / \delta t = (\delta M_{R i,r,l,1} / \delta t) / y_i v_r / \{\rho_r (1 - \theta_{P i,r}) (\pi r_{i,r,l,1}^2)\}$	root extension of primary and secondary axes driven by root mass growth	[C21b]
$\delta L_{i,r,l,2} / \delta t = (\delta M_{R i,r,l,2} / \delta t) v_r / \{\rho_r (1 - \theta_{P i,r}) (\pi r_{i,r,l,2}^2)\}$		[C21c]
$f_{\text{tai}} = T_{ci} \{\exp[\mathbf{B}_v - \mathbf{H}_{av} / (\mathbf{RT}_{ci})]\} / \{1 + \exp[(\mathbf{H}_{dl} - \mathbf{ST}_{ci}) / (\mathbf{RT}_{ci})] + \exp[(\mathbf{ST}_{ci} - \mathbf{H}_{dh}) / (\mathbf{RT}_{ci})]\}$	Arrhenius function for $R_a$	[C22a]
$f_{\text{umi}} = e^{(0.0811 (T_{ci} - 298.15))}$	temperature function for $R_m$	[C22b]

### Root and Mycorrhizal Nutrient Uptake

$$U_{\text{NH}_4^+,i,r,l} = \{U_{\text{wi},r,l}[\text{NH}_4^+] + 2\pi L_{i,r,l} D_{\text{eNH}_4} ([\text{NH}_4^+] - [\text{NH}_4^+_{i,r,l}]) / \ln(d_{i,r,l}/r_{\text{ri},r,l})\}$$

$$= U'_{\text{NH}_4} (U_{\text{O}_2,i,r,l}/U'_{\text{O}_2,i,r,l}) A_{i,r,l} ([\text{NH}_4^+_{i,r,l}] - [\text{NH}_4^+_{\text{mn}}]) / ([\text{NH}_4^+_{i,r,l}] - [\text{NH}_4^+_{\text{mn}}] + K_{\text{NH}_4}) f_{\text{tai},l} f_{\text{Ni},r,l}$$

root N and P uptake from mass flow + diffusion coupled with active uptake of  $\text{NH}_4^+$ ,  $\text{NO}_3^-$  and  $\text{H}_2\text{PO}_4^-$  constrained by  $\text{O}_2$  uptake, as for microbial N and P uptake in [A26] [C23a] [C23b] [C23c] [C23d] [C23e] [C23f]

$$U_{\text{NO}_3^-,i,r,l} = \{U_{\text{wi},r,l}[\text{NO}_3^-] + 2\pi L_{i,r,l} D_{\text{eNO}_3} ([\text{NO}_3^-] - [\text{NO}_3^-_{i,r,l}]) / \ln(d_{i,r,l}/r_{\text{ri},r,l})\}$$

$$= U'_{\text{NO}_3} (U_{\text{O}_2,i,r,l}/U'_{\text{O}_2,i,r,l}) A_{i,r,l} ([\text{NO}_3^-_{i,r,l}] - [\text{NO}_3^-_{\text{mn}}]) / ([\text{NO}_3^-_{i,r,l}] - [\text{NO}_3^-_{\text{mn}}] + K_{\text{NO}_3}) f_{\text{tai},l} f_{\text{Ni},r,l}$$

$$U_{\text{PO}_4^-,i,r,l} = \{U_{\text{wi},r,l}[\text{H}_2\text{PO}_4^-] + 2\pi L_{i,r,l} D_{\text{ePO}_4} ([\text{H}_2\text{PO}_4^-] - [\text{H}_2\text{PO}_4^-_{i,r,l}]) / \ln(d_{i,r,l}/r_{\text{ri},r,l})\}$$

$$= U'_{\text{PO}_4} (U_{\text{O}_2,i,r,l}/U'_{\text{O}_2,i,r,l}) A_{i,r,l} ([\text{H}_2\text{PO}_4^-_{i,r,l}] - [\text{H}_2\text{PO}_4^-_{\text{mn}}]) / ([\text{H}_2\text{PO}_4^-_{i,r,l}] - [\text{H}_2\text{PO}_4^-_{\text{mn}}] + K_{\text{PO}_4}) f_{\text{tai},l} f_{\text{Pi},r,l}$$

$$f_{\text{Ni},r,l} = \sigma_{\text{Ci},r,l} / (\sigma_{\text{Ci},r,l} + \sigma_{\text{Ni},r,l} / K_{\text{INc}})$$

product inhibition of  $U_{\text{NH}_4}$ ,  $U_{\text{NO}_3}$  and  $U_{\text{PO}_4}$  determined by  $\sigma_{\text{N}}$  and  $\sigma_{\text{P}}$  vs.  $\sigma_{\text{C}}$  in roots [C23g] [C23h]

$$f_{\text{Pi},r,l} = \sigma_{\text{Ci},r,l} / (\sigma_{\text{Ci},r,l} + \sigma_{\text{Pi},r,l} / K_{\text{IPc}})$$

### C<sub>4</sub> Gross Primary Productivity

#### C<sub>4</sub> Mesophyll

$$GPP = \sum_{i,j,k,l,m,n,o} (V_{\text{g(m4)}i,j,k,l,m,n,o} - V_{\text{c(m4)}i,j,k,l,m,n,o})$$

[C24]

$$V_{\text{g(m4)}i,j,k,l,m,n,o} = (C_{\text{b}} - C_{\text{i(m4)}i,j,k,l,m,n,o}) / r_{\text{fi},j,k,l,m,n,o}$$

gaseous diffusion [C25]

$$V_{\text{c(m4)}i,j,k,l,m,n,o} = \min\{V_{\text{b(m4)}i,j,k,l,m,n,o}, V_{\text{j(m4)}i,j,k,l,m,n,o}\}$$

mesophyll carboxylation [C26]

$$r_{\text{fi},j,k,l,m,n,o} = r_{\text{ifmini},j,k,l,m,n,o} + (r_{\text{ifmaxi}} - r_{\text{ifmini},j,k,l,m,n,o}) e^{(-\beta \psi_i)}$$

[C27]

$$r_{\text{ifmini},j,k,l,m,n,o} = (C_{\text{b}} - C_{\text{i(m4)}i}) / V_{\text{c0(m4)}i,j,k,l,m,n,o}$$

[C28]

$$V_{\text{b(m4)}i,j,k,l,m,n,o} = V_{\text{bmax(m4)}i,j,k} (C_{\text{c(m4)}i,j,k,l,m,n,o} - \Gamma_{\text{(m4)}i,j,k}) / (C_{\text{c(m4)}i,j,k,l,m,n,o} + K_{\text{c(m4)}i})$$

$\text{CO}_2$ -limited carboxylation [C29]

$$V_{\text{j(m4)}i,j,k,l,m,n,o} = J_{\text{(m4)}i,j,k,l,m,n,o} Y_{\text{(m4)}i,j,k,l,m,n,o}$$

light-limited carboxylation [C30a]

$$Y_{\text{(m4)}i,j,k} = (C_{\text{c(m4)}i,j,k,l,m,n,o} - \Gamma_{\text{(m4)}i,j,k}) / (3.0 C_{\text{c(m4)}i,j,k,l,m,n,o} + 10.5 \Gamma_{\text{(m4)}i,j,k})$$

[C30b]

$$J_{\text{(m4)}i,j,k,l,m,n,o} = (\varepsilon I_{i,l,m,n,o} + J_{\text{max(m4)}i,j,k} - ((\varepsilon I_{i,l,m,n,o} + J_{\text{max(m4)}i,j,k})^2 - 4\alpha\varepsilon I_{i,l,m,n,o} J_{\text{max(m4)}i,j,k})^{0.5}) / (2\alpha)$$

irradiance response function [C31]

$$V_{\text{bmax}(m4)_{i,j,k}} = V_{\text{bmax}(m4)}' [N_{\text{pep}(m4)_{i,j,k}}]' N_{\text{lf}(i,j,k)} A_{\text{lf}(i,j,k)} f_{\text{C}(m4)_{i,j,k}} f_{\psi i} f_{\text{tvi}} \quad \text{PEPc activity} \quad [\text{C32}]$$

$$J_{\text{max}(m4)_{i,j,k}} = J_{\text{max}}' [N_{\text{chl}(m4)_{i,j,k}}]' N_{\text{lf}(i,j,k)} A_{\text{lf}(i,j,k)} f_{\text{C}(m4)_{i,j,k}} f_{\psi i} f_{\text{tvi}} \quad \text{chlorophyll activity} \quad [\text{C33}]$$

$$f_{\text{C}(m4)_{i,j,k}} = 1.0 / (1.0 + [\chi_{\text{C4}(m4)_{i,j,k}}] / K_{\chi_{\text{C4}(m4)}}) \quad \text{C}_4 \text{ product inhibition} \quad [\text{C34}]$$

$$f_{\psi i,j,k,l,m,n,o} = (r_{\text{lfmin}(i,j,k,l,m,n,o)} / r_{\text{lf}(i,j,k,l,m,n,o)})^{0.5} \quad \text{non-stomatal water limitation} \quad [\text{C35}]$$

$$f_{\text{tvi}} = T_{\text{ci}} \{ \exp[B - H_a / (RT_{\text{ci}})] \} / \{ 1 + \exp[(H_{\text{dl}} - ST_{\text{ci}}) / (RT_{\text{ci}})] + \exp[(ST_{\text{ci}} - H_{\text{dh}}) / (RT_{\text{ci}})] \} \quad \text{temperature limitation} \quad [\text{C36}]$$

### *C<sub>4</sub> Mesophyll-Bundle Sheath Exchange*

$$V_{\chi_{\text{C4}(m4)_{i,j,k}}} = \kappa_{\chi_{\text{C4}(m4)}} (\chi_{\text{C4}(m4)_{i,j,k}} W_{\text{lf}(b4)_{i,j,k}} - \chi_{\text{C4}(b4)_{i,j,k}} W_{\text{lf}(m4)_{i,j,k}}) / (W_{\text{lf}(b4)_{i,j,k}} + W_{\text{lf}(m4)_{i,j,k}}) \quad \text{mesophyll-bundle sheath transfer} \quad [\text{C37}]$$

$$V_{\chi_{\text{C4}(b4)_{i,j,k}}} = \kappa_{\chi_{\text{C4}(b4)}} \chi_{\text{C4}(b4)_{i,j,k}} / (1.0 + C_{\text{c}(b4)_{i,j,k}} / K_{\chi_{\text{C4}(b4)}}) \quad \text{bundle sheath decarboxylation} \quad [\text{C38}]$$

$$V_{\phi(b4)_{i,j,k}} = \kappa_{\text{C}_{\text{c}(b4)}} (C_{\text{c}(b4)_{i,j,k}} - C_{\text{c}(m4)_{i,j,k}}) (12 \times 10^{-9}) W_{\text{lf}(b4)_{i,j,k}} \quad \text{bundle sheath-mesophyll leakage} \quad [\text{C39}]$$

$$\delta \chi_{\text{C4}(m4)_{i,j,k}} / \delta t = \sum_{l,m,n,o} V_{\text{c}(m4)_{i,j,k,l,m,n,o}} - V_{\chi_{\text{C4}(m4)_{i,j,k}}} \quad \text{mesophyll carboxylation products} \quad [\text{C40}]$$

$$\delta \chi_{\text{C4}(b4)_{i,j,k}} / \delta t = V_{\chi_{\text{C4}(m4)_{i,j,k}}} - V_{\chi_{\text{C4}(b4)_{i,j,k}}} \quad \text{bundle sheath carboxylation products} \quad [\text{C41}]$$

$$\delta C_{\text{c}(b4)_{i,j,k}} / \delta t = V_{\chi_{\text{C4}(b4)_{i,j,k}}} - V_{\phi(b4)_{i,j,k}} - \sum_{l,m,n,o} V_{\text{c}(b4)_{i,j,k,l,m,n,o}} \quad \text{bundle sheath CO}_2 \text{ concentration} \quad [\text{C42}]$$

### *C<sub>4</sub> Bundle Sheath*

$$V_{\text{c}(b4)_{i,j,k,l,m,n,o}} = \min\{V_{\text{b}(b4)_{i,j,k}}, V_{\text{j}(b4)_{i,j,k,l,m,n,o}}\} \quad \text{bundle sheath carboxylation} \quad [\text{C43}]$$

$$V_{\text{b}(b4)_{i,j,k}} = V_{\text{bmax}(b4)_{i,j,k}} (C_{\text{c}(b4)_{i,j,k}} - \Gamma_{(b4)_{i,j,k}}) / (C_{\text{c}(b4)_{i,j,k}} + K_{\text{c}(b4)_i}) \quad \text{CO}_2\text{-limited carboxylation} \quad [\text{C44}]$$

$$V_{\text{j}(b4)_{i,j,k,l,m,n,o}} = J_{(b4)_{i,j,k,l,m,n,o}} Y_{(b4)_{i,j,k}} \quad \text{light- limited carboxylation} \quad [\text{C45a}]$$

$$Y_{(b4)_{i,j,k}} = (C_{\text{c}(b4)_{i,j,k}} - \Gamma_{(b4)_{i,j,k}}) / (4.5 C_{\text{c}(b4)_{i,j,k}} + 10.5 \Gamma_{(b4)_{i,j,k}}) \quad \text{carboxylation efficiency of } V_{\text{j}(b4)} \quad [\text{C45b}]$$

$$J_{(b4)_{i,j,k,l,m,n,o}} = (\varepsilon I_{i,l,m,n,o} + J_{\text{max}(b4)_{i,j,k}} - ((\varepsilon I_{i,l,m,n,o} + J_{\text{max}(b4)_{i,j,k}})^2 - 4\alpha\varepsilon I_{i,l,m,n,o} J_{\text{max}(b4)_{i,j,k}})^{0.5}) / (2\alpha) \quad \text{irradiance response function} \quad [\text{C46}]$$

$$V_{\text{bmax}(b4)_{i,j,k}} = V_{\text{bmax}(b4)}' [N_{\text{rub}(b4)_{i,j,k}}]' N_{\text{lf}_{i,j,k}} A_{\text{lf}_{i,j,k}} f_{\text{C}(c3)_{i,j,k}} f_{\psi_i} f_{\text{vi}} \quad \text{RuBPC activity} \quad [\text{C47}]$$

$$J_{\text{max}(b4)_{i,j,k}} = J_{\text{max}}' [N_{\text{chl}(b4)_{i,j,k}}]' N_{\text{lf}_{i,j,k}} A_{\text{lf}_{i,j,k}} f_{\text{C}(c3)_{i,j,k}} f_{\psi_i} f_{\text{vi}} \quad \text{chlorophyll activity} \quad [\text{C48}]$$

$$f_{\text{C}(c3)_{i,j,k}} = \min\{ [v_{\text{lf}_{i,j}}] / ([v_{\text{lf}_{i,j}}] + [\chi_{\text{c3}(b4)_{i,j}}] / K_{\text{VIF}}), [\pi_{\text{lf}_{i,j}}] / ([\pi_{\text{lf}_{i,j}}] + [\chi_{\text{c3}(b4)_{i,j}}] / K_{\text{TPF}}) \} \quad \text{C}_3 \text{ product inhibition} \quad [\text{C49}]$$

### *Shoot – Root - Mycorrhizal C, N, P Transfer*

$$Z_{s\text{C}_{i,j-i,r,l}} = g_{s\text{C}_{i,j-i,r,l}} (\sigma_{\text{C}_{i,j}} M_{\text{R}_{i,r,l}} - \sigma_{\text{C}_{i,r,l}} M_{\text{B}_{i,j}}) / (M_{\text{R}_{i,r,l}} + M_{\text{B}_{i,j}}) \quad \text{shoot – root C transfer driven by} \quad [\text{C50}]$$

$$Z_{s\text{N,P}_{i,j-i,r,l}} = g_{s\text{N,P}_{i,j-i,r,l}} (\sigma_{\text{N,P}_{i,j}} \sigma_{\text{C}_{i,r,l}} - \sigma_{\text{N,P}_{i,r,l}} \sigma_{\text{C}_{i,j}}) / (\sigma_{\text{C}_{i,r,l}} + \sigma_{\text{C}_{i,j}}) \quad \text{shoot – root N,P transfer driven by} \quad [\text{C51}]$$

$$Z_{r\text{C}_{i,j-i,r,l}} = g_{r\text{C}_{i,j-i,r,l}} (\sigma_{\text{C}_{i,r,l}} M_{\text{M}_{i,m,l}} - \sigma_{\text{C}_{i,m,l}} M_{\text{R}_{i,r,l}}) / (M_{\text{M}_{i,m,l}} + M_{\text{R}_{i,r,l}}) \quad \text{root – mycorrhizal C transfer} \quad [\text{C52}]$$

$$Z_{r\text{N,P}_{i,j-i,r,l}} = g_{r\text{N,P}_{i,j-i,r,l}} (\sigma_{\text{N,P}_{i,r,l}} \sigma_{\text{C}_{i,m,l}} - \sigma_{\text{N,P}_{i,m,l}} \sigma_{\text{C}_{i,r,l}}) / (\sigma_{\text{C}_{i,m,l}} + \sigma_{\text{C}_{i,r,l}}) \quad \text{root – mycorrhizal N,P transfer} \quad [\text{C53}]$$

$\sigma_{\text{C}}$  concentration gradients  
shoot – root N,P transfer driven by  
 $\sigma_{\text{N,P}}$  concentration gradients  
root – mycorrhizal C transfer  
driven by  $\sigma_{\text{C}}$  conc'n gradients  
root – mycorrhizal N,P transfer  
driven by  $\sigma_{\text{N,P}}$  conc'n gradients

## Definition of Variables in Appendix C

Variable	Definition	Unit	Equation	Value	Reference
<i>subscripts</i>					
<i>i</i>	species or functional type: evergreen, coniferous, deciduous, annual, perennial, C <sub>3</sub> , C <sub>4</sub> , monocot, dicot, legume etc.				
<i>j</i>	branch or tiller				
<i>k</i>	node				
<i>l</i>	soil or canopy layer				
<i>m</i>	leaf azimuth				
<i>n</i>	leaf inclination				
<i>o</i>	leaf exposure (sunlit vs. shaded)				
<i>z</i>	organ including leaf, stem, root <i>r</i> , mycorrhizae <i>m</i>				
<i>variables</i>					

$A$	leaf, root or mycorrhizal surface area	$\text{m}^2 \text{m}^{-2}$	[C1,C6b,C6d,C8b,C21,C23,C32,C33,C47]		
$\beta$	shape parameter for stomatal effects on $\text{CO}_2$ diffusion and non-stomatal effects on carboxylation	$\text{MPa}^{-1}$	[C4 C27,C35,]	-5.0	Grant and Flanagan (2007)
$B$	parameter such that $f_t = 1.0$ at $T_c = 298.15 \text{ K}$		[C36]	17.533	
$B_j$	parameter such that $f_{ij} = 1.0$ at $T_c = 298.15 \text{ K}$		[C10c]	17.363	
$B_{kc}$	parameter such that $f_{tkci} = 1.0$ at $T_c = 298.15 \text{ K}$		[C10d]	22.187	
$B_{ko}$	parameter such that $f_{tkoi} = 1.0$ at $T_c = 298.15 \text{ K}$		[C10e]	8.067	
$B_o$	parameter such that $f_{toi} = 1.0$ at $T_c = 298.15 \text{ K}$		[C10b]	24.221	
$B_v$	parameter such that $f_{tvi} = 1.0$ at $T_c = 298.15 \text{ K}$		[C10a, C22]	26.238	
$C_b$	$[\text{CO}_2]$ in canopy air	$\mu\text{mol mol}^{-1}$	[C2,C5 C25,C28]		
$C_c$	$[\text{CO}_2]$ in canopy chloroplasts in equilibrium with $C_{ii,j,k,l,m,n,o}$	$\mu\text{M}$	[C6a,C7b]		
$C_{c(b4)}$	$[\text{CO}_2]$ in $\text{C}_4$ bundle sheath	$\mu\text{M}$	[C38,C39,C42,C44,C45b]		
$C_{c(m4)}$	$[\text{CO}_2]$ in $\text{C}_4$ mesophyll in equilibrium with $C_{ii,j,k,l,m,n,o}$	$\mu\text{M}$	[C29,C30b,C39]		
$C_i'$	$[\text{CO}_2]$ in canopy leaves when $\psi_{ci} = 0$	$\mu\text{mol mol}^{-1}$	[C5]	$0.70 \times C_b$	Larcher (2001)
$C_i$	$[\text{CO}_2]$ in canopy leaves	$\mu\text{mol mol}^{-1}$	[C2]		
$C_{i(m4)'}'$	$[\text{CO}_2]$ in $\text{C}_4$ mesophyll air when $\psi_{ci} = 0$	$\mu\text{mol mol}^{-1}$	[C28]	$0.45 \times C_b$	
$C_{i(m4)}$	$[\text{CO}_2]$ in $\text{C}_4$ mesophyll air	$\mu\text{mol mol}^{-1}$	[C25]		
$C_{i,j,z=l}$	C content of leaf ( $z = l$ )	$\text{g C m}^{-2}$	[C18]		
$D_{e \text{ NH}_4l}$	effective dispersivity-diffusivity of $\text{NH}_4^+$ during root uptake	$\text{m}^2 \text{h}^{-1}$	[C23]		

$D_{e\text{NO}_3l}$	effective dispersivity-diffusivity of $\text{NO}_3^-$ during root uptake	$\text{m}^2 \text{h}^{-1}$	[C23]		
$D_{e\text{PO}_4l}$	effective dispersivity-diffusivity of $\text{H}_2\text{PO}_4^-$ during root uptake	$\text{m}^2 \text{h}^{-1}$	[C23]		
$D_{r\text{O}_2}$	aqueous diffusivity of $\text{O}_2$ from root aerenchyma to root or mycorrhizal surfaces	$\text{m}^2 \text{h}^{-1}$	[C14d]		
$D_{s\text{O}_2}$	aqueous diffusivity of $\text{O}_2$ from soil to root or mycorrhizal surfaces	$\text{m}^2 \text{h}^{-1}$	[C14d]		
$d_{i,r,l}$	half distance between adjacent roots assumed equal to uptake path length	m	[C23]	$(\pi L_{s,z} / \Delta z)^{-1/2}$	Grant (1998)
$E_{\text{N,P}}$	energy cost of nutrient uptake	$\text{g C g N}^{-1}$ or $\text{P}^{-1}$	[C13]	2.15	Veen (1981)
$f_{\text{C}(c3)}$	$\text{C}_3$ product inhibition of RuBP carboxylation activity in $\text{C}_4$ bundle sheath or $\text{C}_3$ mesophyll	–	[C47,C48,C49]		
$f_{\text{C}(m4)}$	$\text{C}_4$ product inhibition of PEP carboxylation activity in $\text{C}_4$ mesophyll	–	[C32,C33,C34]		
$F_{\text{chl}}$	fraction of leaf protein in chlorophyll	-	[C8b]	0.025	
$f_{i\text{C}}$	N,P inhibition on carboxylation, leaf structural N,P growth	–	[C6a,C7,C11,C12]		
$f_{i\text{N}}$	N inhibition on root N uptake	–	[C23g]		
$f_{i\text{P}}$	P inhibition on root P uptake	–	[C23h]		
$f_{\lambda\text{N}}$	fraction of $X_{\text{mx}}$ N translocated out of leaf or root before litterfall	–	[C19a,c]		
$f_{\lambda\text{P}}$	fraction of $X_{\text{mx}}$ P translocated out of leaf or root before litterfall	–	[C19b,d]		
$F_{\text{rubisco}}$	fraction of leaf protein in rubisco	-	[C6b,d]	0.125	
$f_{\text{ta}}$	temperature effect on $R_{ai,j}$ and $U$	–	[C14, C22,C23]		
$f_{\text{tb}}$	temperature effect on carboxylation	–	[C6b,C10a]		



$f_{ij}$	temperature effect on electron transport		[C8b,C10c]		
$f_{ikc}$	temperature effect on $K_{c_i}$		[C6e,C10d]		Bernacchi et al. (2001,2003)
$f_{iko}$	temperature effect on $K_{o_i}$		[C6e,C10e]		Bernacchi et al. (2001,2003)
$f_{im}$	temperature effect on $R_{mi,j}$	–	[C16, C22b]	$Q_{10} = 2.25$	
$f_{to}$	temperature effect on oxygenation		[C6d,C10b]		
$f_{tv}$	temperature effect on carboxylation	–	[C32,C33,C36,C47,C48]		
$f_{xN}$	inhibition of root or mycorrhizal N exudation	–	[C19f,h]		
$f_{xP}$	inhibition of root or mycorrhizal P exudation	–	[C19g,i]		
$f_{\psi_i}$	non-stomatal water effect on carboxylation	–	[C6a,C7a,C9]		Medrano et al. (2002)
$f_{\psi_i}$	non-stomatal water effect on carboxylation	–	[C32,C33,C35C47,C48]		
$g_{sC}$	conductance for shoot-root C transfer	$h^{-1}$	[C50]	calculated from root depth, axis number	Grant (1998)
$g_{sN,P}$	rate constant for shoot-root N,P transfer	$h^{-1}$	[C51]	0.1	Grant (1998)
$g_{rC}$	rate constant for root-mycorrhizal C transfer	$h^{-1}$	[C52]	0.1	Grant (1998)
$g_{rN,P}$	rate constant for root-mycorrhizal N,P transfer	$h^{-1}$	[C53]	0.1	Grant (1998)
$H_a$	energy of activation	$J mol^{-1}$	[C36]	$57.5 \times 10^3$	
$H_{aj}$	energy of activation for electron transport	$J mol^{-1}$	[C10c]	$43 \times 10^3$	Bernacchi et al. (2001,2003)
$H_{akc}$	parameter for temperature sensitivity of $K_{c_i}$	$J mol^{-1}$	[C10d]	$55 \times 10^3$	Bernacchi et al. (2001,2003)

$H_{ako}$	parameter for temperature sensitivity of $K_{o_i}$	$J mol^{-1}$	[C10e]	$20 \times 10^3$	Bernacchi et al. (2001,2003)
$H_{ao}$	energy of activation for oxygenation	$J mol^{-1}$	[C10b, C22]	$60 \times 10^3$	Bernacchi et al. (2001,2003)
$H_{av}$	energy of activation for carboxylation	$J mol^{-1}$	[C10a, C22]	$65 \times 10^3$	Bernacchi et al. (2001,2003)
$H_{dh}$	energy of high temperature deactivation	$J mol^{-1}$	[C10, C22]	$222.5 \times 10^3$	
$H_{dh}$	energy of high temperature deactivation	$J mol^{-1}$	[C36]	$220 \times 10^3$	
$H_{dl}$	energy of low temperature deactivation	$J mol^{-1}$	[C10, C22]	$198.0 \times 10^3$	
$H_{dl}$	energy of low temperature deactivation	$J mol^{-1}$	[C36]	$190 \times 10^3$	
$[H_2PO_4^-]_{i,r,l}$	concentration of $H_2PO_4^-$ root or mycorrhizal surfaces	$g N m^{-3}$	[C23]		
$[H_2PO_4^-]_{mn}$	concentration of $H_2PO_4^-$ at root or mycorrhizal surfaces below which $U_{PO_4} = 0$	$g N m^{-3}$	[C23]	0.002	Barber and Silberbush, 1984
$I$	irradiance	$\mu mol m^{-2} s^{-1}$	[C8a,]		
$J$	electron transport rate in $C_3$ mesophyll	$\mu mol m^{-2} s^{-1}$	[C7a,C8a]		
$J_{(b4)}$	electron transport rate in $C_4$ bundle sheath	$\mu mol m^{-2} s^{-1}$	[C45a,C46]		
$J_{(m4)}$	electron transport rate in $C_4$ mesophyll	$\mu mol m^{-2} s^{-1}$	[C30a,C31]		
$J_{max}'$	specific electron transport rate at non-limiting $I$ and $25^\circ C$ when $\psi_{ci} = 0$ and nutrients are nonlimiting	$\mu mol g^{-1} s^{-1}$	[C33,C48]	400	
$J_{max(b4)}$	electron transport rate in $C_4$ bundle sheath at non-limiting $I$	$\mu mol m^{-2} s^{-1}$	[C46,C48]		
$J_{max(m4)}$	electron transport rate in $C_4$ mesophyll at non-limiting $I$	$\mu mol m^{-2} s^{-1}$	[C31,C33]		
$J_{max}$	electron transport rate at non-limiting $I$ , $\psi_{ci}$ , temperature and N,P	$\mu mol m^{-2} s^{-1}$	[C8a,C8b]		
$K_{c(b4)}$	Michaelis-Menten constant for carboxylation in $C_4$ bundle sheath	$\mu M$	[C44]	30.0 at $25^\circ C$ and zero $O_2$	Lawlor (1993)

$K_{c(m4)}$	Michaelis-Menten constant for carboxylation in C <sub>4</sub> mesophyll	$\mu\text{M}$	[C29]	3.0 at 25°C	Lawlor (1993)
$K_c$	Michaelis-Menten constant for carboxylation at zero O <sub>2</sub>	$\mu\text{M}$	[C6c,C6e]	12.5 at 25 °C	Farquhar et al. (1980)
$K_c$	Michaelis-Menten constant for carboxylation at ambient O <sub>2</sub>	$\mu\text{M}$	[C6e]		
$K_{iC_N}$	inhibition constant for growth in shoots from $\sigma_C$ vs. $\sigma_N$	$\text{g C g N}^{-1}$	[C11]	100	Grant (1998)
$K_{iC_P}$	inhibition constant for growth in shoots from $\sigma_C$ vs. $\sigma_P$	$\text{g C g P}^{-1}$	[C11]	1000	Grant (1998)
$K_{I_{\gamma C4(b4)}}$	constant for CO <sub>2</sub> product inhibition of C <sub>4</sub> decarboxylation in C <sub>4</sub> bundle sheath	$\mu\text{M}$	[C38]	1000.0	
$K_{I_{\gamma C4(m4)}}$	constant for C <sub>4</sub> product inhibition of PEP carboxylation activity in C <sub>4</sub> mesophyll	$\mu\text{M}$	[C34]	$5 \times 10^6$	
$K_{I_{\nu f}}$	constant for C <sub>3</sub> product inhibition of RuBP carboxylation activity in C <sub>4</sub> bundle sheath or C <sub>3</sub> mesophyll caused by [ $\nu_{fi,j}$ ]	$\text{g C g N}^{-1}$	[C49]	100	
$K_{I_{\pi f}}$	constant for C <sub>3</sub> product inhibition of RuBP carboxylation activity in C <sub>4</sub> bundle sheath or C <sub>3</sub> mesophyll caused by [ $\pi_{fi,j}$ ]	$\text{g C g P}^{-1}$	[C49]	1000	
$K_{iN_C}$	inhibition constant for N uptake in roots from $\sigma_{C_{i,j}}$ vs. $\sigma_{N_j}$	$\text{g N g C}^{-1}$	[C23]	0.1	Grant (1998)
$K_{iP_C}$	inhibition constant for P uptake in roots from $\sigma_{C_{i,j}}$ vs. $\sigma_{P_{i,j}}$ roots	$\text{g P g C}^{-1}$	[C23]	0.01	Grant (1998)
$K_{\lambda N}$	inhibition constant for remobilization of leaf or root N during senescence	$\text{g N g C}^{-1}$	[C19c]	0.1	
$K_{\lambda P}$	inhibition constant for remobilization of leaf or root P during senescence	$\text{g P g C}^{-1}$	[C19d]	0.01	
$K_{NH_4}$	M-M constant for NH <sub>4</sub> <sup>+</sup> uptake at root or mycorrhizal surfaces	$\text{g N m}^{-3}$	[C23]	0.40	Barber and Silberbush, 1984
$K_{NO_3}$	M-M constant for NO <sub>3</sub> <sup>-</sup> uptake at root or mycorrhizal surfaces	$\text{g N m}^{-3}$	[C23]	0.35	Barber and Silberbush, 1984

$K_{PO_4}$	M-M constant for $H_2PO_4^-$ uptake root or mycorrhizal surfaces	$g P m^{-3}$	[C23]	0.125	Barber and Silberbush, 1984 Griffin (1972)
$K_{O_2}$	Michaelis-Menten constant for root or mycorrhizal $O_2$ uptake	$g m^{-3}$	[C14c]	0.064	
$K_o$	inhibition constant for $O_2$ in carboxylation	$\mu M$	[C6c,C6e]	500 at 25 °C	Farquhar et al. (1980)
$K_{xN}$	inhibition constant for exudation of root or mycorrhizal N	$g C g N^{-1}$	[C19h]	1.0	
$K_{xP}$	inhibition constant for exudation of root or mycorrhizal P	$g C g N^{-1}$	[C19i]	10.0	
$L$	root length	$m m^{-2}$	[C14d,C21b,C23]		
$l_C$	C litterfall from leaf or root	$g C m^{-2} h^{-1}$	[C18,C19a,b,C20]		
$l_{N,P}$	N or P litterfall from leaf or root	$g C m^{-2} h^{-1}$	[C19a,b]		
$M_B$	branch C phytomass	$g C m^{-2}$	[C20,C50]		
$M_L$	leaf C phytomass	$g C m^{-2}$	[C12,C21]		
$M_{L_N}, M_{L_R}$	non-remobilizable, remobilizable (protein) leaf C phytomass	$g C m^{-2}$	[C6,C12,C18]		
$M_M$	mycorrhizal C phytomass	$g C m^{-2}$	[C52]		
$M_R$	root C phytomass	$g C m^{-2}$	[C20,C21,C50,C52]		
$M_{iprot}$	leaf protein phytomass calculated from leaf N, P contents	$g N m^{-2}$	[C6b,C6d,C8b,C12]		
N,P	N or P content of organ $z$	$g N m^{-2}$	[C16, C19]		
$N_{leaf}$	maximum leaf structural N content	$g N g C^{-1}$	[C12]	0.10	
$N'_{leaf}$	minimum leaf structural N content	$g N g C^{-1}$	[C12]	$0.33 \times N_{leaf}$	
$N_{lf}$	total leaf N	$g N m^{-2} leaf$	[C32,C33,C47,C48]		
$N_{prot}$	N content of protein remobilized from leaf or root	$g N C^{-1}$	[C12,C19a]	0.4	

$[N_{\text{chl}(b4)}]'$	ratio of chlorophyll N in C <sub>4</sub> bundle sheath to total leaf N	g N g N <sup>-1</sup>	[C48]	0.05	
$[N_{\text{chl}(m4)}]'$	ratio of chlorophyll N in C <sub>4</sub> mesophyll to total leaf N	g N g N <sup>-1</sup>	[C33]	0.05	
$[\text{NH}_4^+_{i,r,l}]$	concentration of NH <sub>4</sub> <sup>+</sup> at root or mycorrhizal surfaces	g N m <sup>-3</sup>	[C23]		
$[\text{NH}_4^+_{\text{mn}}]$	concentration of NH <sub>4</sub> <sup>+</sup> at root or mycorrhizal surfaces below which $U_{\text{NH}_4} = 0$	g N m <sup>-3</sup>	[C23]	0.0125	Barber and Silberbush, 1984
$[\text{NO}_3^-_{i,r,l}]$	concentration of NH <sub>4</sub> <sup>+</sup> at root or mycorrhizal surfaces	g N m <sup>-3</sup>	[C23]		
$[\text{NO}_3^-_{\text{mn}}]$	concentration of NO <sub>3</sub> <sup>-</sup> at root or mycorrhizal surfaces below which $U_{\text{NO}_3} = 0$	g N m <sup>-3</sup>	[C23]	0.03	Barber and Silberbush, 1984
$[N_{\text{pep}(m4)}]'$	ratio of PEP carboxylase N in C <sub>4</sub> mesophyll to total leaf N	g N g N <sup>-1</sup>	[C32]	0.025	
$[N_{\text{rub}(b4)}]'$	ratio of RuBP carboxylase N in C <sub>4</sub> bundle sheath to total leaf N	g N g N <sup>-1</sup>	[C47]	0.025	
O <sub>2q</sub>	aqueous O <sub>2</sub> concentration in root or mycorrhizal aerenchyma	g m <sup>-3</sup>	[C14c,d]		
O <sub>2r</sub>	aqueous O <sub>2</sub> concentration at root or mycorrhizal surfaces	g m <sup>-3</sup>	[C14c,d]		
O <sub>2s</sub>	aqueous O <sub>2</sub> concentration in soil solution	g m <sup>-3</sup>	[C14c,d]		
O <sub>c</sub>	[O <sub>2</sub> ] in canopy chloroplasts in equilibrium with O <sub>2</sub> in atm.	μM	[C6c,C6e]		
<b>P<sub>leaf</sub></b>	maximum leaf structural P content	g P g C <sup>-1</sup>	[C12]	0.10	
<b>P'<sub>leaf</sub></b>	minimum leaf structural P content	g P g C <sup>-1</sup>	[C12]	0.33 x <b>P<sub>leaf</sub></b>	
<b>P<sub>prot</sub></b>	P content of protein remobilized from leaf or root	g P C <sup>-1</sup>	[C12,C19b]	0.04	
$[\pi_{\text{if}}]$	concentration of nonstructural root P uptake product in leaf	g P g C <sup>-1</sup>	[C49]		
<b>θ<sub>P</sub></b>	root or mycorrhizal porosity	m <sup>3</sup> m <sup>-3</sup>	[C21b]	0.1 – 0.5	
<b>R</b>	gas constant	J mol <sup>-1</sup> K <sup>-1</sup>	[C10, C22]	8.3143	
<b>R</b>	gas constant	J mol <sup>-1</sup> K <sup>-1</sup>	[C36]	8.3143	

$R_a$	total autotrophic respiration	$\text{g C m}^{-2} \text{h}^{-1}$	[C13]		
$R_a'$	$R_a$ under nonlimiting $\text{O}_2$	$\text{g C m}^{-2} \text{h}^{-1}$	[C14]		
$R_c'$	specific autotrophic respiration of $\sigma_{C_{i,j}}$ at $T_{ci} = 25^\circ\text{C}$	$\text{g C g C}^{-1} \text{h}^{-1}$	[C14]	0.015	
$R_c$	autotrophic respiration of $\sigma_{C_{i,j}}$ or $\sigma_{C_{i,r,l}}$	$\text{g C m}^{-2} \text{h}^{-1}$	[C13,C14,C17, C15]		
$R_g$	growth respiration	$\text{g C m}^{-2} \text{h}^{-1}$	[C17,C20]		
$r_{lf}$	leaf stomatal resistance	$\text{s m}^{-1}$	[C25,C27,C39]		
$r_{lfmaxi}$	leaf cuticular resistance	$\text{s m}^{-1}$	[C27]		
$r_{lfmini,j,k,l,m,n,o}$	leaf stomatal resistance when $\psi_{ci} = 0$	$\text{s m}^{-1}$	[C27,C28,C35]		
$r_{li,j,k,l,m,n,o}$	leaf stomatal resistance	$\text{s m}^{-1}$	[C2,C4,C9]		
$r_{lmaxi}$	leaf cuticular resistance	$\text{s m}^{-1}$	[C4]		
$r_{lmini,j,k,l,m,n,o}$	leaf stomatal resistance when $\psi_{ci} = 0$	$\text{s m}^{-1}$	[C4,C5,C9]		
$R_m'$	specific maintenance respiration of $\sigma_{C_{i,j}}$ at $T_{ci} = 25^\circ\text{C}$	$\text{g C g N}^{-1} \text{h}^{-1}$	[C16]	0.0115	Barnes et al. (1998)
$R_{mi,j}$	above-ground maintenance respiration	$\text{g C m}^{-2} \text{h}^{-1}$	[C16,C17,C15]		
$r_{qi,r,l}$	radius of root aerenchyma	m	[C14d]		
$r_{ri,r,l}$	root or mycorrhizal radius	m	[C14d,C21b,c,C23 a,c,e]	$1.0 \times 10^{-4}$ or $5.0 \times 10^{-6}$	
$R_{si,j}$	respiration from remobilization of leaf C	$\text{g C m}^{-2} \text{h}^{-1}$	[C13,C15,C18, C20]		
$r_{sl}$	thickness of soil water films	m	[C14d]		
$r_x$	rate constant for root or mycorrhizal exudation	$\text{h}^{-1}$	[C19f,g,h]	0.001	
$\rho_r$	dry matter content of root biomass	$\text{g g}^{-1}$	[C21b]	0.125	

$S$	change in entropy	$\text{J mol}^{-1} \text{K}^{-1}$	[C10, C22]	710	Sharpe and DeMichelle (1977)
$S$	change in entropy	$\text{J mol}^{-1} \text{K}^{-1}$	[C36]	710	
$\sigma_C$	nonstructural C product of $\text{CO}_2$ fixation	$\text{g C g C}^{-1}$	[C11, C19c,d,e,h,i, C23g,h, C50-53]		
$\sigma_N$	nonstructural N product of root uptake	$\text{g N g C}^{-1}$	[C11, C19c,f,h,i C23g,h, C51, C53]		
$\sigma_P$	nonstructural P product of root uptake	$\text{g P g C}^{-1}$	[C11, C19d,g,h,i C23g,h, C51, C53]		
$T_c$	canopy temperature	K	[C10, C22]		
$T_c$	canopy temperature	$^{\circ}\text{C}$	[C36]		
$U_{\text{NH}_4^+, r, l}$	$\text{NH}_4^+$ uptake by roots or mycorrhizae	$\text{g N m}^{-2} \text{h}^{-1}$	[C23]		
$U'_{\text{NH}_4}$	maximum $U_{\text{NH}_4}$ at 25 $^{\circ}\text{C}$ and non-limiting $\text{NH}_4^+$	$\text{g N m}^{-2} \text{h}^{-1}$	[C23]	$5.0 \times 10^{-3}$	Barber and Silberbush, 1984
$U_{\text{NO}_3^-, r, l}$	$\text{NO}_3^-$ uptake by roots or mycorrhizae	$\text{g N m}^{-2} \text{h}^{-1}$	[C23]		
$U'_{\text{NO}_3}$	maximum $U_{\text{NO}_3}$ at 25 $^{\circ}\text{C}$ and non-limiting $\text{NO}_3^-$	$\text{g N m}^{-2} \text{h}^{-1}$	[C23]	$5.0 \times 10^{-3}$	Barber and Silberbush, 1984
$U_{\text{PO}_4^-, r, l}$	$\text{H}_2\text{PO}_4^-$ uptake by roots or mycorrhizae	$\text{g N m}^{-2} \text{h}^{-1}$	[C23]		
$U'_{\text{PO}_4}$	maximum $U_{\text{PO}_4}$ at 25 $^{\circ}\text{C}$ and non-limiting $\text{H}_2\text{PO}_4^-$	$\text{g N m}^{-2} \text{h}^{-1}$	[C23]	$5.0 \times 10^{-3}$	Barber and Silberbush, 1984
$U_{\text{O}_2, r, l}$	$\text{O}_2$ uptake by roots and mycorrhizae under ambient $\text{O}_2$	$\text{g O m}^{-2} \text{h}^{-1}$	[C14b,c, C23b,d,f]		
$U'_{\text{O}_2, l, r}$	$\text{O}_2$ uptake by roots and mycorrhizae under nonlimiting $\text{O}_2$	$\text{g O m}^{-2} \text{h}^{-1}$	[C14b,c, C23b,d,f]		
$U_{w, i, r, l}$	root water uptake	$\text{m}^3 \text{m}^{-2} \text{h}^{-1}$	[C14d, C23]		
$V_{\phi(b4)ij,k}$	$\text{CO}_2$ leakage from $\text{C}_4$ bundle sheath to $\text{C}_4$ mesophyll	$\text{g C m}^{-2} \text{h}^{-1}$	[C39, C42]		

$V_b'$	specific rubisco carboxylation at 25 °C	$\mu\text{mol g}^{-1} \text{rubisco s}^{-1}$	[C6b]	45	Farquhar et al. (1980)
$V_{b(b4)ij,k}$	CO <sub>2</sub> -limited carboxylation rate in C <sub>4</sub> bundle sheath	$\mu\text{mol m}^{-2} \text{s}^{-1}$	[C43,C44]		
$V_{b(m4)ijkl,m,n,o}$	CO <sub>2</sub> -limited carboxylation rate in C <sub>4</sub> mesophyll	$\mu\text{mol m}^{-2} \text{s}^{-1}$	[C26]		
$V_{bij,kl,m,n,o}$	CO <sub>2</sub> -limited leaf carboxylation rate	$\mu\text{mol m}^{-2} \text{s}^{-1}$	[C3,C6]		
$V_{bmax(b4)'}'$	RuBP carboxylase specific activity in C <sub>4</sub> bundle sheath at 25°C when $\psi_{ci} = 0$ and nutrients are nonlimiting	$\mu\text{mol g}^{-1} \text{s}^{-1}$	[C47]	75	
$V_{bmax(b4)ij,k}$	CO <sub>2</sub> -nonlimited carboxylation rate in C <sub>4</sub> bundle sheath	$\mu\text{mol m}^{-2} \text{s}^{-1}$	[C44,C47]		
$V_{bmax(m4)'}'$	PEP carboxylase specific activity in C <sub>4</sub> mesophyll at 25°C when $\psi_{ci} = 0$ and nutrients are nonlimiting	$\mu\text{mol g}^{-1} \text{s}^{-1}$	[C32]	150	
$V_{bmax(m4)ij,k}$	CO <sub>2</sub> -nonlimited carboxylation rate in C <sub>4</sub> mesophyll	$\mu\text{mol m}^{-2} \text{s}^{-1}$	[C29,C32]		
$V_{bmaxi,j,k}$	leaf carboxylation rate at non-limiting CO <sub>2</sub> , $\psi_{ci}$ , $T_c$ and N,P	$\mu\text{mol m}^{-2} \text{s}^{-1}$	[C6a,C6b,C6c]		
$V_{c(b4)ijkl,m,n,o}$	CO <sub>2</sub> fixation rate in C <sub>4</sub> bundle sheath	$\mu\text{mol m}^{-2} \text{s}^{-1}$	[C43]		
$V_{c(m4)ijkl,m,n,o}$	CO <sub>2</sub> fixation rate in C <sub>4</sub> mesophyll	$\mu\text{mol m}^{-2} \text{s}^{-1}$	[C24,C26,C40,C41]		
$V_{c0(m4)ijkl,m,n,o}$	CO <sub>2</sub> fixation rate in C <sub>4</sub> mesophyll when $\psi_{ci} = 0$ MPa	$\mu\text{mol m}^{-2} \text{s}^{-1}$	[C28]		
$V_{ci,j,kl,m,n,o}$	leaf CO <sub>2</sub> fixation rate	$\mu\text{mol m}^{-2} \text{s}^{-1}$	[C1,C3]		
$V_c'_{ijkl,m,n,o}$	leaf CO <sub>2</sub> fixation rate when $\psi_{ci} = 0$	$\mu\text{mol m}^{-2} \text{s}^{-1}$	[C5]		
$V_{g(m4)ijkl,m,n,o}$	CO <sub>2</sub> diffusion rate into C <sub>4</sub> mesophyll	$\mu\text{mol m}^{-2} \text{s}^{-1}$	[C24,C25]		
$V_{gi,j,kl,m,n,o}$	leaf CO <sub>2</sub> diffusion rate	$\mu\text{mol m}^{-2} \text{s}^{-1}$	[C1,C2]		
$V_j'$	specific chlorophyll $e^-$ transfer at 25 °C	$\mu\text{mol g}^{-1} \text{chlorophyll s}^{-1}$	[C8b]	450	Farquhar et al. (1980)
$V_{j(b4)ijkl,m,n,o}$	irradiance-limited carboxylation rate in C <sub>4</sub> bundle sheath	$\mu\text{mol m}^{-2} \text{s}^{-1}$	[C43,C45a]		



$V_{j(m4)i,j,k,l,m,n,o}$	irradiance-limited carboxylation rate in C <sub>4</sub> mesophyll	$\mu\text{mol m}^{-2} \text{s}^{-1}$	[C26,C30a]		
$V_{ji,j,k,l,m,n,o}$	irradiance-limited leaf carboxylation rate	$\mu\text{mol m}^{-2} \text{s}^{-1}$	[C3,C7a]		
$V_o'$	specific rubisco oxygenation at 25 °C	$\mu\text{mol g}^{-1} \text{rubisco s}^{-1}$	[C6d]	9.5	Farquhar et al. (1980)
$V_{\text{omax}i,j,k}$	leaf oxygenation rate at non-limiting O <sub>2</sub> , $\psi_{ci}$ , $T_c$ and N,P	$\mu\text{mol m}^{-2} \text{s}^{-1}$	[C6c,d]		
$V_{\chi C4(b4)i,j,k}$	decarboxylation of C <sub>4</sub> fixation product in C <sub>4</sub> bundle sheath	$\text{g C m}^{-2} \text{h}^{-1}$	[C38,C41,C42]		
$V_{\chi C4(m4)}$	transfer of C <sub>4</sub> fixation product between C <sub>4</sub> mesophyll and bundle sheath	$\text{g C m}^{-2} \text{h}^{-1}$	[C37]		
$[V_{lf}]$	concentration of nonstructural root N uptake product in leaf	$\text{g N g C}^{-1}$	[C49]		
$v_r$	specific volume of root biomass	$\text{m}^3 \text{g}^{-1}$	[C21b]		
$W_{lf(b4)}$	C <sub>4</sub> bundle sheath water content	$\text{g m}^{-2}$	[C37,C39]		
$W_{lf(m4)}$	C <sub>4</sub> mesophyll water content	$\text{g m}^{-2}$	[C37]		
$X_{\text{mx}}$	maximum fraction of remobilizable N or P translocated out of leaf or root during senescence	-	[C19a,b]	0.6	Kimmins (2004)
$x_{i,r,l,C}$	root and mycorrhizal C exudation	$\text{g C m}^{-2} \text{h}^{-1}$	[C19e]		
$x_{i,r,l,N}$	root and mycorrhizal N exudation	$\text{g N m}^{-2} \text{h}^{-1}$	[C19f]		
$x_{i,r,l,P}$	root and mycorrhizal P exudation	$\text{g P m}^{-2} \text{h}^{-1}$	[C19g]		
$Y$	carboxylation yield from electron transport in C <sub>3</sub> mesophyll	$\mu\text{mol CO}_2 \mu\text{mol e}^{-1}$	[C7a,b]		
$Y_{(b4)}$	carboxylation yield from electron transport in C <sub>4</sub> bundle sheath	$\mu\text{mol CO}_2 \mu\text{mol e}^{-1}$	[C45a,b]		
$Y_{(m4)}$	carboxylation yield from electron transport in C <sub>4</sub> mesophyll	$\mu\text{mol CO}_2 \mu\text{mol e}^{-1}$	[C30a,b]		

$Y_g$	fraction of $\sigma_{C_{i,j}}$ used for growth expended as $R_{g_{i,j,z}}$ by organ $z$	$g C g C^{-1}$	[C20]	0.28 ( $z = \text{leaf}$ ), 0.24 ( $z = \text{root and other non-foliar}$ ), 0.20 ( $z = \text{wood}$ )	Waring and Running (1998)
$y$	plant population	$m^{-2}$	[C21]		
$Z_{sC}$	shoot-root C transfer	$g C m^{-2} h^{-1}$	[C50]		
$Z_{sN,P}$	shoot-root N,P transfer	$g N,P m^{-2} h^{-1}$	[C51]		
$Z_{rC}$	root-mycorrhizal C transfer	$g C m^{-2} h^{-1}$	[C52]		
$Z_{rN,P}$	root-mycorrhizal N,P transfer	$g N,P m^{-2} h^{-1}$	[C53]		
$\Gamma$	CO <sub>2</sub> compensation point in C <sub>3</sub> mesophyll	$\mu M$	[C6a,C6c,C7b]		
$\Gamma_{(b4)}$	CO <sub>2</sub> compensation point in C <sub>4</sub> bundle sheath	$\mu M$	[C44,C45b]		
$\Gamma_{(m4)}$	CO <sub>2</sub> compensation point in C <sub>4</sub> mesophyll	$\mu M$	[C29,C30b]		
$\alpha$	shape parameter for response of $J$ to $I$	-	[C8a]	0.7	
$\alpha$	shape parameter for response of $J$ to $I$	-	[C31,C46]	0.75	
$\chi$	area:mass ratio of leaf growth	$m g^{-3}$	[C21]	0.0125	Grant and Hesketh (1992)
$\chi_{C4(b4)}$	non-structural C <sub>4</sub> fixation product in C <sub>4</sub> bundle sheath	$g C m^{-2}$	[C37,C38,C41]		
$\chi_{C4(m4)}$	non-structural C <sub>4</sub> fixation product in C <sub>4</sub> mesophyll	$g C m^{-2}$	[C37,C40]		
$[\chi_{C3(b4)}]$	concentration of non-structural C <sub>3</sub> fixation product in C <sub>4</sub> bundle sheath	$g g^{-1}$	[C49]		
$[\chi_{C4(m4)}]$	concentration of non-structural C <sub>4</sub> fixation product in C <sub>4</sub> mesophyll	$\mu M$	[C34]		
$\epsilon$	quantum yield	$\mu mol e^{-} \mu mol \text{ quanta}^{-1}$	[C8a]	0.45	Farquhar et al. (1980)

$\varepsilon$	quantum yield	$\mu\text{mol e}^- \mu\text{mol}$ $\text{quanta}^{-1}$	[C31,C46]	0.45	Farquhar et al., (1980)
$\kappa_{\text{Cc}(b4)}$	conductance to CO <sub>2</sub> leakage from C <sub>4</sub> bundle sheath	$\text{h}^{-1}$	[C39]	20	
$\psi_t$	canopy turgor potential	MPa	[C4]	1.25 at $\psi_c = 0$	

## Appendix D: Soil Water, Heat, Gas and Solute Fluxes

### *Surface Water Flux*

$Q_{rx(x,y)} = v_{x(x,y)} d_{mx,y} L_{y(x,y)}$	2D Manning equation in x (EW) and y (NS) directions	[D1]
$Q_{ry(x,y)} = v_{y(x,y)} d_{mx,y} L_{x(x,y)}$		
$d_{x,y} = \max(0, d_{w(x,y)} + d_{i(x,y)} - d_{s(x,y)}) d_{w(x,y)} / (d_{w(x,y)} + d_{i(x,y)})$	surface water depth	[D2]
$v_{x(x,y)} = R^{0.67} s_{x(x,y)}^{0.5} / z_{r(x,y)}$	runoff velocity over E slope	[D3]
$v_{y(x,y)} = R^{0.67} s_{y(x,y)}^{0.5} / z_{r(x,y)}$	runoff velocity over S slope	
$v_{x(x,y)} = -R^{0.67} s_{x(x,y)}^{0.5} / z_{r(x,y)}$	runoff velocity over W slope	
$v_{y(x,y)} = -R^{0.67} s_{y(x,y)}^{0.5} / z_{r(x,y)}$	runoff velocity over N slope	
$\Delta(d_{w(x,y)} A_{x,y}) / \Delta t = Q_{r,x(x,y)} - Q_{r,x+1(x,y)} + Q_{r,y(x,y)} - Q_{r,y+1(x,y)} + P - E_{x,y} - Q_{wz(x,y,l)}$	2D kinematic wave theory for overland flow	[D4]
$R = s_r d_m / [2 (s_r^2 + 1) 0.5]$	wetted perimeter	[D5a]
$s_{x(x,y)} = 2 \text{ abs}[(Z + d_s + d_m)_{x,y} - (Z + d_s + d_m)_{x+1,y}] / (L_{x(x,y)} + L_{x(x+1,y)})$	2D slope from topography and pooled surface water in x (EW) and y (NS) directions	[D5b]
$s_{y(x,y)} = 2 \text{ abs}[(Z + d_s + d_m)_{x,y} - (Z + d_s + d_m)_{x,y+1}] / (L_{y(x,y)} + L_{y(x,y+1)})$		
$LE_l = L (e_a - e_{l(T_l, \psi_l)}) / r_{al}$	evaporation from surface litter	[D6a]
$LE_s = L (e_a - e_{s(T_s, \psi_s)}) / r_{as}$	evaporation from soil surface	[D6b]

### *Subsurface Water Flux*

$Q_{wx(x,y,z)} = K'_x (\psi_{sx,y,z} - \psi_{sx+1,y,z})$	3D Richard's or Green-Ampt equation depending on saturation of source or target cell in x (EW),	[D7]
$Q_{wy(x,y,z)} = K'_y (\psi_{sx,y,z} - \psi_{sx,y+1,z})$		

$$Q_{wz(x,y,z)} = K'_z (\psi_{sx,y,z} - \psi_{sx,y,z+1})$$

$$\Delta \theta_{w,x,y,z} / \Delta t = (Q_{wx(x,y)} - Q_{wx+1(x,y)} + Q_{wy(x,y)} - Q_{wy+1(x,y)} + Q_{wz(x,y)} - Q_{wz+1(x,y)} + Q_{f(x,y,z)}) / L_{z(x,y,z)}$$

$$K'_x = 2 K_{x,y,z} K_{x+1,y,z} / (K_{x,y,z} L_{x(x+1,y,z)} + K_{x+1,y,z} L_{x(x,y,z)})$$

$$= 2 K_{x,y,z} / (L_{x(x+1,y,z)} + L_{x(x,y,z)})$$

$$= 2 K_{x+1,y,z} / (L_{x(x+1,y,z)} + L_{x(x,y,z)})$$

$$K'_y = 2 K_{x,y,z} K_{x,y+1,z} / (K_{x,y,z} L_{y(x,y+1,z)} + K_{x,y+1,z} L_{y(x,y,z)})$$

$$= 2 K_{x,y,z} / (L_{y(x,y+1,z)} + L_{y(x,y,z)})$$

$$= 2 K_{x,y+1,z} / (L_{y(x,y+1,z)} + L_{y(x,y,z)})$$

$$K'_z = 2 K_{x,y,z} K_{x,y,z+1} / (K_{x,y,z} L_{z(x,y,z+1)} + K_{x,y,z+1} L_{z(x,y,z)})$$

$$= 2 K_{x,y,z} / (L_{z(x,y,z+1)} + L_{z(x,y,z)})$$

$$= 2 K_{x,y,z+1} / (L_{z(x,y,z+1)} + L_{z(x,y,z)})$$

$y$  (NS) and  $z$  (vertical) directions

3D water transfer plus freeze-thaw [D8]

in direction  $x$  if source and [D9a]

destination cells are unsaturated  
in direction  $x$  if source cell is [D9b]

saturated  
in direction  $x$  if destination cell is

saturated [D9a]

in direction  $y$  if source and  
destination cells are unsaturated [D9b]

in direction  $y$  if source cell is

saturated  
in direction  $y$  if destination cell is

saturated [D9a]

in direction  $z$  if source and  
destination cells are unsaturated [D9b]

in direction  $z$  if source cell is

saturated  
in direction  $z$  if destination cell is

saturated

### Exchange with Water Table

$$Q_{tx(x,y,z)} = K_{x,y,z} [\psi' - \psi_{sx,y,z} + 0.01 (d_{zx,y,z} - d_t)] / (L_{tx} + 0.5 L_{x(x,y,z)})$$

$$Q_{ty(x,y,z)} = K_{x,y,z} [\psi' - \psi_{sx,y,z} + 0.01 (d_{zx,y,z} - d_t)] / (L_{ty} + 0.5 L_{y(x,y,z)})$$

if  $\psi_{sx,y,z} > \psi' + 0.01(d_{zx,y,z} - d_t)$  for [D10]  
all depths  $z$  from  $d_{zx,y,z}$  to  $d_t$   
or if  $d_{zx,y,z} > d_t$

### Heat Flux

$$R_n + LE + H + G = 0$$

for each canopy, snow, residue and [D11]  
soil surface, depending on  
exposure

$$G_{x(x,y,z)} = 2 \kappa_{(x,y,z),(x+1,y,z)} (T_{(x,y,z)} - T_{(x+1,y,z)}) / (L_{x(x,y,z)} + L_{x(x+1,y,z)}) + c_w T_{(x,y,z)} Q_{wx(x,y,z)}$$

3D conductive – convective heat [D12]  
flux among snowpack, surface  
residue and soil layers in  $x$  (EW),  $y$

$$G_{y(x,y,z)} = 2 \kappa_{(x,y,z),(x,y+1,z)} (T_{(x,y,z)} - T_{(x,y+1,z)}) / (L_{y(x,y,z)} + L_{y(x,y+1,z)}) + c_w T_{(x,y,z)} Q_{wy(x,y,z)}$$

$$G_{z(x,y,z)} = 2 \kappa_{(x,y,z),(x,y,z+I)} (T_{(x,y,z)} - T_{(x,y,z+I)}) / (L_z(x,y,z) + L_z(x,y,z+I)) + c_w T_{(x,y,z)} Q_{wz(x,y,z)}$$

(NS) and z (vertical) directions

$$G_{x(x-I,y,z)} - G_{x(x,y,z)} + G_{y(x,y,I-z)} - G_{y(x,y,z)} + G_{z(x,y,z-I)} - G_{z(x,y,z)} + LQ_{f(x,y,z)} + c_{(x,y,z)} (T_{(x,y,z)} - T'_{(x,y,z)}) / \Delta t = 0$$

3D general heat flux equation in snowpack, surface residue and soil layers [D13]

### Gas Flux

$$Q_{dsyx,y,z} = a_{gsx,y,z} D_{d\gamma} (S'_{\gamma} f_{d,x,y,z} [\gamma_{gs}]_{x,y,z} - [\gamma_{ss}]_{x,y,z})$$

volatilization – dissolution between aqueous and gaseous [D14a]

$$Q_{dryx,y,z} = a_{grx,y,z} D_{d\gamma} (S'_{\gamma} f_{d,x,y,z} [\gamma_{gr}]_{x,y,z} - [\gamma_{sr}]_{x,y,z})$$

phases in soil and root [D14b]

$$Q_{gsyzx,y,I} = g_{ax,y} \{ [\gamma_a] - \{ 2 [\gamma_{gs}]_{x,y,I} D_{gsyz(x,y,I)} / L_z(x,y,I) + g_{ax,y} [\gamma_a] \} / \{ 2 D_{gsyz(x,y,I)} / L_z(x,y,I) + g_{ax,y} \} \}$$

volatilization – dissolution [D15a]

$$Q_{dsyx,y,I} = a_{gsx,y,I} D_{d\gamma} (S'_{\gamma} f_{d,x,y,I} [\gamma_a] - [\gamma_{ss}]_{x,y,I})$$

between gaseous and aqueous phases at the soil surface (z = I) and the atmosphere [D15b]

$$Q_{gsyx(x,y,z)} = - Q_{wx(x,y,z)} [\gamma_{gs}]_{x,y,z} + 2 D_{gsyx(x,y,z)} ([\gamma_{gs}]_{x,y,z} - [\gamma_{gs}]_{x+I,y,z}) / (L_x(x,y,z) + L_x(x+I,y,z))$$

3D convective - conductive gas flux among soil layers in x (EW), y [D16a]

$$Q_{gsyy(x,y,z)} = - Q_{wy(x,y,z)} [\gamma_{gs}]_{x,y,z} + 2 D_{gsyy(x,y,z)} ([\gamma_{gs}]_{x,y,z} - [\gamma_{gs}]_{x,y+I,z}) / (L_y(x,y,z) + L_y(x,y+I,z))$$

(NS) and z (vertical) directions, [D16b]

$$Q_{gsyz(x,y,z)} = - Q_{wz(x,y,z)} [\gamma_{gs}]_{x,y,z} + 2 D_{gsyz(x,y,z)} ([\gamma_{gs}]_{x,y,z} - [\gamma_{gs}]_{x,y,z+I}) / (L_z(x,y,z) + L_z(x,y,z+I))$$

[D16c]

$$Q_{gryz(x,y,z)} = D_{gryz(x,y,z)} ([\gamma_{gr}]_{x,y,z} - [\gamma_a]) / \Sigma_{1,z} L_z(x,y,z)$$

convective - conductive gas flux between roots and the atmosphere [D16d]

$$D_{gsyx(x,y,z)} = D'_{gr} f_{gx,y,z} [0.5 (\theta_{gx,y,z} + \theta_{gx+I,y,z})]^{0.67} / \theta_{psx,y,z}$$

gasous diffusivity as a function of air-filled porosity in soil [D17a]

$$D_{gsyy(x,y,z)} = D'_{gr} f_{gy,y,z} [0.5 (\theta_{gx,y,z} + \theta_{gx,y+I,z})]^{0.67} / \theta_{psx,y,z}$$

[D17b]

$$D_{gsyz(x,y,z)} = D'_{gr} f_{gz,y,z} [0.5 (\theta_{gx,y,z} + \theta_{gx,y,z+I})]^{0.67} / \theta_{psx,y,z}$$

[D17c]

$$D_{gryz(x,y,z)} = D'_{gr} f_{gx,y,z} \theta_{prx,y,z}^{1.33} A_r(x,y,z) / A_{x,y}$$

gasous diffusivity as a function of air-filled porosity in roots [D17d]

$$Q_{byz} = \min[0.0, \{ (44.64 \theta_{wx,y,z} 273.16 / T_{(x,y,z)}) - \Sigma_{\gamma} ([\gamma_s]_{x,y,z} / (S'_{\gamma} f_{d,x,y,z} M_{\gamma})) \} ]$$

$$([\gamma_s]_{x,y,z} / (S'_{\gamma} f_{d,x,y,z} M_{\gamma})) / \Sigma_{\gamma} ([\gamma_s]_{x,y,z} / (S'_{\gamma} f_{d,x,y,z} M_{\gamma})) S'_{\gamma} f_{d,x,y,z} M_{\gamma} V_{x,y,z}$$

bubbling (-ve flux) when total of all partial gas pressures exceeds atmospheric pressure [D18]

### Solute Flux

$$Q_{syx}(x,y,z) = -Q_{wx}(x,y,z) [\gamma_s]_{x,y,z} + 2 D_{syx}(x,y,z) ([\gamma_s]_{x,y,z} - [\gamma_s]_{x,y,z+I}) / (L_x(x,y,z) + L_x(x,y,z+I))$$

3D convective - dispersive solute flux among soil layers in x (EW), y (NS) and z (vertical) directions [D19]

$$Q_{syy}(x,y,z) = -Q_{wy}(x,y,z) [\gamma_s]_{x,y,z} + 2 D_{syy}(x,y,z) ([\gamma_s]_{x,y,z} - [\gamma_s]_{x,y,z+I}) / (L_y(x,y,z) + L_y(x,y,z+I))$$

$$Q_{syz}(x,y,z) = -Q_{wz}(x,y,z) [\gamma_s]_{x,y,z} + 2 D_{syz}(x,y,z) ([\gamma_s]_{x,y,z} - [\gamma_s]_{x,y,z+I}) / (L_z(x,y,z) + L_z(x,y,z+I))$$

$$D_{syx}(x,y,z) = D_{qx}(x,y,z) |Q_{wx}(x,y,z)| + D'_{sy} f_{sx,y,z} [0.5(\theta_{wx,y,z} + \theta_{wx+I,y,z})] \tau$$

$$D_{syy}(x,y,z) = D_{qy}(x,y,z) |Q_{wy}(x,y,z)| + D'_{sy} f_{sx,y,z} [0.5(\theta_{wx,y,z} + \theta_{wx+I,y,z})] \tau$$

$$D_{syz}(x,y,z) = D_{qz}(x,y,z) |Q_{wz}(x,y,z)| + D'_{sy} f_{sx,y,z} [0.5(\theta_{wx,y,z} + \theta_{wx+I,y,z})] \tau$$

$$D_{qx}(x,y,z) = 0.5 \alpha (L_x(x,y,z) + L_x(x,y,z+I))^\beta$$

aqueous dispersivity as functions of water flux and water-filled porosity [D20]

$$D_{qy}(x,y,z) = 0.5 \alpha (L_y(x,y,z) + L_y(x,y,z+I))^\beta$$

$$D_{qz}(x,y,z) = 0.5 \alpha (L_z(x,y,z) + L_z(x,y,z+I))^\beta$$

dispersivity as a function of water flow length [D21]

### Definition of Variables in Appendix D

Variable	Definition	Unit	Equation	Value	Reference
<i>subscripts</i>					
x	grid cell position in west to east direction				
y	grid cell position in north to south direction				
z	grid cell position in vertical direction			z = 0: surface residue, z = 1 to n: soil layers	
<i>variables</i>					
A	area of landscape position	m <sup>2</sup>	[D17c]		
A <sub>r</sub>	root cross-sectional area of landscape position	m <sup>2</sup>	[D17c]		
a <sub>gr</sub>	air-water interfacial area in roots	m <sup>2</sup> m <sup>-2</sup>	[D14b]		

$a_{gs}$	air-water interfacial area in soil	$m^2 m^{-2}$	[D14a,D15b]		Skopp (1985)
$\alpha$	dependence of $D_q$ on $L$	-	[D21]		
$\beta$	dependence of $D_q$ on $L$	-	[D21]		
$c$	heat capacity of soil	$MJ m^{-2} ^\circ C^{-1}$	[D13]		
$c_w$	heat capacity of water	$MJ m^{-3} ^\circ C^{-1}$	[D12]	4.19	
$D_{d\gamma}$	volatilization - dissolution transfer coefficient for gas $\gamma$	$m^2 h^{-1}$	[D14,D15a]		
$D_{g\gamma}$	gaseous diffusivity of gas $\gamma$ in roots	$m^2 h^{-1}$	[D16d,D17d]		Luxmoore et al. (1970a,b)
$D_{gs\gamma}$	gaseous diffusivity of gas $\gamma$ in soil	$m^2 h^{-1}$	[D15a,D16a,b,c,D17a,b,c]		Millington and Quirk (1960)
$D'_{g\gamma}$	diffusivity of gas $\gamma$ in air at 0 °C	$m^2 h^{-1}$	[D17]	$6.43 \times 10^{-2}$ for $\gamma = O_2$	Campbell (1985)
$D_q$	dispersivity	m	[D20,D21]		
$D_{s\gamma}$	aqueous diffusivity of gas or solute $\gamma$	$m^2 h^{-1}$	[D19,D20]		
$D'_{s\gamma}$	diffusivity of gas $\gamma$ in water at 0 °C	$m^2 h^{-1}$	[D20]	$8.57 \times 10^{-6}$ for $\gamma = O_2$	Campbell (1985)
$d_m$	depth of mobile surface water	m	[D1,D2,D5a,D6]		
$d_i$	depth of surface ice	m	[D2]		
$d_s$	maximum depth of surface water storage	m	[D2,D5b]		
$d_t$	depth of external water table	m	[D10]		
$d_w$	depth of surface water	m	[D1,D2]		
$d_z$	depth to mid-point of soil layer	m	[D10]		
$E$	evaporation or transpiration flux	$m^3 m^{-2} h^{-1}$	[D4,D11]		



$e_a$	atmospheric vapor density	$\text{m}^3 \text{m}^{-3}$	[D6]	
$e_{l(T_l, \psi_l)}$	surface litter vapor density at current $T_l$ and $\psi_l$	$\text{g m}^{-3}$	[D6a]	
$e_{s(T_s, \psi_s)}$	soil surface vapor density at current $T_s$ and $\psi_s$	$\text{g m}^{-3}$	[D6b]	
$f_{d_\gamma}$	temperature dependence of $S'_\gamma$	-	[D14,D15b,D18]	Wilhelm et al. (1977)
$f_{g_\gamma}$	temperature dependence of $D'_{g\gamma}$	-	[D17]	Campbell (1985)
$f_{s_\gamma}$	temperature dependence of $D'_{s\gamma}$	-	[D20]	Campbell (1985)
$G$	soil surface heat flux	$\text{m}^3 \text{m}^{-2} \text{h}^{-1}$	[D11]	
$G_x, G_y, G_z$	soil heat flux in $x, y$ or $z$ directions	$\text{MJ m}^{-2} \text{h}^{-1}$	[D12,D13]	
$g_a$	boundary layer conductance	$\text{m h}^{-1}$	[D15a]	
$\gamma$	gas ( $\text{H}_2\text{O}, \text{CO}_2, \text{O}_2, \text{CH}_4, \text{NH}_3, \text{N}_2\text{O}, \text{N}_2, \text{H}_2$ ) or solute (from appendix E)		[D14,D15]	
$[\gamma_a]$	atmospheric concentration of gas $\gamma$	$\text{g m}^{-3}$	[D15,D16d]	
$[\gamma_{gr}]$	gaseous concentration of gas $\gamma$ in roots	$\text{g m}^{-3}$	[D14b,D16d]	
$[\gamma_{gs}]$	gaseous concentration of gas $\gamma$ in soil	$\text{g m}^{-3}$	[D14a,D15a,D16a, D16b,D16c]	
$[\gamma_{sr}]$	aqueous concentration of gas $\gamma$ in roots	$\text{g m}^{-3}$	[D14b]	
$[\gamma_{ss}]$	aqueous concentration of gas $\gamma$ in soil	$\text{g m}^{-3}$	[D14a,D15b,D18, D19]	
$H$	sensible heat flux	$\text{MJ m}^{-2} \text{h}^{-1}$	[D11]	
$K$	hydraulic conductivity	$\text{m}^2 \text{MPa}^{-1} \text{h}^{-1}$	[D9,D10]	Green and Corey (1971)
$K'_x, K'_y, K'_z$	hydraulic conductance in $x, y$ or $z$ directions	$\text{m MPa}^{-1} \text{h}^{-1}$	[D7,D9]	
$\kappa$	thermal conductivity	$\text{MJ m}^{-1} \text{h}^{-1} \text{ }^\circ\text{C}^{-1}$	[D12]	de Vries (1963)

$L_t$	distance from boundary to external water table in $x$ or $y$ directions	m	[D10]	
$L_x, L_y, L_z$	length of landscape element in $x$ , $y$ or $z$ directions	m	[D1,D5b,D8,D9,D10,D12,D15a,D16,D19]	
$LE_1$	latent heat flux from surface litter	[D6a]	$\text{MJ m}^{-2} \text{h}^{-1}$	
$LE_s$	latent heat flux from soil surface	[D6b]	$\text{MJ m}^{-2} \text{h}^{-1}$	
$L$	latent heat of evaporation	$\text{MJ m}^{-3}$	[D6,D11,D13]	2460
$M_\gamma$	atomic mass of gas $\gamma$	$\text{g mol}^{-1}$	[D18]	
$P$	precipitation flux	$\text{m}^3 \text{m}^{-2} \text{h}^{-1}$	[D4]	
$Q_{\text{byz}}$	bubbling flux	$\text{g m}^{-2} \text{h}^{-1}$	[D18]	
$Q_{\text{dry}}$	volatilization – dissolution of gas $\gamma$ between aqueous and gaseous phases in roots	$\text{g m}^{-2} \text{h}^{-1}$	[D14b]	
$Q_{\text{ds}\gamma}$	volatilization – dissolution of gas $\gamma$ between aqueous and gaseous phases in soil	$\text{g m}^{-2} \text{h}^{-1}$	[D14a,D15b]	
$Q_f$	freeze-thaw flux (thaw +ve)	$\text{m}^3 \text{m}^{-2} \text{h}^{-1}$	[D8,D13]	
$Q_{\text{gr}\gamma}$	gaseous flux of gas $\gamma$ between roots and the atmosphere	$\text{g m}^{-2} \text{h}^{-1}$	[D16d]	
$Q_{\text{gs}\gamma}$	gaseous flux of gas $\gamma$ in soil	$\text{g m}^{-2} \text{h}^{-1}$	[D15a,D16a,b,c]	
$Q_{\text{tx}}, Q_{\text{ty}}$	surface water flow in $x$ or $y$ directions	$\text{m}^3 \text{m}^{-2} \text{h}^{-1}$	[D1,D4]	
$Q_{\text{sy}}$	aqueous flux of gas or solute $\gamma$	$\text{g m}^{-2} \text{h}^{-1}$	[D19]	
$Q_t$	water flux between boundary grid cell and external water table in $x$ or $y$ directions	$\text{m}^3 \text{m}^{-2} \text{h}^{-1}$	[D10]	
$Q_{\text{wx}}, Q_{\text{wy}}, Q_{\text{wz}}$	subsurface water flow in $x$ , $y$ or $z$ directions	$\text{m}^3 \text{m}^{-2} \text{h}^{-1}$	[D4,D7,D8,D12,D16,D19,D20]	
$\theta_g$	air-filled porosity	$\text{m}^3 \text{m}^{-3}$	[D17a,b,c]	

$\theta_{pr}$	root porosity	$m^3 m^{-3}$	[D17d]	dryland spp. 0.10 wetland spp. 0.20	Luxmoore et al. (1970a,b)
$\theta_{ps}$	soil porosity	$m^3 m^{-3}$	[D17a,b,c]		
$\theta_w$	water-filled porosity	$m^3 m^{-3}$	[D8,D18,D20]		
$R$	ratio of cross-sectional area to perimeter of surface flow	m	[D3,D5a]		
$R_n$	net radiation	$MJ m^{-2} h^{-1}$	[D11]		
$r_{al}$	surface litter boundary layer resistance	$m h^{-1}$	[D6a]		
$r_{as}$	Soil surface boundary layer resistance	$m h^{-1}$	[D6b]		
$S'_\gamma$	Ostwald solubility coefficient of gas $\gamma$ at 30 °C	-	[D14,D15b,D18]	0.0293 for $\gamma = O_2$	Wilhelm et al. (1977)
$s_r$	slope of channel sides during surface flow	$m m^{-1}$	[D5a]		
$s_x, s_y$	slope in $x$ or $y$ directions	$m m^{-1}$	[D3,D5b]		
$T$	soil temperature	°C	[D12,D18]		
$\tau$	tortuosity	-	[D20]		
$v_x, v_y$	velocity of surface flow in $x$ or $y$ directions	$m h^{-1}$	[D1,D3]		
$\psi'$	soil water potential at saturation	MPa	[D10]	$5.0 \times 10^{-3}$	
$\psi_s$	soil water potential	MPa	[D7,D10]		
$Z$	surface elevation	m	[D5b]		
$z_r$	Manning's roughness coefficient	$m^{-1/3} h$	[D3]	0.01	

## Appendix E: Solute Transformations

### *Precipitation - Dissolution Equilibria*

$\text{Al}(\text{OH})_{3(s)} \Leftrightarrow (\text{Al}^{3+}) + 3 (\text{OH}^-)$	(amorphous $\text{Al}(\text{OH})_3$ )	-33.0	[E1] <sup>1</sup>
$\text{Fe}(\text{OH})_{3(s)} \Leftrightarrow (\text{Fe}^{3+}) + 3 (\text{OH}^-)$	(soil Fe)	-39.3	[E2]
$\text{CaCO}_{3(s)} \Leftrightarrow (\text{Ca}^{2+}) + (\text{CO}_3^{2-})$	(calcite)	-9.28	[E3]
$\text{CaSO}_{4(s)} \Leftrightarrow (\text{Ca}^{2+}) + (\text{SO}_4^{2-})$	(gypsum)	-4.64	[E4]
$\text{AlPO}_{4(s)} \Leftrightarrow (\text{Al}^{3+}) + (\text{PO}_4^{3-})$	(variscite)	-22.1	[E5] <sup>2</sup>
$\text{FePO}_{4(s)} \Leftrightarrow (\text{Fe}^{3+}) + (\text{PO}_4^{3-})$	(strengite)	-26.4	[E6]
$\text{Ca}(\text{H}_2\text{PO}_4)_{2(s)} \Leftrightarrow (\text{Ca}^{2+}) + 2 (\text{H}_2\text{PO}_4^-)$	(monocalcium phosphate)	-1.15	[E7] <sup>3</sup>
$\text{CaHPO}_{4(s)} \Leftrightarrow (\text{Ca}^{2+}) + (\text{HPO}_4^{2-})$	(monetite)	-6.92	[E8]
$\text{Ca}_5(\text{PO}_4)_3\text{OH}_{(s)} \Leftrightarrow 5 (\text{Ca}^{2+}) + 3 (\text{PO}_4^{3-}) + (\text{OH}^-)$	(hydroxyapatite)	-58.2	[E9]

### *Cation Exchange Equilibria*<sup>4</sup>

$\text{X-Ca} + 2 (\text{NH}_4^+) \Leftrightarrow 2 \text{X-NH}_4 + (\text{Ca}^{2+})$		1.00	[E10]
$3 \text{X-Ca} + 2 (\text{Al}^{3+}) \Leftrightarrow 2 \text{X-Al} + 3 (\text{Ca}^{2+})$		1.00	[E11]
$\text{X-Ca} + (\text{Mg}^{2+}) \Leftrightarrow \text{X-Mg} + (\text{Ca}^{2+})$		0.60	[E12]
$\text{X-Ca} + 2 (\text{Na}^+) \Leftrightarrow 2 \text{X-Na} + (\text{Ca}^{2+})$		0.16	[E13]
$\text{X-Ca} + 2 (\text{K}^+) \Leftrightarrow 2 \text{X-K} + (\text{Ca}^{2+})$		3.00	[E14]
$\text{X-Ca} + 2 (\text{H}^+) \Leftrightarrow 2 \text{X-H} + (\text{Ca}^{2+})$		1.00	[E15]

<sup>1</sup> Round brackets denote solute activity. Numbers in italics denote log *K* (precipitation-dissolution, ion pairs), Gapon coefficient (cation exchange) or log *c* (anion exchange).

<sup>2</sup> All equilibrium reactions involving N and P are calculated for both band and non-band volumes if a banded fertilizer application has been made. These volumes are calculated dynamically from diffusive transport of soluble N and P.

<sup>3</sup> May only be entered as fertilizer, not considered to be naturally present in soils.

<sup>4</sup> X- denotes surface exchange site for cation or anion adsorption.

### Anion Adsorption Equilibria

$X-OH_2^+ \Leftrightarrow X-OH + (H^+)$	-7.35	[E16]
$X-OH \Leftrightarrow X-O^- + (H^+)$	-8.95	[E17]
$X-H_2PO_4 + H_2O \Leftrightarrow X-OH_2^+ + (H_2PO_4^-)$	-2.80	[E18]
$X-H_2PO_4 + (OH^-) \Leftrightarrow X-OH + (H_2PO_4^-)$	4.20	[E19]
$X-HPO_4^- + (OH^-) \Leftrightarrow X-OH + (HPO_4^{2-})$	2.60	[E20]

### Organic Acid Equilibria

$X-COOH \Leftrightarrow X-COO^- + (H^+)$	-5.00	[E21]
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### Ion Pair Equilibria

$(NH_4^+) \Leftrightarrow (NH_3)_{(g)} + (H^+)$	-9.24	[E22]
$H_2O \Leftrightarrow (H^+) + (OH^-)$	-14.3	[E23]
$(CO_2)_{(g)} + H_2O \Leftrightarrow (H^+) + (HCO_3^-)$	-6.42	[E24]
$(HCO_3^-) \Leftrightarrow (H^+) + (CO_3^{2-})$	-10.4	[E25]
$(AlOH^{2+}) \Leftrightarrow (Al^{3+}) + (OH^-)$	-9.06	[E26]
$(Al(OH)_2^+) \Leftrightarrow (AlOH^{2+}) + (OH^-)$	-10.7	[E27]
$(Al(OH)_3^0) \Leftrightarrow (Al(OH)_2^+) + (OH^-)$	-5.70	[E28]
$(Al(OH)_4^-) \Leftrightarrow (Al(OH)_3^0) + (OH^-)$	-5.10	[E29]
$(AlSO_4^+) \Leftrightarrow (Al^{3+}) + (SO_4^{2-})$	-3.80	[E30]
$(FeOH^{2+}) \Leftrightarrow (Fe^{3+}) + (OH^-)$	-12.1	[E31]
$(Fe(OH)_2^+) \Leftrightarrow (FeOH^{2+}) + (OH^-)$	-10.8	[E32]
$(Fe(OH)_3^0) \Leftrightarrow (Fe(OH)_2^+) + (OH^-)$	-6.94	[E33]
$(Fe(OH)_4^-) \Leftrightarrow (Fe(OH)_3^0) + (OH^-)$	-5.84	[E34]
$(FeSO_4^+) \Leftrightarrow (Fe^{3+}) + (SO_4^{2-})$	-4.15	[E35]
$(CaOH^+) \Leftrightarrow (Ca^{2+}) + (OH^-)$	-1.90	[E36]

$(\text{CaCO}_3^0) \Leftrightarrow (\text{Ca}^{2+}) + (\text{CO}_3^{2-})$	-4.38	[E37]
$(\text{CaHCO}_3^+) \Leftrightarrow (\text{Ca}^{2+}) + (\text{HCO}_3^-)$	-1.87	[E38]
$(\text{CaSO}_4^0) \Leftrightarrow (\text{Ca}^{2+}) + (\text{SO}_4^{2-})$	-2.92	[E39]
$(\text{MgOH}^+) \Leftrightarrow (\text{Mg}^{2+}) + (\text{OH}^-)$	-3.15	[E40]
$(\text{MgCO}_3^0) \Leftrightarrow (\text{Mg}^{2+}) + (\text{CO}_3^{2-})$	-3.52	[E41]
$(\text{MgHCO}_3^+) \Leftrightarrow (\text{Mg}^{2+}) + (\text{HCO}_3^-)$	-1.17	[E42]
$(\text{MgSO}_4^0) \Leftrightarrow (\text{Mg}^{2+}) + (\text{SO}_4^{2-})$	-2.68	[E43]
$(\text{NaCO}_3^-) \Leftrightarrow (\text{Na}^+) + (\text{CO}_3^{2-})$	-3.35	[E44]
$(\text{NaSO}_4^-) \Leftrightarrow (\text{Na}^+) + (\text{SO}_4^{2-})$	-0.48	[E45]
$(\text{KSO}_4^-) \Leftrightarrow (\text{K}^+) + (\text{SO}_4^{2-})$	-1.30	[E46]
$(\text{H}_3\text{PO}_4) \Leftrightarrow (\text{H}^+) + (\text{H}_2\text{PO}_4^-)$	-2.15	[E47]
$(\text{H}_2\text{PO}_4^-) \Leftrightarrow (\text{H}^+) + (\text{HPO}_4^{2-})$	-7.20	[E48]
$(\text{HPO}_4^{2-}) \Leftrightarrow (\text{H}^+) + (\text{PO}_4^{3-})$	-12.4	[E49]
$(\text{FeH}_2\text{PO}_4^{2+}) \Leftrightarrow (\text{Fe}^{3+}) + (\text{H}_2\text{PO}_4^-)$	-5.43	[E50]
$(\text{FeHPO}_4^+) \Leftrightarrow (\text{Fe}^{3+}) + (\text{HPO}_4^{2-})$	-10.9	[E51]
$(\text{CaH}_2\text{PO}_4^+) \Leftrightarrow (\text{Ca}^{2+}) + (\text{H}_2\text{PO}_4^-)$	-1.40	[E52]
$(\text{CaHPO}_4^0) \Leftrightarrow (\text{Ca}^{2+}) + (\text{HPO}_4^{2-})$	-2.74	[E53]
$(\text{CaPO}_4^-) \Leftrightarrow (\text{Ca}^{2+}) + (\text{PO}_4^{3-})$	-6.46	[E54]
$(\text{MgHPO}_4^0) \Leftrightarrow (\text{Mg}^{2+}) + (\text{HPO}_4^{2-})$	-2.91	[E55]

## Appendix F: Symbiotic N<sub>2</sub> Fixation

### *Microbial Growth*

$R_{\max i,l} = M_{ni,l} R' [\chi_{ni,l}] / ([\chi_{ni,l}] + K_{\chi n}) f_t f_{NP}$	respiration demand	[F1]
$f_t = T_l \{ \exp[B - H_a / (R T_l)] \} / \{ 1 + \exp[(H_{dl} - ST_l) / (R T_l)] + \exp[(ST_l - H_{dh}) / (R T_l)] \}$	Arrhenius function	[F2]
$f_{NP} = \min\{ [N_{ni,l}] / [N_n'], [P_{ni,l}] / [P_n'] \}$	N or P limitation	[F3]
$R_{i,l} = R_{\max i,l} (V_{O_2 i,l} / V_{O_2 \max i,l})$	O <sub>2</sub> limitation	[F4]
$V_{O_2 \max i,l} = 2.67 R_{\max i,l}$	O <sub>2</sub> demand	[F5]
$V_{O_2 i,l} = V_{O_2 \max i,l} [O_{2ri,l}] / ([O_{2ri,l}] + K_{O_2 r})$	equilibrate O <sub>2</sub> uptake with supply	[F6a]
$= 2\pi L_{ri,l} D_{sO_2} ([O_{2l}] - [O_{2ri,l}]) / \ln((r_{ri,l} + r_{wl}) / r_{ri,l})$		[F6b]
$R_{mi,l} = R_m N_{ni,l} f_{tm}$	maintenance respiration	[F7]
$f_{tm} = e^{[y(T_l - 298.16)]}$	temperature function	[F8]
$R_{gi,l} = \max\{0.0, R_{i,l} - R_{mi,l}\}$	growth + fixation respiration	[F9]
$R_{si,l} = \max\{0.0, R_{mi,l} - R_{i,l}\}$	microbial senescence	[F10]
$L_{Ci,l} = R_{si,l} \min\{M_{ni,l} / (2.5 N_{ni,l}), M_{ni,l} / (25.0 P_{ni,l})\}$	microbial C litterfall	[F11]

### *N<sub>2</sub> Fixation*

$V_{N_2 i,l} = \min\{R_{gi,l} E_{N_2}' f_{CP}, M_{ni,l} [N_n'] - N_{ni,l}\} [N_{2ri,l}] / ([N_{2ri,l}] + K_{N_2 r})$	rate of N <sub>2</sub> fixation	[F12]
$f_{CP} = \min\{[\chi_{ni,l}] / (1.0 + [v_{ni,l}] / K_{I\chi n}), [\pi_{ni,l}] / (1.0 + [v_{ni,l}] / K_{I\pi n})\}$	product inhibition of N <sub>2</sub> fixation	[F13]
$R_{N_2 i,l} = V_{N_2 i,l} / E_{N_2}'$	fixation respiration	[F14]
$U_{\chi i,l} = (R_{gi,l} - R_{N_2 i,l}) / (1 - Y_n')$	growth respiration	[F15]

$\delta M_{ni,l} / \delta t = U_{i,l} Y_n' - L_{Ci,l}$	microbial C growth		[F16]
$\delta N_{ni,l} / \delta t = \delta M_{ni,l} / \delta t \min\{v_{ni,l} / \chi_{ni,l}, [N_n']\}$	microbial N growth	$\delta M_{ndi,l} / \delta t > 0$	[F17a]
$\delta N_{ni,l} / \delta t = N_{ni,l} / M_{ni,l} \delta M_{ni,l} / \delta t$	microbial N growth	$\delta M_{ndi,l} / \delta t < 0$	[F17b]
$\delta P_{ni,l} / \delta t = \delta M_{ni,l} / \delta t \min\{\pi_{ni,l} / \chi_{ni,l}, [P_n']\}$	microbial P growth	$\delta M_{ndi,l} / \delta t > 0$	[F18a]
$\delta P_{ni,l} / \delta t = P_{ni,l} / M_{ni,l} \delta M_{ni,l} / \delta t$	microbial P growth	$\delta M_{ndi,l} / \delta t < 0$	[F18b]
$L_{Ni,l} = \text{abs}(\delta N_{ni,l} / \delta t)$	microbial N litterfall	$\delta N_{ndi,l} / \delta t < 0$	[F19]
$L_{Pi,l} = \text{abs}(\delta P_{ni,l} / \delta t)$	microbial P litterfall	$\delta P_{ndi,l} / \delta t < 0$	[F20]

### *Nodule – Root Exchange*

$V_{\chi i,l} = \kappa (\chi_{ri,l} M_{ni,l} - \chi_{ni,l} M_{ri,l}) / (M_{ni,l} + M_{ri,l})$	nodule–root C exchange		[F21]
$V_{v i,l} = \kappa (v_{ri,l} \chi_{ni,l} - v_{ni,l} \chi_{ri,l}) / (\chi_{ni,l} + \chi_{ri,l})$	nodule–root N exchange		[F22]
$V_{\pi i,l} = \kappa (\pi_{ri,l} \chi_{ni,l} - \pi_{ni,l} \chi_{ri,l}) / (\chi_{ni,l} + \chi_{ri,l})$	nodule–root P exchange		[F23]
$\delta \chi_{ni,l} / \delta t = V_{\chi i,l} - \min\{R_{mi,l}, R_{i,l}\} - R_{N2i,l} - U_{\chi i,l} + F_{LC1} L_{Ci,l}$	nodule nonstructural C		[F24]
$\delta v_{ni,l} / \delta t = V_{v i,l} - \delta N_{ni,l} / \delta t + V_{N2i,l} + F_{LN1} L_{Ni,l}$	nodule nonstructural N		[F25]
$\delta \pi_{ni,l} / \delta t = V_{\pi i,l} - \delta P_{ni,l} / \delta t + F_{LP1} L_{Pi,l}$	nodule nonstructural P		[F26]

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### Definition of Variables in Appendix F

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Variable	Definition	Units	Equations	Input Values	Reference
$B$	parameter such that $f_i = 1.0$ at $T_i = 298.15$ K		F2	17.533	



$\chi_{ni,l}$	nodule nonstructural C	$\text{g m}^{-2}$	F17a,F18a,F21,F22,B23,B24		
$[\chi_{ni,l}]$	nodule nonstructural C concentration	$\text{g g}^{-1}$	F1,F13		
$\chi_{ri,l}$	root nonstructural C	$\text{g m}^{-2}$	F21,F22,F23		
$D_{sO_2}$	diffusivity of aqueous $O_2$	$\text{m}^2 \text{h}^{-1}$	F6b		
$E_{N_2}'$	direct energy cost of $N_2$ fixation	$\text{g N g C}^{-1}$	F12,F14	0.25	Gutschick, (1981), Voisin et al., (2003)
$F_{LCI}$	fraction of nodule C litterfall remobilized as nonstructural C	-	F24		
$F_{LNI}$	fraction of nodule N litterfall remobilized as nonstructural N	-	F25		
$F_{LPI}$	fraction of nodule P litterfall remobilized as nonstructural P	-	F26		
$f_{CP}$	effect of nodule nonstructural C or P content on $N_2$ fixation	-	F12,F13		
$f_{NP}$	effect of nodule N or P content on respiration	-	F1,F3		
$f_t$	temperature function for nodule respiration	-	F1,F2		
$f_{tm}$	temperature function for nodule maintenance respiration	-	F7,F8		
$H_a$	energy of activation	$\text{J mol}^{-1}$	F2	$57.5 \times 10^3$	
$H_{dh}$	energy of high temperature deactivation	$\text{J mol}^{-1}$	F2	$220 \times 10^3$	
$H_{dl}$	energy of low temperature deactivation	$\text{J mol}^{-1}$	F2	$190 \times 10^3$	
$K_{\chi n}$	Michaelis-Menten constant for nodule respiration of $\chi_{ndi,l}$	$\text{g g}^{-1}$	F1	0.01	
$K_{I_{\chi n}}$	inhibition constant for nonstructural N:C on $N_2$ fixation	$\text{g g}^{-1}$	F13	10	
$K_{I_{r_n}}$	inhibition constant for nonstructural N:P on $N_2$ fixation	$\text{g g}^{-1}$	F13	1000	

$K_{N_2r}$	Michaelis-Menten constant for nodule $N_2$ uptake	$g\ N\ m^{-3}$	F12	0.14
$K_{O_2r}$	Michaelis-Menten constant for nodule $O_2$ uptake	$g\ O\ m^{-3}$	F6a	
$\kappa$	rate constant for nonstructural C,N,P exchange between root and nodule	$h^{-1}$	F21,F22,F23	
$L_{ri,l}$	root length	$m\ m^{-2}$	F6b	
$L_{Ci,l}$	nodule C litterfall	$g\ C\ m^{-2}\ h^{-1}$	F11,F16,F24	
$L_{Ni,l}$	nodule N litterfall	$g\ N\ m^{-2}\ h^{-1}$	F19,F25	
$L_{Pi,l}$	nodule P litterfall	$g\ P\ m^{-2}\ h^{-1}$	F20,F26	
$M_{ni,l}$	nodule structural C	$g\ C\ m^{-2}$	F1,F11,F12,F16 ,F17,F18,F21	
$M_{ri,l}$	root structural C	$g\ C\ m^{-2}$	F21	
$[N_n']$	maximum nodule structural N concentration	$g\ N\ g\ C^{-1}$	F3,F12	0.1
$N_{ni,l}$	nodule structural N	$g\ N\ m^{-2}$	F7,F11,F12,F17,F 19,F25	
$[N_{ni,l}]$	nodule structural N concentration	$g\ N\ g\ C^{-1}$	F3,F17a	
$[N_{2ri,l}]$	rhizosphere aqueous $N_2$ concentration	$g\ N\ m^{-3}$	F12	
$v_{ni,l}$	nodule nonstructural N	$g\ N\ m^{-2}$	F17a,F22,F25	
$v_{ri,l}$	root nonstructural N	$g\ N\ m^{-2}$	F22	
$[v_{ni,l}]$	nodule concentration of nonstructural N	$g\ g^{-1}$	F13,F17a	
$[O_{2ri,l}]$	rhizosphere aqueous $O_2$ concentration	$g\ O\ m^{-3}$	F6a,b	
$[O_{2l}]$	soil aqueous $O_2$ concentration	$g\ O\ m^{-3}$	F6b	

$[P_n']$	maximum nodule structural P concentration	$\text{g P g C}^{-1}$	F3,F18a	0.01
$P_{ni,l}$	nodule structural P	$\text{g P m}^{-2}$	F18a,F20,F26	
$[P_{ni,l}]$	nodule structural P concentration	$\text{g P g C}^{-1}$	F3,F11	
$\pi_{ni,l}$	nodule nonstructural P	$\text{g P m}^{-2}$	F18a,F23,F26	
$\pi_{ri,l}$	root nonstructural P	$\text{g P m}^{-2}$	F23	
$[\pi_{ni,l}]$	nodule concentration of nonstructural P	$\text{g g}^{-1}$	F13	
$R$	gas constant	$\text{J mol}^{-1} \text{K}^{-1}$	F2	8.3143
$R_{gi,l}$	nodule growth respiration	$\text{g C m}^{-2} \text{h}^{-1}$	F9,F12,F15	
$R'$	specific nodule respiration at 25°C, and non-limiting $\text{O}_2$ , $\chi_{ndi,l}$ , $V_{ndi,l}$ and $\pi_{ndi,l}$	$\text{h}^{-1}$	F1	0.125
$R_{i,l}$	nodule respiration under ambient $\text{O}_2$	$\text{g C m}^{-2} \text{h}^{-1}$	F4,F9,F10,F24	
$R_m$	specific nodule maintenance respiration at 25°C	$\text{g C g C}^{-1} \text{h}^{-1}$	F7	
$R_{maxi,l}$	nodule respiration under non-limiting $\text{O}_2$	$\text{g C m}^{-2} \text{h}^{-1}$	F1,F4,F5	
$R_{mi,l}$	nodule maintenance respiration	$\text{g C m}^{-2} \text{h}^{-1}$	F7,F9,F10,F24	
$R_{N_2i,l}$	nodule respiration for $\text{N}_2$ fixation	$\text{g C m}^{-2} \text{h}^{-1}$	F14,F15,F24	
$R_{si,l}$	nodule senescence respiration	$\text{g C m}^{-2} \text{h}^{-1}$	F9,F11	
$r_{ri,l}$	root radius	m	F6b	
$r_{wl}$	radius of soil water films	m	F6b	
$S$	change in entropy	$\text{J mol}^{-1} \text{K}^{-1}$	F2	710
$T_l$	soil temperature	K	F2,F8	

$U_{\chi i,l}$	uptake of nodule nonstructural C for growth	$\text{g C m}^{-2} \text{h}^{-1}$	F15,F16,F24	
$V_{\chi i,l}$	nonstructural C transfer between root and nodule	$\text{g C m}^{-2} \text{h}^{-1}$	F21,F24	
$V_{\nu i,l}$	nonstructural N transfer between root and nodule	$\text{g N m}^{-2} \text{h}^{-1}$	F22,F25	
$V_{\text{N}_2 i,l}$	$\text{N}_2$ fixation	$\text{g N m}^{-2} \text{h}^{-1}$	F12,F14,F25	
$V_{\text{O}_2 \text{maxi},l}$	$\text{O}_2$ uptake by nodules under non-limiting $\text{O}_2$	$\text{g O m}^{-2} \text{h}^{-1}$	F4,F5,F6a	
$V_{\text{O}_2 i,l}$	$\text{O}_2$ uptake by nodules under ambient $\text{O}_2$	$\text{g O m}^{-2} \text{h}^{-1}$	F4,F6	
$V_{\bar{m},l}$	nonstructural P transfer between root and nodule	$\text{g P m}^{-2} \text{h}^{-1}$	F23,F26	
$Y_n'$	nodule growth yield	$\text{g C g C}^{-1}$	F15,F16	0.67
$y$	shape parameter for $f_{\text{tm}}$	-	F8	0.081

## Appendix G: CH<sub>4</sub> Production and Consumption

### *Anaerobic Fermenters and H<sub>2</sub> Producing Acetogens*

$R_{i,f} = \{R'f M_{i,f,a} [DOC_{i,c}] / (Kf ((1 + [O_2] / K_i) + [DOC_{i,c}]))\} f_t$	respiration by fermenters		[G1]
$DOC_{i,c} \rightarrow 0.67 A_{i,c} + 0.33 CO_2-C + 0.11 H_2$	partition respiration products		[G2]
$U_{i,f,c} = R_{mi,f} + (R_{i,f} - R_{mi,f}) (1.0 + Y_f)$	uptake by fermenters	[ $R_{i,f} > R_{mi,f}$ ]	[G3a]
$U_{i,f,c} = R_{i,f}$		[ $R_{i,f} < R_{mi,f}$ ]	[G3b]
$Y_f = -\Delta G_f / E_M$	growth yield of fermentation		[G4]
$\Delta G_f = \Delta G'f + \{R T \ln([H_2] / [H_2']^4)\}$	free energy change of fermentation		[G5]
$\delta M_{i,f,j,c} / \delta t = F_j U_{i,f,c} - F_j R_{i,f} - D_{i,f,j,c}$	growth of fermenters	[ $R_{i,f} > R_{mi,f}$ ]	[G6a]
$\delta M_{i,f,j,c} / \delta t = F_j U_{i,f,c} - R_{mi,f,j} - D_{i,f,j,c}$		[ $R_{i,f} < R_{mi,f}$ ]	[G6b]

### *Acetotrophic Methanogens*

$R_{i,m} = \{R'_m M_{i,m,a} [A_{i,c}] / (K_m + [A_{i,c}])\} f_t$	respiration by acetotrophic methanogens		[G7]
$A_{i,c} \rightarrow 0.50 CH_4-C + 0.50 CO_2-C$	partition respiration products		[G8]
$U_{i,m,c} = R_{mi,m} + (R_{i,m} - R_{mi,m}) (1.0 + Y_m)$	uptake by acetotrophic methanogens	[ $R_{i,m} > R_{mi,m}$ ]	[G9a]
$U_{i,m,c} = R_{i,m}$		[ $R_{i,m} < R_{mi,m}$ ]	[G9b]
$-Y_m = -\Delta G'_m / E_M$	growth yield of acetotrophic methanogenesis		[G10]
$\delta M_{i,m,j,c} / \delta t = F_j U_{i,m,c} - F_j R_{i,m} - D_{i,m,j,c}$	growth of acetotrophic methanogens	[ $R_{i,m} > R_{mi,m}$ ]	[G11a]

$$\delta M_{i,m,j,c} / \delta t = F_j U_{i,m,c} - R_{mi,m,j} - D_{i,m,j,c} \quad [R_{i,m} < R_{mi,m}] \quad [G11b]$$

### *Hydrogenotrophic Methanogens*

$$R_h = \{R'_h M_{h,a} [H_2] / (K_h + [H_2]) [CO_2] / (K_c + [CO_2])\} f_t \quad \text{respiration by hydrogenotrophic methanogens} \quad [G12]$$



$$U_{h,c} = R_{mh} + (R_h - R_{mh}) (1.0 + Y_h) \quad \text{uptake by hydrogenotrophic methanogens} \quad [R_h > R_{mh}] \quad [G14a]$$

$$U_{h,c} = R_h \quad [R_h < R_{mh}] \quad [G14b]$$

$$Y_h = -\Delta G_h / E_C \quad \text{growth yield of hydrogenotrophic methanogenesis} \quad [G15]$$

$$\Delta G_h = \Delta G'_h - \{R T \ln([H_2] / [H_2']^4)\} \quad \text{free energy change of} \quad [G16]$$

$$\delta M_{h,j,c} / \delta t = F_j U_{h,c} - F_j R_h - D_{h,j,c} \quad \text{hydrogenotrophic methanogenesis} \quad [R_h > R_{mh}] \quad [G17a]$$

$$\delta M_{h,j,c} / \delta t = F_j U_{h,c} - R_{mh,j} - D_{h,j,c} \quad \text{growth of hydrogenotrophic methanogens} \quad [R_h < R_{mh}] \quad [G17b]$$

### *Autotrophic Methanotrophs*

$$X'_t = \{X'_t M_{t,a} [CH_4] / (K_t + [CH_4])\} f_t \quad CH_4 \text{ oxidation by methanotrophs under non-limiting } O_2 \quad [G18]$$

$$R'_t = X'_t Y_{tR} \quad \text{respiration by methanotrophs under non-limiting } O_2 \quad [G19]$$

$$Y_{tR} = -\Delta G'_t / E_G \quad \text{energy yield from } CH_4 \text{ oxidation} \quad [G20]$$

$$X_t = X'_t f_{O_2t} \quad CH_4 \text{ oxidation by methanotrophs under ambient } O_2 \quad [G21a]$$

$$R_t = R'_t f_{O_2t} \quad \text{respiration by methanotrophs under ambient } O_2 \quad [G21b]$$



$\text{CH}_2\text{O} - \text{C} + 2.67 \text{O}_2 \rightarrow \text{CO}_2 - \text{C} + 1.5 \text{H}_2\text{O}$	O <sub>2</sub> requirements for respiration by methanotrophs		[G24]
$U_{t,c} = R_{mt} + (R_t - R_{mt}) (1.0 + Y_{tG})$	uptake by methanotrophs	$[R_t > R_{mt}]$	[G25a]
$U_{t,c} = R_t$		$[R_t < R_{mt}]$	[G25b]
$Y_{tG} = -\Delta G'_c / E_M$	growth yield of methanotrophy		[G26]
$\delta M_{t,j,c} / \delta t = F_j U_{t,c} - F_j R_t - D_{t,j,c}$		$[R_t > R_{mt}]$	[G27a]
$\delta M_{t,j,c} / \delta t = F_j U_{t,c} - R_{mt,j} - D_{t,j,c}$		$[R_t < R_{mt}]$	[G27b]

### Definition of Variables in Appendix G

Variable	Definition	Units	Equations	Input Values	Reference
A	acetate	g C m <sup>-2</sup>	[G2]		
[A]	aqueous concentration of acetate	g C m <sup>-3</sup>	[G7]		
<i>a</i>	descriptor for <i>j</i> = active component of <i>M<sub>i</sub></i>				
[CH <sub>4</sub> ]	aqueous concentration of CH <sub>4</sub>	g C m <sup>-3</sup>	[G18]		
[CO <sub>2</sub> ]	aqueous concentration of CO <sub>2</sub>	g C m <sup>-3</sup>	[G12]		
<i>D<sub>h,j,c</sub></i>	decomposition of hydrogenotrophic methanogens	g C m <sup>-2</sup> h <sup>-1</sup>	[G17]		
<i>D<sub>i,f,j,c</sub></i>	decomposition of fermenters and acetogens	g C m <sup>-2</sup> h <sup>-1</sup>	[G6]		
<i>D<sub>i,m,j,c</sub></i>	decomposition of acetotrophic methanogens	g C m <sup>-2</sup> h <sup>-1</sup>	[G11]		
<i>D<sub>t,j,c</sub></i>	decomposition of autotrophic methanotrophs	g C m <sup>-2</sup> h <sup>-1</sup>	[G27]		
E <sub>C</sub>	energy required to construct new M from CO <sub>2</sub>	kJ g C <sup>-1</sup>	[G15]	75	

$E_G$	energy required to transform CH <sub>4</sub> into organic C	kJ g C <sup>-1</sup>	[G20]	23.5	Anthony (1982)
$E_M$	energy required to construct new M from organic C	kJ g C <sup>-1</sup>	[G4,G10,G26]	25	
$F_j$	partitioning coefficient for $j$ in $M_{i,n,j}$		[G6,G11,G17,G27]		
$f$	descriptor for fermenters and acetogens in each $M_i$				
$f_{O_2t}$	ratio of O <sub>2</sub> uptake to O <sub>2</sub> requirement for CH <sub>4</sub> oxidation		[G21a,b]		
$f_t$	temperature function for growth-related processes (dimensionless)		[G1,G7,G12]		
$\Delta G'_c$	free energy change of C oxidation-O <sub>2</sub> reduction	kJ g C <sup>-1</sup>	[G26]	-37.5	Brock and Madigan (1991)
$\Delta G_f$	free energy change of fermentation plus acetogenesis	kJ g P <sub>i,c</sub> <sup>-1</sup>	[G4,G5]		
$\Delta G'_f$	$\Delta G_f$ when [H <sub>2</sub> ] = [H <sub>2</sub> ']	kJ g P <sub>i,c</sub> <sup>-1</sup>	[G5]	-4.43	Brock and Madigan (1991), Schink (1997)
$\Delta G_h$	free energy change of hydrogenotrophic methanogenesis	kJ g CO <sub>2</sub> -C <sup>-1</sup>	[G15,G16]		
$\Delta G'_h$	free energy change of hydrogenotrophic methanogenesis when [H <sub>2</sub> ] = [H <sub>2</sub> ']	kJ g CO <sub>2</sub> -C <sup>-1</sup>	[G16]	-0.27	Brock and Madigan (1991)
$\Delta G'_m$	free energy change of acetotrophic methanogenesis	kJ g A <sub>i,c</sub> <sup>-1</sup>	[G10]	-1.03	Brock and Madigan (1991), Schink (1997)
$\Delta G'_t$	free energy change of CH <sub>4</sub> oxidation by methanotrophs	kJ g CH <sub>4</sub> -C <sup>-1</sup>	[G20]	-9.45	Brock and Madigan (1991)



[H <sub>2</sub> ]	aqueous concentration of H <sub>2</sub>	g H m <sup>-3</sup>	[G5,G12,G16]		
[H <sub>2</sub> ']	aqueous concentration of H <sub>2</sub> when $\Delta G_h = \Delta G'_h$ and $\Delta G_f = \Delta G'_f$	g H m <sup>-3</sup>	[G5,G16]	150 x 10 <sup>-6</sup>	Brock and Madigan (1991)
<i>h</i>	descriptor for hydrogenotrophic methanogens in each M <sub><i>i</i></sub>				
<i>i</i>	descriptor for organic matter-microbe complex ( <i>i</i> = plant residue, manure, particulate OM <sub><i>i</i></sub> , or humus)				
<i>j</i>	descriptor for structural or kinetic components for each functional type within each M <sub><i>j</i></sub> ( <i>e.g.</i> <i>a</i> = <i>active</i> )				
<b>K<sub><i>c</i></sub></b>	M-M constant for uptake of CO <sub>2</sub> by hydrogenotrophic methanogens	g C m <sup>-3</sup>	[G12]	0.12	
<b>K<sub><i>f</i></sub></b>	M-M constant for uptake of DOC <sub><i>i,c</i></sub> by fermenters and acetogens	g C m <sup>-3</sup>	[G1]	12	McGill et al. (1981)
<b>K<sub><i>i</i></sub></b>	inhibition constant for O <sub>2</sub> on fermentation	g O m <sup>-3</sup>	[G1]	0.064	
<b>K<sub><i>h</i></sub></b>	M-M constant for uptake of H <sub>2</sub> by hydrogenotrophic methanogens	g H m <sup>-3</sup>	[G12]	0.01	Mosey (1983), Robinson and Tiedje (1982)
<b>K<sub><i>m</i></sub></b>	M-M constant for uptake of A <sub><i>i,c</i></sub> by acetotrophic methanogens	g C m <sup>-3</sup>	[G7]	12	Smith and Mah (1978), Zehnder et al. (1980)
<b>K<sub><i>t</i></sub></b>	M-M constant for uptake of CH <sub>4</sub> by methanotrophs	g C m <sup>-3</sup>	[G18]	3 x 10 <sup>-3</sup>	Conrad (1984)
<i>k</i>	descriptor for elemental fraction within each <i>j</i> ( <i>j</i> = <i>c</i> , <i>n</i> or <i>p</i> )				
<b>M</b>	microbial communities	g C m <sup>-2</sup>			
<b>M<sub><i>h</i></sub></b>	hydrogenotrophic methanogen community	g C m <sup>-2</sup>	[G12,G17]		
<b>M<sub><i>i,f</i></sub></b>	fermenter and acetogenic community	g C m <sup>-2</sup>	[G1,G6]		

$M_{i,m}$	acetotrophic methanogen community	$\text{g C m}^{-2}$	[G7,G11]		
$M_t$	autotrophic methanotrophic community	$\text{g C m}^{-2}$	[G18,G27]		
$m$	descriptor for acetotrophic methanogens in each $M_j$				
DOC	soluble organic matter	$\text{g C m}^{-2}$	[G2]		
[DOC]	aqueous concentration of soluble organic matter	$\text{g C m}^{-3}$	[G1]		
$R$	gas constant	$\text{kJ mol}^{-1} \text{K}^{-1}$	[G5,G16]	$8.3143 \times 10^{-3}$	
$R'_f$	specific respiration by fermenters and acetogens at saturating $[P_{i,c}]$ , 30 °C and zero water potential	$\text{g C g } M_{i,f,a}^{-1} \text{h}^{-1}$	[G1]	0.4	Lawrence (1971), Wofford et al. (1986)
$R_h$	CO <sub>2</sub> reduction by hydrogenotrophic methanogens	$\text{g C m}^{-2} \text{h}^{-1}$	[G12,G13,G14,G17,G18]		
$R'_h$	specific CO <sub>2</sub> reduction by hydrogenotrophic methanogens at saturating $[\text{H}_2]$ and $[\text{CO}_2]$ , and at 30 °C and zero water potential	$\text{g C g } M_{h,a}^{-1} \text{h}^{-1}$	[G12]	0.12	Shea et al. (1968), Zehnder and Wuhrmann (1977)
$R_{i,f}$	respiration of hydrolysis products by fermenters and acetogens	$\text{g C m}^{-2} \text{h}^{-1}$	[G1,G2,G3,G6]		
$R_{i,m}$	respiration of acetate by acetotrophic methanogens	$\text{g C m}^{-2} \text{h}^{-1}$	[G7,G8,G9,G11]		
$R'_m$	specific respiration by acetotrophic methanogens at saturating $[A_{i,c}]$ , 30 °C and zero water potential	$\text{g C g } M_{i,m,a}^{-1} \text{h}^{-1}$	[G7]	0.20	Smith and Mah (1980)
$R_{m,h,j}$	maintenance respiration by hydrogenotrophic methanogens	$\text{g C m}^{-2} \text{h}^{-1}$	[G14,G17]		
$R_{m,i,f,j}$	maintenance respiration by fermenters and acetogens	$\text{g C m}^{-2} \text{h}^{-1}$	[G3,G6]		
$R_{m,i,m,j}$	maintenance respiration by acetotrophic methanogens	$\text{g C m}^{-2} \text{h}^{-1}$	[G9,G11]		

$R_{mt,j}$	maintenance respiration by methanotrophs	$\text{g C m}^{-2} \text{h}^{-1}$	[G25,G27]		
$R_t$	CH <sub>4</sub> oxidation by methanotrophs for respiration	$\text{g C m}^{-2} \text{h}^{-1}$	[G21b,G23,G24, G25,G27a]		
$R'_t$	CH <sub>4</sub> oxidation by methanotrophs for respiration at saturating O <sub>2</sub>	$\text{g C m}^{-2} \text{h}^{-1}$	[G19,G21b]		
T	soil temperature	K	[G5,G16]		
$t$	descriptor for autotrophic methanotrophs				
$U_{h,c}$	rate of CO <sub>2</sub> uptake by $M_h$	$\text{g C m}^{-2} \text{h}^{-1}$	[G14,G17,G18]		
$U_{i,f,k}$	rate of DOC <sub><math>i,k</math></sub> uptake by $M_{i,f}$	$\text{g C m}^{-2} \text{h}^{-1}$	[G3,G6]		
$U_{i,m,c}$	rate of A <sub><math>i,c</math></sub> uptake by $M_{i,m}$	$\text{g C m}^{-2} \text{h}^{-1}$	[G9,G11]		
$U_{t,c}$	rate of CH <sub>4</sub> uptake by $M_t$	$\text{g C m}^{-2} \text{h}^{-1}$	[G25,G27]		
$X_t$	CH <sub>4</sub> oxidation by methanotrophs	$\text{g C m}^{-2} \text{h}^{-1}$	[G21a,G22]		
$X'_t$	CH <sub>4</sub> oxidation by methanotrophs at saturating O <sub>2</sub>	$\text{g C m}^{-2} \text{h}^{-1}$	[G1,G2,G4a]		
$X'_t$	specific CH <sub>4</sub> oxidation by methanotrophs at saturating O <sub>2</sub> , 30 °C and zero water potential	$\text{g C g}^{-1} \text{h}^{-1}$	[G18]	0.5	Conrad (1984)
$Y_f$	biomass yield from fermentation and acetogenic reactions	$\text{g } M_{i,f} \text{ g DOC}_{i,c}^{-1}$	[G3,G4]		
$Y_h$	biomass yield from hydrogenotrophic methanogenic reaction	$\text{g } M_h \text{ g CO}_2\text{-C}^{-1}$	[G14,G15,G18]		
$Y_m$	biomass yield from acetotrophic methanogenic reaction	$\text{g } M_{i,m} \text{ g A}_{i,c}^{-1}$	[G9,G10]		
$Y_{tG}$	biomass yield from methanotrophic growth respiration	$\text{g } M_t\text{-C g CH}_4\text{-C}^{-1}$	[G25a,G26]		
$Y_{tR}$	ratio of CH <sub>4</sub> respired vs. CH <sub>4</sub> oxidized by methanotrophs	$\text{g C g C}^{-1}$	[G19,G20]		

## Appendix H: Inorganic N Transformations

### *Mineralization and Immobilization of Ammonium by All Microbial Populations*

$$I_{\text{NH}_4^{i,n,j}} = (M_{i,m,j,C} C_{\text{Nj}} - M_{i,m,j,N}) \quad (I_{\text{NH}_4^{i,n,j}} < 0) \quad [\text{H1a}]$$

$$I_{\text{NH}_4^{i,n,j}} = (M_{i,m,j,C} C_{\text{Nj}} - M_{i,m,j,N}) [\text{NH}_4^+] / ([\text{NH}_4^+] + K_{\text{NH}_4m}) \quad (I_{\text{NH}_4^{i,n,j}} > 0) \quad [\text{H1b}]$$

$$I_{\text{NO}_3^{i,n,j}} = (M_{i,m,j,C} C_{\text{Nj}} - (M_{i,m,j,N} + I_{\text{NH}_4^{i,n,j}})) [\text{NO}_3^-] / ([\text{NO}_3^-] + K_{\text{NO}_3m}) \quad (I_{\text{NO}_3^{i,n,j}} > 0) \quad [\text{H1b}]$$

### *Oxidation of DOC and Reduction of Oxygen by Heterotrophs*

$$X'_{\text{DOC},i,h} = \{X'_{\text{DOC}} M_{i,h,a} [\text{DOC}_i] / ([\text{DOC}_i] + K_{\text{Xh}})\} f_t \quad [\text{H2}]$$

$$R'_{\text{O}_2,i,h} = \mathbf{RQ}_C X'_{\text{DOC},i,h} \quad [\text{H3}]$$

$$R_{\text{O}_2,i,h} = 4\pi n M_{i,h,a} D_{\text{sO}_2} ([\text{O}_{2s}] - [\text{O}_{2mi,h}]) [r_m r_w / (r_w - r_m)] \quad [\text{H4a}]$$

$$= R'_{\text{O}_2,i,h} [\text{O}_{2mi,h}] / ([\text{O}_{2mi,h}] + K_{\text{O}_2h}) \quad [\text{H4b}]$$

$$X_{\text{DOC},i,h} = X'_{\text{DOC},i,h} R_{\text{O}_2,i,h} / R'_{\text{O}_2,i,h} \quad [\text{H5}]$$

### *Oxidation of DOC and Reduction of Nitrate, Nitrite and Nitrous Oxide by Denitrifiers*

$$R'_{\text{NO}_3^{i,d}} = E_{\text{NO}_x} f_e (R'_{\text{O}_2^{i,d}} - R_{\text{O}_2^{i,d}}) \quad [\text{H6}]$$

$$R_{\text{NO}_3^{i,d}} = R'_{\text{NO}_3^{i,d}} [\text{NO}_3^-] / ([\text{NO}_3^-] + K_{\text{NO}_3d}) \quad [\text{H7}]$$

$$R_{\text{NO}_2^{i,d}} = (R'_{\text{NO}_3^{i,d}} - R_{\text{NO}_3^{i,d}}) [\text{NO}_2^-] / ([\text{NO}_2^-] + K_{\text{NO}_2d}) \quad [\text{H8}]$$

$$R_{\text{N}_2\text{O},i,d} = 2 (R'_{\text{NO}_3^{i,d}} - R_{\text{NO}_3^{i,d}} - R_{\text{NO}_2^{i,d}}) [\text{N}_2\text{O}] / ([\text{N}_2\text{O}] + K_{\text{N}_2\text{O}d}) \quad [\text{H9}]$$

$$X_{\text{DOC},i,d} = X_{\text{DOC},i,d} (\text{from } [\text{H5}]) + F_{\text{NO}_x} (R_{\text{NO}_3^{i,d}} + R_{\text{NO}_2^{i,d}}) + F_{\text{N}_2\text{O}} R_{\text{N}_2\text{O},i,d} \quad [\text{H10}]$$

### *Oxidation of Ammonia and Reduction of Oxygen by Nitrifiers*

$$X'_{\text{NH}_3^{i,n}} = X'_{\text{NH}_3} M_{i,n,a} \{[\text{NH}_3\text{S}] / ([\text{NH}_3\text{S}] + K_{\text{NH}_3n})\} \{[\text{CO}_2\text{S}] / ([\text{CO}_2\text{S}] + K_{\text{CO}_2})\} f_t \quad [\text{H11}]$$

$$R'_{\text{O}_2^{i,n}} = \mathbf{RQ}_{\text{NH}_3} X'_{\text{NH}_3^{i,n}} + \mathbf{RQ}_C X'_{\text{Ci},n} \quad [\text{H12}]$$

$$R_{\text{O}_2^{i,n}} = 4\pi n M_{i,n,a} D_{\text{sO}_2} (r_m r_w / (r_w - r_m)) ([\text{O}_{2s}] - [\text{O}_{2mi,n}]) \quad [\text{H13a}]$$

$$= R'_{\text{O}_2^{i,n}} [\text{O}_{2mi,n}] / ([\text{O}_{2mi,n}] + K_{\text{O}_2n}) \quad [\text{H13b}]$$

$$X_{\text{NH}_3^{i,n}} = X'_{\text{NH}_3^{i,n}} R_{\text{O}_2^{i,n}} / R'_{\text{O}_2^{i,n}} \quad [\text{H14}]$$

### *Oxidation of Nitrite and Reduction of Oxygen by Nitrifiers*

$$X'_{\text{NO}_2^{i,o}} = X'_{\text{NO}_2} M_{i,o,a} \{[\text{NO}_2^-] / ([\text{NO}_2^-] + K_{\text{NO}_2o})\} \{[\text{CO}_2\text{S}] / ([\text{CO}_2\text{S}] + K_{\text{CO}_2})\} f_t \quad [\text{H15}]$$

$$R'_{\text{O}_2^{i,o}} = \mathbf{RQ}_{\text{NO}_2} X'_{\text{NO}_2^{i,o}} + \mathbf{RQ}_C X'_{\text{Ci},o} \quad [\text{H16}]$$

$$R_{O_2i,o} = 4\pi n M_{i,o,a} D_{sO_2} (r_m r_w / (r_w - r_m)) ([O_{2s}] - [O_{2mi,o}]) \quad [H17a]$$

$$= R'_{O_2i,o} [O_{2mi,o}] / ([O_{2mi,o}] + K_{O_2o}) \quad [H17b]$$

$$X_{NO_2i,o} = X'_{NO_2i,o} R_{O_2i,o} / R'_{O_2i,o} \quad [H18]$$

*Oxidation of Ammonia and Reduction of Nitrite by Nitrifiers*

$$R'_{NO_2i,n} = E_{NO_x} f_e (R'_{O_2i,n} - R_{O_2i,n}) \quad [H19]$$

$$R_{NO_2i,n} = R'_{NO_2i,n} \{ [NO_2^-] / ([NO_2^-] + K_{NO_2n}) \} \{ [CO_{2s}] / ([CO_{2s}] + K_{CO_2}) \} \quad [H20]$$

$$X_{NH_3i,n} = X_{NH_3i,n} \text{ (from [H14])} + 0.33 R_{NO_2i,n} \quad [H21]$$

**Definition of Variables in Appendix H**

Name	Definition	Units	Equations	Input Values	Reference
<i>Subscripts</i>					
<i>a</i>	active component of $M_{i,m}$				
<i>d</i>	heterotrophic denitrifier population (subset of <i>h</i> )				
<i>h</i>	heterotrophic community (subset of <i>m</i> )				
<i>i</i>	substrate-microbe complex				
<i>j</i>	kinetic components of $M_{i,m}$				
<i>m</i>	all microbial communities				
<i>n</i>	autotrophic ammonia oxidizer population (subset of <i>m</i> )				
<i>o</i>	autotrophic nitrite oxidizer population (subset of <i>m</i> )				
<i>Variables</i>					
$C_{Nj}$	maximum ratio of $M_{i,m,j,N}$ to $M_{i,m,j,C}$ maintained by $M_{i,m,j}$	$g\ N\ g\ C^{-1}$	[H1]	0.22 and 0.13 for <i>j</i> = labile and resistant	
$[CO_{2s}]$	CO <sub>2</sub> concentration in soil solution	$g\ C\ m^{-3}$	[H11,H15,H20]		
$[DOC_i]$	concentration of dissolved decomposition products	$g\ C\ m^{-3}$	[H2]		
$D_{sO_2}$	aqueous dispersivity-diffusivity of O <sub>2</sub>	$m^2\ h^{-1}$	[H4,H13,H17]		
$E_{NO_x}$	e <sup>-</sup> accepted by NO <sub>x</sub> vs. O <sub>2</sub> when oxidizing DOC	$g\ N\ g\ O_2^{-1}$	[H6,H19]	28/32 = 0.875	

$F_{NO_x}$	e <sup>-</sup> donated by C vs. e <sup>-</sup> accepted by NO <sub>x</sub> when oxidizing DOC	g C g N <sup>-1</sup>	[H10]	12/28 = 0.43	
$F_{N_2O}$	e <sup>-</sup> donated by C vs. e <sup>-</sup> accepted by N <sub>2</sub> O when oxidizing DOC	g C g N <sup>-1</sup>	[H10]	6/28 = 0.215	
$f_e$	fraction of electrons not accepted by O <sub>2</sub> transferred to N oxides	-	[H6,H19]	0.25	Koike and Hattori (1975)
$f_t$	temperature function for microbial processes	-	[H2,H11]		
$I_{NH_4i,n,j}$	mineralization ( $I_{NH_4i,n,j} < 0$ ) or immobilization ( $I_{NH_4i,n,j} > 0$ ) of NH <sub>4</sub> <sup>+</sup> by $M_{i,n,j,C}$	g N m <sup>-2</sup> h <sup>-1</sup>	[H1]		
$I_{NO_3i,n,j}$	immobilization ( $I_{NO_3i,n,j} > 0$ ) of NO <sub>3</sub> <sup>-</sup> by $M_{i,n,j,C}$	g N m <sup>-2</sup> h <sup>-1</sup>	[H1]		
$K_{CO_2}$	Michaelis-Menten constant for reduction of CO <sub>2</sub> S by $M_{i,n,a}$ and $M_{i,o,a}$	g C m <sup>-3</sup>	[H11,H15,H20]	0.15	
$K_{NH_3n}$	M-M constant for oxidation of NH <sub>3</sub> S by nitrifiers	g N m <sup>-3</sup>	[H11]	0.01	Suzuki et al. (1974)
$K_{NH_4m}$	M-M constant for microbial NH <sub>4</sub> <sup>+</sup> uptake	g N m <sup>-3</sup>	[H1]	0.35	
$K_{NO_2d}$	M-M constant for reduction of NO <sub>2</sub> <sup>-</sup> by denitrifiers	g N m <sup>-3</sup>	[H8]	3.5	Yoshinari et al. (1977)
$K_{NO_2n}$	M-M constant for reduction of NO <sub>2</sub> <sup>-</sup> by nitrifiers	g N m <sup>-3</sup>	[H20]	3.5	
$K_{NO_2o}$	M-M constant for oxidation of NO <sub>2</sub> <sup>-</sup> by nitrifiers	g N m <sup>-3</sup>	[H15]	10	
$K_{NO_3d}$	M-M constant for reduction of NO <sub>3</sub> <sup>-</sup> by denitrifiers	g N m <sup>-3</sup>	[H7]	3.5	Yoshinari et al. (1977)
$K_{N_2Od}$	M-M constant for reduction of N <sub>2</sub> O by denitrifiers	g N m <sup>-3</sup>	[H9]	0.35	Yoshinari et al. (1977)
$K_{O_2h}$	M-M constant for reduction of O <sub>2</sub> s by heterotrophs	g O <sub>2</sub> m <sup>-3</sup>	[H4b]	0.064	Griffin (1972)
$K_{O_2n}$	M-M constant for reduction of O <sub>2</sub> s by NH <sub>3</sub> oxidizers	g O <sub>2</sub> m <sup>-3</sup>	[H13b]	0.32	Focht and Verstraete (1977)
$K_{O_2o}$	M-M constant for reduction of O <sub>2</sub> s by NO <sub>2</sub> <sup>-</sup> oxidizers	g O <sub>2</sub> m <sup>-3</sup>	[H17b]	0.32	Focht and Verstraete (1977)

$K_{xh}$	M-M constant for oxidation of DOC by heterotrophs	$\text{g C m}^{-3}$	[H2]	12	(McGill et al., 1981)
$M_{i,h,a}$	active biomass of heterotrophs	$\text{g C m}^{-2}$	[H2,H7]		
$M_{i,n,a}$	active biomass of $\text{NH}_3$ oxidizers	$\text{g C m}^{-2}$	[H11,H13]		
$M_{i,m,j,C}$	C biomass of microbial population $M_{i,m,j}$	$\text{g C m}^{-2}$	[H1]		
$M_{i,m,j,N}$	N biomass of microbial population $M_{i,m,j}$	$\text{g N m}^{-2}$	[H1]		
$M_{i,o,a}$	active biomass of $\text{NO}_2^-$ oxidizers	$\text{g C m}^{-2}$	[H15,H17]		
$[\text{NH}_{3s}]$	concentration of $\text{NH}_3$ in soil solution	$\text{g N m}^{-3}$	[H11]		
$[\text{NH}_4^+]$	concentration of $\text{NH}_4^+$ in soil solution	$\text{g N m}^{-3}$	[H1]		
$[\text{NO}_2^-]$	concentration of $\text{NO}_2^-$ in soil solution	$\text{g N m}^{-3}$	[H8,H15,H20]		
$[\text{NO}_3^-]$	concentration of $\text{NO}_3^-$ in soil solution	$\text{g N m}^{-3}$	[H7]		
$[\text{N}_2\text{O}]$	concentration of $\text{N}_2\text{O}$ in soil solution	$\text{g N m}^{-3}$	[H9]		
$n$	number of microbes	$\text{g}^{-1}$	[H13,H17]		
$[\text{O}_{2mi,h}]$	$\text{O}_2$ concentration at heterotrophic surfaces	$\text{g O}_2 \text{ m}^{-3}$	[H7]		
$[\text{O}_{2mi,n}]$	$\text{O}_2$ concentration at $\text{NH}_3$ oxidizer surfaces	$\text{g O}_2 \text{ m}^{-3}$	[H13]		
$[\text{O}_{2mi,o}]$	$\text{O}_2$ concentration at $\text{NO}_2^-$ oxidizer surfaces	$\text{g O}_2 \text{ m}^{-3}$	[H17]		
$[\text{O}_{2s}]$	$\text{O}_2$ concentration in soil solution	$\text{g O}_2 \text{ m}^{-3}$	[H7,H13,H17]		
$R_{\text{NO}_2i,d}$	$\text{NO}_2^-$ reduction by denitrifiers	$\text{g N m}^{-2} \text{ h}^{-1}$	[H8,H9,H10]		
$R'_{\text{NO}_2i,n}$	rate of $\text{NO}_2^-$ reduction by $\text{NH}_3$ oxidizers under non-limiting $[\text{NO}_2^-]$ and $[\text{CO}_{2s}]$	$\text{g N m}^{-2} \text{ h}^{-1}$	[H19,H20]		
$R_{\text{NO}_2i,n}$	rate of $\text{NO}_2^-$ reduction by $\text{NH}_3$ oxidizers under ambient $[\text{NO}_2^-]$ and $[\text{CO}_{2s}]$	$\text{g N m}^{-2} \text{ h}^{-1}$	[H20,H21]		
$R'_{\text{NO}_3i,d}$	$\text{NO}_3^-$ reduction by denitrifiers under non-limiting $[\text{NO}_3^-]$	$\text{g N m}^{-2} \text{ h}^{-1}$	[H6,H7,H8,H9]		
$R_{\text{NO}_3i,d}$	$\text{NO}_3^-$ reduction by denitrifiers under ambient $[\text{NO}_3^-]$	$\text{g N m}^{-2} \text{ h}^{-1}$	[H7,H8,H9,H10]		
$R_{\text{N}_2\text{O}i,d}$	$\text{N}_2\text{O}$ reduction by denitrifiers	$\text{g N m}^{-2} \text{ h}^{-1}$	[H9,H10]		
$R'_{\text{O}_2i,d}$	rate of $\text{O}_{2s}$ reduction by denitrifiers under non-limiting $[\text{O}_{2s}]$	$\text{g O}_2 \text{ m}^{-2} \text{ h}^{-1}$	[H6]		
$R_{\text{O}_2i,d}$	rate of $\text{O}_{2s}$ reduction by denitrifiers under ambient $[\text{O}_{2s}]$	$\text{g O}_2 \text{ m}^{-2} \text{ h}^{-1}$	[H6]		

$R'_{O_2i,h}$	rate of $O_{2S}$ reduction by heterotrophs under non-limiting [ $O_{2S}$ ]	$g O_2 m^{-2} h^{-1}$	[H3,H4,H5]		
$R_{O_2i,h}$	rate of $O_{2S}$ reduction by heterotrophs under ambient [ $O_{2S}$ ]	$g O_2 m^{-2} h^{-1}$	[H4,H5]		
$R'_{O_2i,n}$	rate of $O_{2S}$ reduction by $NH_3$ oxidizers under non-limiting [ $O_{2S}$ ]	$g O_2 m^{-2} h^{-1}$	[H12,H13,H14,H19]		
$R_{O_2i,n}$	rate of $O_{2S}$ reduction by $NH_3$ oxidizers under ambient [ $O_{2S}$ ]	$g O_2 m^{-2} h^{-1}$	[H13,H14,H19]		
$R'_{O_2i,o}$	rate of $O_{2S}$ reduction by $NO_2^-$ oxidizers under non-limiting [ $O_{2S}$ ]	$g O_2 m^{-2} h^{-1}$	[H16,H17,H18]		
$R_{O_2i,o}$	rate of $O_{2S}$ reduction by $NO_2^-$ oxidizers under ambient [ $O_{2S}$ ]	$g O_2 m^{-2} h^{-1}$	[H17,H18]		
<b>RQ<sub>C</sub></b>	respiratory quotient for reduction of $O_2$ coupled to oxidation of C	$g O_2 g C^{-1}$	[H3,H12,H16]	2.67	Brock and Madigan (1991)
<b>RQ<sub>NH<sub>3</sub></sub></b>	respiratory quotient for reduction of $O_2$ coupled to oxidation of $NH_3$	$g O_2 g N^{-1}$	[H12]	3.43	Brock and Madigan (1991)
<b>RQ<sub>NO<sub>2</sub></sub></b>	respiratory quotient for reduction of $O_2$ coupled to oxidation of $NO_2^-$	$g O_2 g N^{-1}$	[H16]	1.14	Brock and Madigan (1991)
$r_m$	radius of microbial sphere	m	[H4,H13,H17]		
$r_w$	radius of $r_m$ + water film at current soil water potential	m	[H4,H13,H17]		from $\psi_s$ according to Kemper (1966)
$X'_{Ci,n}$	rate of C oxidation by $NH_3$ oxidizers under non-limiting [ $O_{2S}$ ]	$g C m^{-2} h^{-1}$	[H12]		
$X'_{Ci,o}$	rate of C oxidation by $NO_2^-$ oxidizers under non-limiting [ $O_{2S}$ ]	$g C m^{-2} h^{-1}$	[H16]		
$X'_{DOC}$	specific rate of DOC oxidation by heterotrophs at 25 °C under non-limiting [DOC] and [ $O_{2S}$ ]	$g C g C^{-1} h^{-1}$	[H2]	0.125	Shields et al. (1973)
$X'_{DOCi,h}$	rate of DOC oxidation by heterotrophs under non-limiting [ $O_{2S}$ ]	$g N m^{-2} h^{-1}$	[H2,H3,H5]		
$X_{DOCi,h}$	rate of DOC oxidation by heterotrophs under ambient [ $O_{2S}$ ]	$g N m^{-2} h^{-1}$	[H5]		



$X_{\text{DOC}i,d}$	rate of DOC oxidation by heterotrophs under ambient $[\text{O}_{2s}]$ and $[\text{NO}_x]$	$\text{g N m}^{-2} \text{ h}^{-1}$	[H10]		
$X'_{\text{NH}_3}$	specific rate of $\text{NH}_3$ oxidation by $\text{NH}_3$ oxidizers at 25 °C under non-limiting $[\text{O}_{2s}]$	$\text{g N g C}^{-1} \text{ h}^{-1}$	[H11]]	0.625	Belser and Schmidt (1980)
$X_{\text{NH}_3i,n}$	rate of $\text{NH}_3$ oxidation by $\text{NH}_3$ oxidizers coupled with reduction of $\text{O}_2 + \text{NO}_2^-$ under ambient $[\text{O}_{2s}]$	$\text{g N m}^{-2} \text{ h}^{-1}$	[H14,H21]		
$X'_{\text{NH}_3i,n}$	rate of $\text{NH}_3$ oxidation by $\text{NH}_3$ oxidizers under non-limiting $[\text{O}_{2s}]$	$\text{g N m}^{-2} \text{ h}^{-1}$	[H11,H12,H14]		
$X'_{\text{NO}_2i,o}$	rate of $\text{NO}_2^-$ oxidation by $\text{NO}_2^-$ oxidizers under non-limiting $[\text{O}_{2s}]$	$\text{g N m}^{-2} \text{ h}^{-1}$	[H15,H16,H18]		
$X_{\text{NO}_2i,o}$	rate of $\text{NO}_2^-$ oxidation by $\text{NO}_2^-$ oxidizers coupled with reduction of $\text{O}_2$ under ambient $[\text{O}_{2s}]$	$\text{g N m}^{-2} \text{ h}^{-1}$	[H18]		
$X'_{\text{NO}_2}$	specific rate of $\text{NO}_2^-$ oxidation by $\text{NO}_2^-$ oxidizers at 25 °C under non-limiting $[\text{O}_{2s}]$	$\text{g N g C}^{-1} \text{ h}^{-1}$	[H15]	2.5	Belser (1977)

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