

1           **Steady-state solutions for subsurface chlorophyll maximum in**  
2           **stratified water columns with a bell-shape vertical profile of**  
3           **chlorophyll**

4                           X. Gong, J. Shi, H. W. Gao, X. H. Yao

5           Key Laboratory of Marine Environment and Ecology (Ministry of Education of  
6           China), Ocean University of China, Qingdao 266100, China

7           Correspondence to: H. W. Gao (hwgao@ouc.edu.cn)

8           **Abstract:**

9           A bell-shape vertical profile of chlorophyll a (Chl a) concentration, conventionally  
10          referred to as Subsurface Chlorophyll Maximum (SCM) phenomenon, has frequently  
11          been observed in stratified oceans and lakes. This profile is assumed to be a general  
12          Gaussian distribution in this study. By substituting the general Gaussian function into  
13          ecosystem dynamical equations, the steady-state solutions for SCM characteristics  
14          (i.e., SCM layer depth, thickness, and intensity) in various scenarios are derived.  
15          These solutions indicate that: 1) The maximum concentration of Chl a occurs at or  
16          below the depth of maximum growth rates of phytoplankton located at the transition  
17          from nutrient limitation to light limitation, and the depth of SCM layer deepens  
18          logarithmically with an increase in surface light intensity; 2) The shape of SCM layer  
19          (thickness and intensity) is mainly affected by nutrient supply, but independent of  
20          surface light intensity; 3) The intensity of SCM layer is proportional to the diffusive  
21          flux of nutrients from below, getting stronger as a result of this layer being shrunk by  
22          a higher light attenuation coefficient or a larger sinking velocity of phytoplankton. **In**  
23          **addition, the limitation and potential application of the analytical solutions were also**  
24          **presented.**

## 25 **1 Introduction**

26 Vertical profiles of chlorophyll a (Chl a) concentration in lakes, coastal seas and open  
27 oceans are highly variable. However, a bell-shape vertical profile of Chl a,  
28 conventionally referred to as Subsurface Chlorophyll Maximum (SCM) phenomenon,  
29 has been frequently observed in stratified water columns, e.g., it occurred through the  
30 whole year in tropical and subtropical oceans while it existed only during summer in  
31 temperate and high latitude oceanic zones. The subsurface biomass maxima (SBMs)  
32 are also common in stratified water columns. The chlorophyll-to-biomass ratio  
33 generally increases with depth in the euphotic zone. Thus, SCMs may not necessarily  
34 represent SBMs (Cullen, 1982; Fennel and Boss, 2003) and are usually deeper than  
35 SBMs (Fennel and Boss, 2003; Hodges and Rudnick, 2004). However, both the  
36 subsurface maxima in chlorophyll and biomass are usually formed in certain regions  
37 of the water column where two opposing resource (light and nutrient) gradients  
38 combined with vertically heterogeneous turbulent mixing is amenable for survival of  
39 phytoplankton. Thus, SCMs are approximately equal to SBMs in many studies  
40 (Klausmeier and Litchman, 2001; Sharples et al., 2001; Huisman et al., 2006; Raybov  
41 et al., 2010). Fennel and Boss (2003) reported that the photoacclimation of  
42 phytoplankton can be another important reason for forming a SCM in oligotrophic  
43 waters.

44 The SCM phenomenon can be characterized by the thickness, depth, and intensity of  
45 SCM layer (SCML) (Beckmann and Hense, 2007). On-site observations (Platt et al.,  
46 1988; Sharples et al., 2001; Deksheniaks et al., 2001; Mellard et al., 2011) showed  
47 that the SCML occurred relatively shallow (1-50 m) and was thin (several centimeters  
48 to a few meters) in lakes and coastal seas, but the concentration of Chl a was high  
49 (1-100 mg/m<sup>3</sup>). In open oceans, the SCML was deeper (80-130 m) and thicker (tens  
50 of meters) while the concentration of Chl a was relatively low (<1 mg/m<sup>3</sup>) (Anderson,  
51 1969; Platt et al., 1988).

52 SCM has attracted much attention because of the significant contribution of SCML to  
53 the total biomass and primary production in the whole water column (Cullen and  
54 Eppley, 1981; Weston et al., 2005; Siswanto et al., 2005; Hanson et al., 2007;  
55 Sullivan et al., 2010). Pérez et al. (2006) showed that 65-75% of the total Chl a in a  
56 water column of the Atlantic subtropical gyres was presented in SCML and the layer

57 thickness was approximately 50 m. Weston et al. (2005) reported that the SCML  
58 accounted for 58% of the water column primary production in the central North Sea,  
59 although the layer thickness was less than 5 m. Sullivan et al. (2010) found that the  
60 fraction of Chl a in the SCML (thickness <3 m) out of the total water column ranged  
61 from 33% to 47% in the Monterey Bay.

62 Many numerical studies have been conducted to link the thickness, depth and  
63 intensity of the SCML to various environmental parameters (Jamart et al., 1979;  
64 Varela et al., 1994; Klausmeier and Litchman, 2001; Hodges and Rudnick, 2004;  
65 Huisman et al., 2006; Beckmann and Hense, 2007). The thickness of the SCML  
66 mainly depends on the degree of vertical mixing in lakes (Klausmeier and Litchman,  
67 2001). In oligotrophic oceans, light attenuation coefficient is the key factor in  
68 determining the SCML depth (Varela et al., 1994; Hodges and Rudnick, 2004;  
69 Beckmann and Hense, 2007) and the intensity of the SCML depends strongly on  
70 sinking velocity of phytoplankton and vertical diffusivity rather than growth rate of  
71 phytoplankton (Hodges and Rudnick, 2004; Beckmann and Hense, 2007). However,  
72 the thickness, depth and intensity of SCML are very sensitive to variations of  
73 environmental parameters. Therefore, the relationships obtained from a particular  
74 case may not be applicable for other cases. To understand the general relationships  
75 between SCM phenomenon and environmental parameters, the analytical solution for  
76 dynamic ecosystem equations is needed.

77 The algae game theoretical model, pioneered by Klausmeier and Litchmann (2001),  
78 was perhaps the first one to derive the depth and intensity of SCML, although the  
79 SCML is assumed to be infinitely thin. They adopted a delta function to approximate  
80 the phytoplankton distribution in this thin layer. Yoshiyama et al. (2009) used this  
81 model to examine more than one species competing for limiting nutrients and light  
82 below the surface mixed layer. Mellard et al. (2011) included stratification into this  
83 model. However, the SCML was still confined to an infinitely thin layer. In fact,  
84 many observations showed that the thickness of SCML can reach as high as 100 m in  
85 oceans (Platt et al., 1988). For those cases, the assumption of an infinite thickness of  
86 SCML is contradictory to the observations.

87 In this study, we assume that the vertical profile of Chl a can be approximately treated  
88 as a general Gaussian function, instead of a delta function. This parameterizing  
89 approach was proposed firstly by Lewis et al. (1983), and has been widely used to fit

90 vertical profiles of Chl a (Platt et al., 1988; Weston et al., 2005; Ardyna et al., 2013).  
 91 By incorporating the general Gaussian function into the ecosystem dynamical  
 92 equations, we derive the steady-state solutions for the thickness, depth, and intensity  
 93 of SCML in various scenarios and examine their dependence on environmental  
 94 parameters, such as light attenuation coefficient, vertical diffusivity, sinking velocity  
 95 of phytoplankton.

## 96 **2 Methods**

### 97 *2.1 Models*

98 The SCML occurs below the surface mixed layer, where the light attenuated from  
 99 above and nutrients supplied from the deep water match best for phytoplankton  
 100 growth (Fig. 1). The partial differential equations for phytoplankton and nutrients  
 101 dynamics in which light and nutrients are two major limited factors (Eqs. 1, 2) (Riley  
 102 et al., 1949; Lewis et al., 1986; Gabric and Parslow, 1989; Huisman et al., 2006;  
 103 Liccardo et al., 2013) were adopted in this study. Moreover, the photoacclimation of  
 104 phytoplankton was not considered here and the Chl a distribution is supposed to  
 105 represent the distribution of phytoplankton biomass. This is a significant  
 106 simplification. In fact, phytoplankton increases inter-cellular pigment concentration  
 107 when light level decreases (Fennel and Boss, 2003).

$$108 \quad \frac{\partial P}{\partial t} = \mu_m \min(f(I), g(N))P - \varepsilon P - w \frac{\partial P}{\partial z} + \frac{\partial}{\partial z} \left( K_v \frac{\partial P}{\partial z} \right), \quad (1)$$

$$109 \quad \frac{\partial N}{\partial t} = -\mu_m \min(f(I), g(N))P + \alpha \varepsilon P + \frac{\partial}{\partial z} \left( K_v \frac{\partial N}{\partial z} \right), \quad (2)$$

110 where  $P$  denotes the Chl a concentration,  $N$  is the limiting nutrient concentration.  
 111 Usually, the unit of Chl a concentration is  $\text{mg m}^{-3}$ , the concentrations of  
 112 phytoplankton and the limiting nutrients are in unit of  $\text{mmol N m}^{-3}$ . A ratio of 1.59 g  
 113 chlorophyll per mol nitrogen (Cloern et al., 1995; Oschlies, 2001) is thereby used for  
 114 unit conversion.  $\mu_m$  is the maximum growth rate of phytoplankton,  $\varepsilon$  is the loss  
 115 rate of phytoplankton (including respiration, mortality, zooplankton grazing),  $\alpha$  is  
 116 the recycling rate of dead phytoplankton ( $0 \leq \alpha \leq 1$ ).  $w$  is the sinking velocity of  
 117 phytoplankton, which is non-negative in the chosen coordinate system and assumed  
 118 to be constant with depths.  $K_v$  is the vertical turbulent diffusivity and it is much

119 larger within the surface mixed layer than that beneath. Here,  $K_v$  depends on depth  
 120 in the following way (Hodges and Rudnick, 2004; Mellard et al., 2011):

$$121 \quad K_v = \begin{cases} K_{v1} & 0 < z < z_s, \\ K_{v2} & z_s < z < z_b, \end{cases} \quad (3)$$

122 where  $z_s$  is the depth of surface mixed layer,  $z_b$  is the bottom of water column or the  
 123 location where the Chl a concentration reduces to nearly zero below the euphotic  
 124 zone. We assume  $K_{v1}$ ,  $K_{v2}$  are constant and  $K_{v1}$  is large enough to homogenize the Chl  
 125 a and nutrient concentrations in the surface mixed layer.

126 A gradual transition from the surface mixed layer to the deep one written in terms of a  
 127 generalized Fermi function is adopted (Ryabov et al., 2010), that is,  $K_v(z) = K_{v2} +$   
 128  $\frac{K_{v1} - K_{v2}}{1 + e^{(z - z_s)/l}}$ , where parameter  $l$  characterizes the width of the transient layer. In our  
 129 study, we assumed this transient layer is finitely thin.

130 The growth limited function  $\min(f(I), g(N))$  for light  $I$  and nutrients  $N$  is:

$$131 \quad \min(f(I), g(N)) = \min\left(\frac{I(z)}{K_I + I(z)}, \frac{N(z)}{K_N + N(z)}\right), \quad (4)$$

132 where  $K_I$  and  $K_N$  denote the half-saturation constants of light and nutrients,  
 133 respectively. The net growth rate,  $\mu_m \min(f(I), g(N)) - \varepsilon$ , is positive only if both the  
 134 light limiting term  $\mu_m f(I)$  and nutrient limiting term  $\mu_m g(N)$  are larger than the  
 135 loss rate  $\varepsilon$ .

136 Light intensity is assumed to decrease exponentially with depth according to  
 137 Lambert-Beer's law, i.e.,

$$138 \quad I(z) = I_0 \exp(-K_d z), \quad (5)$$

139 where  $I_0$  is the surface light intensity and  $K_d$  is the light attenuation coefficient (Morel,  
 140 1988). Assuming a constant  $K_d$ , we ignore the effects of the self-shading and the  
 141 dissolved and particulate material on the attenuation coefficient.

142 The zero-flux boundary conditions for the phytoplankton at the surface and bottom of  
 143 the water column are used. Furthermore, we assume a zero-flux boundary condition  
 144 for nutrients at the surface, while nutrients are replenished from below. That is,

$$\begin{cases} K_{v1} \frac{\partial P}{\partial z} - wP = 0, & K_{v1} \frac{\partial N}{\partial z} = 0, & \text{at } z = 0, \\ K_{v2} \frac{\partial P}{\partial z} - wP = 0, & K_{v2} \frac{\partial N}{\partial z} = K_{v2} \frac{\partial N}{\partial z} \Big|_{z=z_b}, & \text{at } z = z_b. \end{cases} \quad (6)$$

146 In addition, Lewis et al. (1983) first proposed a general Gaussian distribution function  
147 (Eq. 7) to model the nonlinear feature of observed vertical Chl a profiles. In this study,  
148 this function is adopted to represent the bell-shape vertical distribution of Chl a (Fig.  
149 1).

$$P(z) = P_{\max} e^{-\frac{(z-z_m)^2}{2\sigma^2}} \quad 0 \leq z \leq z_b, \quad (7)$$

151 where  $P(z)$  is Chl a concentration as a function of depth  $z$ , and  $P_{\max} = \frac{h}{\sigma\sqrt{2\pi}}$ . The  
152 three Gaussian parameters ( $h$ ,  $z_m$ ,  $\sigma$ ) can vary to characterize the SCM phenomenon.  
153 Thus  $h$  is the vertical integrated Chl a over the entire water column,  $z_m$  is the depth of  
154 the maximum Chl a (the peak of the bell-shape), and  $\sigma$  is the standard deviation of  
155 Gaussian function, which controls the width of the SCML.

## 156 2.2 Three SCM characteristics

157 The thickness of SCML can characterize the vertical extent of Chl a distribution  
158 below the surface mixed layer. It is still debatable how to best define the thickness of  
159 SCML. One easy definition is to use the width between two locations below and  
160 above the Chl a peak, where Chl a is a certain fraction (e.g. 50%,  $100(e^{-1/2})\%$ ) of the  
161 maximum Chl a (Platt et al., 1988; Pérez et al., 2006). Some studies bounded the  
162 layer by sharp vertical gradients in Chl a above and below the peak (Prairie et al.,  
163 2011). Others defined the upper and lower boundary of SCML by ad hoc choices.  
164 Pedrós-Alió et al. (1999) proposed the SCML from the depth of the surface mixed  
165 layer to the lower maximum gradient in the slope of the Chl a profile. Hanson et al.  
166 (2007) defined that the upper boundary of the SCML was the minimum gradient  
167 criterion of  $0.02 \text{ mg Chl a m}^{-1}$  and the lower was the base of the euphotic zone.  
168 Beckmann and Hense (2007) proposed to define the boundaries of SCML by the  
169 existence of two community compensation depths in the water column, which were  
170 located at the depths of two maximum phytoplankton gradients in phytoplankton  
171 biomass.

172 Building on the study by Beckmann and Hense (2007), the locations of the maximum

173 phytoplankton gradients are defined as the boundaries of SCML in this study. That is,

174 
$$\left. \frac{d^2 P}{dz^2} \right|_{z=z_u, z_l} = 0, \quad (8)$$

175 where  $z_u$  and  $z_l$  are the upper and lower boundary of SCML, respectively.

176 By substituting Eq. (7) into this equality, we obtain  $z_u = z_m - \sigma$ ,  $z_l = z_m + \sigma$ . Thus,  
177 the thickness of SCML can thereby be expressed as  $2\sigma$ .

178 From Eq. (8) and the steady state of Eq. (1), one gets the following equality at the  
179 boundaries of SCML:

180 
$$\left( \mu_m \min(f(I), g(N))P - \varepsilon P - w \frac{dP}{dz} \right) \Big|_{z=z_u, z_l} = 0. \quad (9)$$

181 That is, the boundary of SCML is located at the depth where there is the balance  
182 between phytoplankton growth and all losses (including the divergence of the sinking  
183 flux  $w \frac{dP}{dz}$  and the loss  $\varepsilon$  due to mortality, respiration, and grazing), named the  
184 community compensation depth (Ono et al., 2001). Thus, this definition reflects the  
185 physical-biological ecosystem dynamics associated with SCML.

186 As described in Eq. (7), the depth of the SCML is defined as  $z_m$ , that is, the location  
187 of the point-wise maximum value of Chl a.

188 The third quantity, i.e. the intensity of SCML, refers to the maximum value of Chl a  
189 ( $P_{\max}$  in Eq. 7) in the water column.

### 190 2.3 Approach used in this study

191 Previous numerical studies (Huisman et al., 2006; Ryabov et al., 2010) showed that  
192 the ecosystem dynamical model (Eqs. 1 and 2) can approximately reproduce the  
193 bell-shape feature of the vertical Chl a profile (Fig. 1). We substitute the general  
194 Gaussian function of the vertical Chl a profile (Eq. 7) into Eqs. (1) and (2) to derive  
195 explicit relationships between three characteristics of SCM and the environmental  
196 parameters.

197 Firstly, by substituting the general Gaussian function of  $P(z)$  with the steady-state  
198 version of Eq. (1), we obtain that below the surface mixed layer the net growth rate of  
199 phytoplankton can be expressed as follows

200 
$$\mu_m \min(f(I), g(N)) - \varepsilon = -\frac{K_{v2}}{\sigma^4} \left( z - z_m + \frac{w\sigma^2}{2K_{v2}} \right)^2 + \frac{w^2}{4K_{v2}} + \frac{K_{v2}}{\sigma^2}. \quad (10)$$

201 Letting  $\mu_m \min(f(I), g(N)) - \varepsilon = 0$ , we get the two compensation depths,  $z_{c1}$ ,  $z_{c2}$ , by  
 202 solving Eq. (10):

203 
$$z_{c1} = z_m - \frac{w\sigma^2}{2K_{v2}} - \sqrt{\left( \frac{w\sigma^2}{2K_{v2}} \right)^2 + \sigma^2}, \quad z_{c2} = z_m - \frac{w\sigma^2}{2K_{v2}} + \sqrt{\left( \frac{w\sigma^2}{2K_{v2}} \right)^2 + \sigma^2}. \quad (11)$$

204 Clearly, the inequality  $\mu_m \min(f(I), g(N)) - \varepsilon > 0$  is satisfied in the interval  $(z_{c1}, z_{c2})$ .  
 205 This indicates that the subsurface net production occurs only between the two  
 206 compensation depths where the growth rate  $\mu_m \min(f(I), g(N))$  equals the loss rate  
 207  $\varepsilon$ . Beckmann and Hense (2007) found similar results by numerical modeling and  
 208 emphasized the often overlooked fact that an SCML has to have two compensation  
 209 depths.

210 From Eq. (11), we obtain  $z_{c1} \leq z_m - \sigma$  and  $z_m \leq z_{c2} \leq z_m + \sigma$  (Fig. 1). Especially,  
 211  $z_{c1} = z_m - \sigma$ , and  $z_{c2} = z_m + \sigma$  when the sinking velocity of phytoplankton  $w$  is too  
 212 small to be considered. This result is identical to that of Beckmann and Hense (2007)  
 213 for neglecting sinking velocity of phytoplankton.

214 Hence, according to the property of quadratic function, there exists a depth  $z_0$  within  
 215 the two compensation depths,

216 
$$z_0 = z_m - \frac{w\sigma^2}{2K_{v2}}, \quad (12)$$

217 such that the net growth rate of phytoplankton is at its maximum, i.e.,

218 
$$\max(\mu_m \min(f(I), g(N)) - \varepsilon) \Big|_{z_0} = \frac{K_{v2}}{\sigma^2} + \frac{w^2}{4K_{v2}}. \quad (13)$$

219 In other words, the maximum in net growth rates of phytoplankton occurs at the  
 220 depth of  $z_0$ .

221 We define  $T = \sigma^2 / K_{v2}$  as the characteristic vertical mixing time scale in the SCML of  
 222 thickness  $\sigma$  (Bowdon, 1985; Gabric and Parslow, 1989). Let the length scale be  
 223  $L = 2K_{v2} / w$ , which determines the scale height of the phytoplankton distribution  
 224 (Ghosal and Mandre, 2003). Thus, the right hand terms of Eq. (13) can be rewritten



225 as  $1/T+w/(2L)$ . In other words, the maximum net growth rate of phytoplankton,  
 226  $\max(\mu_m \min(f(I), g(N)) - \varepsilon)$ , is determined by the vertical mixing time scale ( $T$ ) and  
 227 the time taken by a phytoplankton sinking ( $w$ ) through lengths ( $2L$ ).

228 Equation (12) also shows that  $z_m \geq z_0$ , that is, the depth of SCML lies at or below  
 229 the depth for phytoplankton having the maximum growth rate. Observations in the  
 230 Southern California Bight have supported this (Cullen and Eppley, 1981).  
 231 Particularly,  $z_m = z_0$  approximately holds when either the sinking velocity ( $w$ ) or  
 232 Gaussian parameter  $\sigma$  is very small. For non-sinking phytoplankton, i.e.,  $w \rightarrow 0$ ,  
 233 numerical modeling can support this equality (Beckmann and Hense, 2007). When  
 234 parameter  $\sigma$  is assumed to be infinitely thin, the equality is obviously correct, which  
 235 has been used to solve for the equilibrium depth and intensity of an infinitely thin  
 236 layer (Klausmeier and Litchman, 2001; Yoshiyama et al., 2009; Mellard et al., 2011).

237 In this special case ( $z_m = z_0$ ), some studies found that the depth of SCML is at the  
 238 location of equal limitation by nutrients and light (Klausmeier and Litchman, 2001;  
 239 Yoshiyama et al., 2009; Mellard et al., 2011). In this study, we further infer that when  
 240  $z_m > z_0$ , the depth of SCML is located at where phytoplankton growth is limited by  
 241 light (Appendix A).

242 According to Eqs. (12) and (A2), the growth of phytoplankton is light-limited at and  
 243 below the depth of SCML. Therefore, for  $z = z_m$  and  $z = z_m + \sigma$ , the net growth rate  
 244 of phytoplankton (Eq. 10) can be expressed as following, respectively:

$$245 \quad \mu_m f(I)|_{z=z_m} - \varepsilon = K_{v2} / \sigma^2 \quad (14)$$

$$246 \quad \mu_m f(I)|_{z=z_m+\sigma} - \varepsilon = -w / \sigma \quad (15)$$

247 At the depth of  $z_m$ , the net growth rate of phytoplankton (Eq. 14) is determined by  
 248 the vertical mixing time,  $T$ , while the time taken by phytoplankton sinking through  
 249 half-length of SCML,  $w/\sigma$ , controls the net growth rate of phytoplankton (Eq. 15) at  
 250 the lower boundary of SCML ( $z_m + \sigma$ ).

251 In addition, from Eqs. (12) and (A2) we obtain that the upper compensation depth,  $z_{c1}$ ,  
 252 is the location where the growth limited by nutrients,  $\mu_m g(N)$ , equals the loss rate,

253  $\varepsilon$ , while the lower compensation depth,  $z_{c2}$ , represents the depth where the growth  
 254 limited by light,  $\mu_m f(I)$ , equals the loss rate,  $\varepsilon$ .

### 255 3 Results

#### 256 3.1 Analytic solutions of three SCM characteristics

257 By substituting the growth limitation function for light (Eqs. 4 and 5) into Eqs. (14)  
 258 or (15), we obtain the expression of parameter  $z_m$ , i.e.,

$$259 \quad z_m = \frac{1}{K_d} \ln \left[ \left( \frac{\mu_m}{\varepsilon + K_{v2}/\sigma^2} - 1 \right) \frac{I_0}{K_I} \right] \quad (16)$$

260 or

$$261 \quad z_m = \frac{1}{K_d} \ln \left[ \left( \frac{\mu_m}{\varepsilon - w/\sigma} - 1 \right) \frac{I_0}{K_I} \right] - \sigma. \quad (17)$$

262 The occurrence for a SCM requires  $z_m > 0$ . Requiring a positive solution for Eq.

263 (16), we obtain  $\left( \frac{\mu_m}{\varepsilon + K_{v2}/\sigma^2} - 1 \right) \frac{I_0}{K_I} > 1$ , i.e.,  $(\mu_m f(I_0) - \varepsilon) \sigma^2 > K_{v2}$ . For any  $\sigma > 0$ , we

264 get  $\mu_m f(I_0) > \varepsilon$ . That is, the necessary condition for the existence of SCM is

265  $\mu_m f(I_0) > \varepsilon$ , which is identical with the result of Fennel and Boss (2003) when  
 266 vertical sinking is constant as a function of depth in their model.

267 Subtracting Eqs. (16) and (17), and rearranging, we obtain the expression of  
 268 parameter  $\sigma$ :

$$269 \quad \left( \frac{\mu_m}{\mu_m - \varepsilon + \frac{w}{\sigma}} - 1 \right) e^{K_d \sigma} = \frac{\mu_m}{\mu_m - \varepsilon - \frac{K_{v2}}{\sigma^2}} - 1 \quad (18)$$

270 Thus far, we have obtained the theoretical relationships between Gaussian parameter

271  $\sigma$ ,  $z_m$  and environmental parameters (Eqs. 16-18). To derive the relationship between

272 Gaussian parameter  $h$  and environmental parameters, we now return to Eqs. (1) and

273 (2). In steady state, adding these two equations leads to:

$$274 \quad (1 - \alpha) \varepsilon P + w \frac{dP}{dz} = \frac{d^2}{dz^2} (K_v (P + N)) \quad (19)$$

275 Note that this relationship holds irrespective of the form of growth limiting function.

276 Integrating this equation from the surface to bottom boundary ( $z_b$ ) and using  
 277 boundary conditions (Eq. 6) gives:

$$278 \quad (1-\alpha)\varepsilon \int_0^{z_b} P(z)dz = K_{v2} \left. \frac{dN}{dz} \right|_{z=z_b} \quad (20)$$

279 When the recycling processes do not immediately convert dead phytoplankton back  
 280 into dissolved nutrients below the surface mixed layer, i.e.,  $\alpha \neq 1$  (For  $\alpha = 1$ , the  
 281 detailed derivation for the intensity of SCML is presented at Appendix B), one gets  
 282 the total Chl a in the water column:

$$283 \quad h = \frac{K_{v2} \left. \frac{dN}{dz} \right|_{z=z_b}}{(1-\alpha)\varepsilon} \quad (21)$$

284 The intensity of SCML is

$$285 \quad P_{\max} = \frac{K_{v2} \left. \frac{dN}{dz} \right|_{z=z_b}}{\sqrt{2\pi}\sigma(1-\alpha)\varepsilon} \quad (22)$$

286 Obviously, both the total Chl a in the water column and the intensity of SCML are  
 287 proportional to the flux of nutrients from below ( $K_{v2} \left. \frac{dN}{dz} \right|_{z=z_b}$ ), which is determined  
 288 by the diffusivity below the surface mixed layer and the nutrients gradient at the  
 289 bottom of water column. Varela et al. (1994) also found a similar result by  
 290 simulations.

### 291 *3.2 Influences of environmental parameters on SCM characteristics*

292 We now investigate how the steady-state thickness, depth, and intensity of SCML  
 293 depend on environmental parameters. Because the analytic solutions for SCML depth  
 294 and intensity depend on Gaussian parameter  $\sigma$  and environmental parameters, we first  
 295 examine the influence of environmental parameters on parameter  $\sigma$ .

296 Equation (18) shows that the thickness of SCML is independent of sea surface light  
 297 intensity ( $I_0$ ). This is consistent with numerical simulations (Beckmann and Hense,  
 298 2007). This result also suggests that seasonal variation of SCML thickness has no  
 299 relation with light intensity. Thus, it is not surprising that the empirical model poorly  
 300 predicted parameter  $\sigma$  by using season as an important factor (Richardson et al.,  
 301 2003).

302 To illustrate the effects of other model parameters ( $K_d$ ,  $K_{v2}$ ,  $\mu_m$ ,  $\varepsilon$ ,  $w$ ) on the parameter  
 303  $\sigma$ , we need to obtain informative algebraic expression of  $\sigma$ . To simplify, by Taylor  
 304 expanding  $e^{K_d\sigma}$  at  $\sigma=0$  and truncating the Taylor series after the linear term, i.e.,  
 305  $e^{K_d\sigma} = 1 + K_d\sigma + o(\sigma^2)$ , Eq. (18) can thereby be rewritten as:

$$306 \quad \sigma^3 - \frac{w}{\varepsilon}\sigma^2 - \frac{\varepsilon K_d K_{v2} + \mu_m w}{\varepsilon K_d (\mu_m - \varepsilon)}\sigma = \frac{K_{v2}(\mu_m/K_d - w)}{\varepsilon(\mu_m - \varepsilon)}. \quad (23)$$

307 According to the properties of a cubic function, we know that Eq. (23) has one and  
 308 only one positive real root  $\sigma$ , when  $\frac{K_{v2}(\mu_m/K_d - w)}{\varepsilon(\mu_m - \varepsilon)} \geq 0$ . Because  $\mu_m f(I_0) > \varepsilon$  and  
 309  $0 < f(I_0) < 1$ , so  $\mu_m > \varepsilon$ . Thus, when the maximum phytoplankton growth rate ( $\mu_m$ )  
 310 within one penetration depth ( $1/K_d$ ) is larger than sinking velocity of phytoplankton,  
 311 i.e.,  $\mu_m/K_d - w \geq 0$ , there exists a non-negative value of parameter  $\sigma$ , which  
 312 increases with increasing  $\frac{K_{v2}(\mu_m/K_d - w)}{\varepsilon(\mu_m - \varepsilon)}$ .

313 Using dimensional analysis, Klausmeier and Litchman (2001) found that the degree  
 314 of turbulence determines the thickness of SCML. Our analytical result shows that the  
 315 thickness of SCML increases with increasing vertical diffusivity below the surface  
 316 mixed layer ( $K_{v2}$ ). In addition, the SCML thickness decreases with increasing sinking  
 317 velocity of phytoplankton ( $w$ ) and light attenuation coefficient ( $K_d$ ).

318 The right hand term in Eq. (23),  $\frac{K_{v2}(\mu_m/K_d - w)}{\varepsilon(\mu_m - \varepsilon)}$ , can be rearranged as

$$319 \quad \frac{K_{v2}(\mu_m/K_d - w)}{-(\varepsilon - \mu_m/2)^2 + \mu_m^2/4}. \text{ Thus, the effect of loss rate } (\varepsilon) \text{ on parameter } \sigma \text{ depends on } \mu_m/2.$$

320 Note that  $\mu_m f(I_0) > \varepsilon$  once the SCM occurs. When the surface light intensity  $I_0$  is  
 321 smaller than or equals to the half-saturation constant for light  $K_I$ , i.e.,  $f(I_0) \leq 0.5$ ,  
 322 then  $0 < \varepsilon < \mu_m f(I_0) \leq \mu_m/2$ , thus,  $\sigma$  decreases with increasing  $\varepsilon$ . Conversely, when  
 323  $f(I_0) > 0.5$ , for  $\varepsilon \geq \mu_m/2$ ,  $\sigma$  increases with increasing  $\varepsilon$ ; for  $\varepsilon < \mu_m/2$ ,  $\sigma$  decreases  
 324 with increasing  $\varepsilon$ . In summary, for smaller loss rates ( $\varepsilon < \mu_m/2$ ), decreased  $\varepsilon$  leads to  
 325 a thicker SCML, while for larger loss rates ( $\varepsilon \geq \mu_m/2$ ), decreased  $\varepsilon$  leads to a thinner  
 326 SCML.

327 Equation (16) can be rewritten as:

328

$$z_m = \frac{1}{K_d} \ln(AI_0), \quad (24)$$

329 where  $A = \frac{1}{K_I} \left( \frac{\mu_m}{\varepsilon + K_{v2}/\sigma^2} - 1 \right)$ . Clearly, from Eq. (18) we know  $A$  does not depend on

330 surface light intensity ( $I_0$ ), thus we infer that the depth of SCML increases  
 331 logarithmically with increasing  $I_0$ . In other words, the SCML gets deeper due to the  
 332 seasonal increase of  $I_0$ , and remains almost unchanged when the surface light  
 333 intensity increases to a certain degree. Observations at the HOT (Hawaii Ocean  
 334 Time-series) site in the eastern Pacific and the SEATS (South East Asia Time-series  
 335 Station) station in the South China Sea showed a significant seasonal variation of  
 336 SCML depth (Chen et al., 2006; Hense and Beckmann, 2008). Hense and Beckmann  
 337 (2008) explained the deepening of SCML depth in spring at HOT site by the seasonal  
 338 increase of the light intensity. Modeling sensitivity analyses also showed that an  
 339 increase in the surface light intensity yields a deeper SCML (Jamart et al., 1979;  
 340 Varela et al., 1994; Beckmann and Hense, 2007).

341 Determining the effect of vertical diffusivity below the surface mixed layer ( $K_{v2}$ ) on  
 342 the steady-state SCML intensity is more difficult. Increased  $K_{v2}$  increases parameter  
 343  $\sigma$  (Eq. 23) and the diffusive flux of nutrients from below (Eq. 22), however, this  
 344 parameter has opposite effects on  $P_{\max}$  (Eq. 22). Rearranged Eq. (23) we obtain

$$345 \quad \frac{K_{v2}}{\sigma} = \frac{(\mu_m - \varepsilon)\varepsilon}{(\mu_m/K_d - w)/\sigma^2 + \varepsilon/\sigma} + \frac{(\mu_m - \varepsilon)w}{(\mu_m/K_d - w)/\sigma + \varepsilon} - \frac{\mu_m w/K_d}{\mu_m/K_d - w + \varepsilon\sigma}. \quad (25)$$

346 Clearly, all the three terms in the right hand of this equality increase due to the  
 347 increasing  $\sigma$  by a higher  $K_{v2}$ . Therefore, it can be inferred that increased vertical  
 348 diffusivity below the surface mixed layer ( $K_{v2}$ ) leads to a stronger SCML intensity  
 349 ( $P_{\max}$ ).

350 The influences of various parameters on SCM characteristics determined by Eqs.  
 351 (16)-(18), (21) and (22) are summarized in Table 1. For example, increased light  
 352 levels (increasing surface light intensity  $I_0$ , decreasing attenuation coefficient  $K_d$ ) or  
 353 increased light competitive ability (decreasing half-saturation constant for light  $K_I$ )  
 354 moves the SCML deeper; increased nutrients supply (increasing vertical diffusivity  
 355 below the surface mixed layer  $K_{v2}$  and loss rate of phytoplankton  $\varepsilon$ ) moves the layer  
 356 toward the surface. The shape of SCML (thickness and intensity) is mainly

357 influenced by nutrients supply ( $K_{v2}$  and  $\varepsilon$ ). The intensity of SCML becomes weaker  
358 as a result of expanding the SCML by a lower sinking velocity of phytoplankton ( $w$ )  
359 and a smaller light attenuation coefficient ( $K_d$ ).

#### 360 **4 Discussion**

361 Considering the two compartment system (nutrients and Chl a) in steady state and a  
362 general Gaussian function for vertical Chl a concentration, we derived the analytical  
363 solution for the fundamental relationships between SCM characteristics and various  
364 parameters. Three special sceneries, limitation and implications of this study were  
365 discussed below.

##### 366 *4.1 Three special sceneries*

367 Equation (18) indicates that the parameter  $\sigma$  is affected by changes in the vertical  
368 diffusivity below the surface mixed layer ( $K_{v2}$ ), the sinking velocity of phytoplankton  
369 ( $w$ ) and the light attenuation coefficient ( $K_d$ ), which inversely affects depth and  
370 intensity of SCML (Eqs. 16, 17, and 22). Thus, three special situations of the  
371 theoretical solutions for SCM characteristics are discussed below.

372 Firstly, the term  $K_{v2}/\sigma^2$  in the right hand of Eq. (18) is neglected. This special  
373 situation occurs either when the vertical diffusivity below the surface mixed layer is  
374 too small to be considered ( $K_{v2} \rightarrow 0$ ), or when  $K_{v2}/\sigma^2$  is much smaller than  $\mu_m - \varepsilon$ ,  
375 i.e., the mixing time scale ( $T = \sigma^2/K_{v2}$ ) below the surface mixed layer is much longer  
376 than the time taken by net growth of phytoplankton,  $(\mu_m - \varepsilon)^{-1}$ . Indeed, in the  
377 seasonal thermocline, vertical turbulent diffusive time scales can vary from weeks to  
378 months for phytoplankton displacements as small as several meters (Denman and  
379 Gargett, 1983). The value of  $(\mu_m - \varepsilon)^{-1}$  used in many studies is usually from 0.1 to 5  
380 days (Gabric and Parslow, 1989; Klausmeier and Litchman, 2001; Huisman et al.,  
381 2006).

382 In this situation, from Eq. (14), the growth rate at SCML depth can be expressed as:

$$383 \quad \mu_m f(I)|_{z=z_m} = \varepsilon. \quad (26)$$

384 In regions with a low vertical diffusivity, Fennel and Boss (2003) derived that, at the  
385 SCML depth, the growth rate of phytoplankton is equal to the loss rate and the  
386 divergence of phytoplankton due to changes in the sinking velocity. Clearly, Eq. (26)

387 is identical to that of Fennel and Boss (2003) for constant sinking velocity of  
388 phytoplankton.

389 In this situation, the depth of SCML can be derived from Eq. (16), i.e.,

$$390 \quad z_m = \frac{1}{K_d} \ln \frac{(\mu_m - \varepsilon) I_0}{\varepsilon K_I}. \quad (27)$$

391 It indicates the SCML depth is directly proportional to the light penetration depth  
392 ( $1/K_d$ ). Beckmann and Hense (2007) have found a similar result by statistical analysis  
393 of numerical modeling.

394 The right hand term of Eq. (27) can be rewritten as  $\frac{1}{K_d} \ln \frac{I_0}{I^*}$  by letting  $I^* = \frac{\varepsilon K_I}{\mu_m - \varepsilon}$ ,  
395 where  $\mu_m f(I^*) = \varepsilon$ . Under the assumption of infinitely thin SCML ( $\sigma \rightarrow 0$ ),  
396 Klausmeier and Litchman (2001) also have derived Eq. (27) by setting the vertical  
397 diffusivity for phytoplankton as zero, i.e.,  $K_v = 0$ , in poorly mixed waters. Here, we  
398 go further to obtain the approximate expression of the thickness of SCML from Eq.  
399 (23), that is,

$$400 \quad 2\sigma = \frac{w}{\varepsilon} + \sqrt{\left(\frac{w}{\varepsilon}\right)^2 + \frac{w}{K_d(\varepsilon - \varepsilon^2/\mu_m)}}. \quad (28)$$

401 Obviously, the thickness of SCML increases with an increase in the sinking velocity  
402 of phytoplankton ( $w$ ), and with a decrease in the maximal growth rate ( $\mu_m$ ) and the  
403 light attenuation coefficient ( $K_d$ ).

404 The second special situation occurs when the term  $w/\sigma$  in the left hand of Eq. (18) is  
405 neglected. This special case occurs in regions where phytoplankton sinking velocity  
406 is very low ( $w \rightarrow 0$ ), or when  $w/\sigma$  is much smaller than  $\mu_m - \varepsilon$ , i.e., the time taken by  
407 phytoplankton sinking through half-length of SCML,  $(w/\sigma)^{-1}$ , is much longer than the  
408 time taken by net growth of phytoplankton,  $(\mu_m - \varepsilon)^{-1}$ . Phytoplankton sinking  
409 velocities exhibit a range of values depending on physical and physiological  
410 phenomena (e.g., size and shape of the cell). In the environment, estimates of sinking  
411 velocity vary from 0 to 9 m per day (Gabric and Parslow, 1989; Huisman and  
412 Sommeijer, 2002). Thus, the latter special scenarios (i.e.,  $w/\sigma \ll \mu_m - \varepsilon$ ) can indeed  
413 occur.

414 In this situation, according to Eq. (15), the net growth rate at the lower boundary of  
 415 SCML can be expressed as

$$416 \quad \mu_m f(I)|_{z=z_m+\sigma} - \varepsilon = 0. \quad (29)$$

417 That is, the lower boundary of SCML,  $z_m+\sigma$ , is located at the compensation depth.

418 In this situation, the depth of SCML can be derived from Eq. (17), i.e.,

$$419 \quad z_m = \frac{1}{K_d} \ln \frac{(\mu_m - \varepsilon) I_0}{\varepsilon K_l} - \sigma. \quad (30)$$

420 Compared with Eq. (27), we know that the depth of SCML is shallower in this special  
 421 case than that in the case of neglecting the influence of vertical diffusivity below the  
 422 surface mixed layer on SCM. This result implies that the displacement ( $\sigma$ ) of SCML  
 423 depth is the result of combined influences of vertical diffusivity and sinking velocity  
 424 of phytoplankton.

425 In this situation, from Eq. (23), we have

$$426 \quad \sigma \left( \sigma + \sqrt{\frac{K_{v2}}{\mu_m - \varepsilon}} \right) \left( \sigma - \sqrt{\frac{K_{v2}}{\mu_m - \varepsilon}} \right) = \frac{\mu_m K_{v2}}{(\mu_m - \varepsilon) \varepsilon K_d}. \quad (31)$$

427 The SCML thickens with a larger vertical diffusivity below the surface mixed layer  
 428 ( $K_{v2}$ ), a smaller growth rate ( $\mu_m$ ) or a lower light attenuation coefficient ( $K_d$ ).  
 429 Especially, when  $K_{v2} = 0$ , we have  $\sigma = 0$ . In other words, for non-sinking  
 430 phytoplankton ( $w \rightarrow 0$ ), when the vertical diffusivity below the surface mixed layer is  
 431 very small ( $K_{v2} \rightarrow 0$ ), the SCML disappears. This indicates that there must be a  
 432 vertical diffusion window sustaining non-sinking phytoplankton species in deep  
 433 waters.

434 The third special situation occurs when  $K_d \sigma$  (i.e.,  $\sigma / (K_d)^{-1}$ ) is too small to be  
 435 considered in Eq. (18). This may occur in clear waters where the light attenuation  
 436 coefficient is very small ( $K_d \rightarrow 0$ ), or in regions where the light penetration depth  
 437 ( $1/K_d$ ) is much larger than a half-width of SCML ( $\sigma$ ). Very narrow (from several to  
 438 tens of centimeters) SCML has been observed in clear, stratified lakes where the light  
 439 penetration depths were from several to tens of meters (Fee, 1976; Camacho, 2006).

440 In this situation, Eq. (18) can be modified to



441  $w\sigma + K_{v2} = 0.$  (32)

442 Clearly, when  $K_{v2} = 0$ ,  $w=0$ , this equation has infinitely many solutions. This means  
443 in stable, clear waters with a predominance of small cells, the deep SCML can occur  
444 with different thicknesses. For example, in the basin of South China Sea,  $<3 \mu\text{m}$   
445 phytoplankton (such as *Prochlorococcus*, *Synechococcus*, picoeukaryotes, etc.) are  
446 the dominant species in SCMLs (Takahashi and Hori, 1984; Liu et al., 2007) with  
447 variable thicknesses (Lee Chen, 2005; Chen et al., 2006).

#### 448 **4.2 Limitation and potential application**

449 To make the complex problem (SCM phenomenon) tractable, the ecosystem  
450 dynamical equations adopted in this study are judiciously simplified. For example, a  
451 constant eddy diffusivity is assumed in the surface mixed layer and below this layer,  
452 respectively. Many processes (turbulence, internal waves, storms, slant-wise and  
453 vertical convection) in upper ocean dynamics are not captured in the model system.  
454 The assumption of steady state will be broken during episodic events of strong  
455 physical forcing, nutrient injection, or blooms (Fennel and Boss, 2003). Similarly the  
456 biological representation is also extremely limited. We neglect food-web and  
457 microbial loop dynamics (detritus, dissolved organic matter, and zooplankton are not  
458 included explicitly), and assume all loss processes, except sinking, to be linearly  
459 proportional to phytoplankton. The sinking velocity of phytoplankton is assumed to  
460 be constant with depths, excluding the effects of temperature and density gradients.  
461 Our model also neglects some feedback mechanisms, like the effect of phytoplankton  
462 on light attenuation. Although these are important aspects that could be included,  
463 their addition is unlikely to change our conclusions qualitatively (Fennel and Boss,  
464 2003).

465 In a stratified water column with a well-mixed surface layer on top of a poorly mixed  
466 subsurface layer, a general Gaussian function of vertical Chl a profile represents the  
467 distribution of which the surface Chl a concentration is nearly zero, the maximum of  
468 Chl a is significantly deeper than the base of surface mixed layer, and the vertical  
469 gradient of Chl a is identically zero at the transition between the two layers. The  
470 assumption of a general Gaussian profile can be broken in several ways. If nutrient  
471 input to the mixed layer due to riverine inputs, surface runoff, or atmospheric  
472 deposition, was considered in the ecosystem, then the surface concentration of Chl a

473 should be positive (Mellard et al. 2011). If the depth of surface mixed layer  $z_s$  is large,  
474 this allows another way for the surface Chl a concentration being positive by  
475 extracting some of the Chl a from the SCML (Beckman and Hense, 2007), then the  
476 vertical gradient of Chl a may not be identically zero at the transition between the  
477 two layers.

478 Under the assumption of a constant loss rate, the lower compensation depth we got  
479 from Eq. (11), the location where the growth rate of phytoplankton limited by light  
480 equals the loss rate, is similar to the popular definition of compensation depth given  
481 by Sverdrup (1953), below which no net growth occurs. This assumption is in the  
482 heart of the Sverdrup's critical depth model and we now understand that it has  
483 significant limitations (Behrenfeld and Boss, 2014). Particularly, the treatment of  
484 grazing loss, is, in the least, an oversimplification, though many numerical models  
485 used a similar one (e.g., Klausmeier and Litchman, 2001; Fennel and Boss, 2003;  
486 Huisman et al., 2006). Grazing loss depends strongly on Chl a concentration (it is an  
487 encounter based process) and, given that zooplankton can move, or, in the least, grow  
488 faster where more food is available, is unlikely to have a constant concentration  
489 distribution (Behrenfeld and Boss, 2014).

490 Our model suggests that the condition for the existence of a SCM is the growth rate  
491 under the limitation of light intensity,  $\mu_m f(I_0)$ , is larger than the loss rate,  $\epsilon$ , in  
492 stratified water columns. Fennel and Boss (2003) found a similar result and pointed  
493 out that this condition for a SCM is general. Many numerical studies have reproduced  
494 the SCM phenomenon, of which the condition of SCM occurrence met with variable  
495 values of the sinking velocity of phytoplankton and the mixing diffusivity  
496 (Klausmeier and Litchman, 2001; Huisman et al., 2006; Mellard et al., 2011).

497 Our two compartment system model reproduces some of the results of the more  
498 complex model with three compartments (phytoplankton, nutrients, and detritus,  
499 Beckmann and Hense, 2007). For example, our model predicts that with fully  
500 recycling of the dead phytoplankton, the total Chl a concentration in water columns  
501 depends on the sinking velocity of phytoplankton and the vertical diffusivity, but  
502 independent on the growth rate and the loss rate of phytoplankton. Beckmann and  
503 Hense (2007) found similar results. Here, we go further to point out an interesting  
504 finding that the derivations of the total Chl a are irrespective of the form of the

505 growth limiting function. Since growth functional forms in phytoplankton models are  
506 still debated in the literature (Haney, 1996; Ayata et al., 2013), this will be most  
507 helpful to estimate the vertical integrated Chl a and primary production.

508 The relationships (in previous sections and in Appendices A and B) we derived can  
509 be used to compute missing model parameters (such as maximum growth rate  $\mu_m$ ,  
510 loss rate  $\varepsilon$ , recycling rate  $\alpha$ ) which are difficult to obtain by on-site observation, if  
511 estimates of others are available. For example, Eq. (B4) allows us to obtain an  
512 estimate of the sinking velocity of phytoplankton from the measurement of SCM  
513 thickness and intensity, the nutrient concentration at water column depth, and the  
514 vertical diffusivity below the surface mixed layer.

515 Our analytic solutions can in principle be tested through a comparison with  
516 observations: for example, the shape of profiles (the SCML thickness, depth, and  
517 intensity), expressed by the characteristic relationships (Eqs. 16-18, 22 and B4), the  
518 vertical integral of total subsurface Chl a concentration (Eqs. 21 and B3), the  
519 consistency of independent field estimates for sinking velocity, vertical diffusivity,  
520 recycling rate and loss rate (Eqs. 21-22 and B3-B4).

521 We retrieve the three SCM characteristics from Eqs. (16-18, and 22) by combining  
522 remote sensing data (annual averaged values of surface light intensity  $I_0$  and light  
523 attenuation coefficient  $K_d$ ) and some parameters from published field and numerical  
524 studies (e.g., sinking velocity of phytoplankton  $w$ , vertical diffusivity below the  
525 surface mixed layer  $K_{v2}$ , loss rate  $\varepsilon$ , maximum growth rate  $\mu_m$ ). Table 2 lists the  
526 values of model parameters at three time-series stations in different ocean regions, i.e.,  
527 the SEATS station, the HOT station, and the Bermuda Atlantic Time-Series Study  
528 (BATS) site in the Sargasso Sea, and the corresponding references. The estimated  
529 results and the observed values of the SCML thickness, depth and intensity at the  
530 three stations are shown in Fig. 2.

531 The estimated depths and thicknesses of the SCML agree reasonably well with the  
532 observations at all three stations. However, the intensities of the SCML are poorly  
533 estimated, implying that other mechanisms supplying nutrients for the SCML, except  
534 upward diffusivity, for phytoplankton growth (Williams et al., 2013). This is the first  
535 try to estimate the depth, thickness and intensity of the SCML using parameters from  
536 satellite data and field studies. We note that the kinematic solution assumed (Eq. 7) is,

537 at best, an approximate solution of the dynamical Eqs. (1-2). Even though  
538 disagreements could be associated with uncertainties from several sources, this type  
539 of try would give some idea of how real-world data could be incorporated into the  
540 model and thus be applied to the field (Pitarch et al. 2014).

## 541 **5 Summary**

542 A general Gaussian function is assumed to represent a bell-shape vertical distribution  
543 of Chl a in stratified water columns. The function is incorporated into the ecosystem  
544 dynamical equations to determine three steady-state SCM characteristics and examine  
545 their dependence on environmental parameters such as vertical diffusivity, sinking  
546 velocity of phytoplankton, light attenuation coefficient.

547 The maximum Chl a concentration occurs at or below the location of the maximum  
548 growth rates of phytoplankton determined by the vertical mixing time scale and the  
549 time taken by a phytoplankton sinking through the length scale.

550 The depth of the SCML in steady state deepens logarithmically with an increase in  
551 surface light intensity, but shoals with increasing light attenuation coefficient,  
552 increasing vertical diffusivity below the surface mixed layer, increasing loss rate of  
553 phytoplankton, and with decreasing sinking velocity of phytoplankton.

554 The shape of the SCML (thickness and intensity) is mainly influenced by nutrients  
555 supply, but independent of sea surface light intensity. The SCML gets thicker and  
556 stronger with a higher vertical diffusivity below the surface mixed layer. The  
557 intensity of SCML in steady state weakens as a result of expanding the SCML by a  
558 smaller sinking velocity of phytoplankton and a lower light attenuation coefficient.

559 In regions with a low vertical diffusivity, the SCML depth is inversely proportional to  
560 light attenuation coefficient, and is deeper than that in regions dominated by  
561 non-sinking phytoplankton. In clear and stable waters with a predominance of small  
562 cells, deeper SCMLs can occur with different thicknesses.

563 Upon potential risk of climate change, it is critical to accurately estimate the global  
564 and regional SCML-related primary production. However, the SCM characteristics  
565 cannot be detected by remote sensing satellites, which will restrict the application of  
566 satellite data in estimating primary production in a large temporal and spatial scale.  
567 The relationships we derived might help to estimate depth-integrated primary  
568 production using available data from satellite observations (incident light and light

569 attenuation coefficient) when appropriate vertical estimates of growth rate and loss  
570 rate of phytoplankton, sinking velocity of phytoplankton and vertical diffusivity were  
571 adopted based on observations or model results. Again, the solutions could also help  
572 to compute environmental parameters that are difficult to obtain from on-site  
573 observation.

574

575 **Appendix A**

576 In steady state, the net nutrient flux at any given depth ( $z$ ) is equals to the net  
 577 nutrients consumption by phytoplankton, then from steady-state of Eq. (2) we obtain  
 578 Eq. (A1) below the surface mixed layer:

$$579 \quad \int (\mu_m \min(f(I), g(N)) - \alpha \varepsilon) P(z) dz \approx K_{v2} \left. \frac{dN(z)}{dz} \right|_z \quad (\text{A1})$$

580 If  $\mu_m \min(f(I), g(N)) - \varepsilon > 0$ , then  $\mu_m \min(f(I), g(N)) - \alpha \varepsilon > 0$  for  $0 < \alpha \leq 1$ , we will  
 581 have  $\frac{dN}{dz} > 0$ . That is,  $N(z)$  will increase with depth below the surface mixed layer.

582 From the properties of the quadratic function in the right hand of Eq. (10), we have  
 583  $\mu_m \min(f(I), g(N)) - \varepsilon > 0$  on the interval  $(z_{c1}, z_{c2})$ . Hence, we have  
 584  $\mu_m \min(f(I), g(N)) - \alpha \varepsilon > 0$  for  $0 < \alpha \leq 1$ , then  $dN/dz > 0$ . In other words,  $N(z)$   
 585 increases with depth on the interval  $(z_{c1}, z_{c2})$ .

586 According to Eq. (4), we know that  $g(N)$  is a monotonic increasing function on  
 587 interval  $(z_{c1}, z_{c2})$ , and  $f(I)$  is a monotonic decreasing function on interval  $(z_{c1}, z_{c2})$ .

588 Note that we have known that the stable SCML occurs in stratified water column only  
 589 when the growth of phytoplankton in the surface mixed layer is nutrient-limited  
 590 (Mellard et al., 2011; Ryabov et al., 2010). In other words, the limitation by nutrients  
 591  $g(N)$  is less than the limitation by light  $f(I)$  within the surface mixed layer, i.e.,  $g(N) <$   
 592  $f(I)$  for  $0 \leq z \leq z_s$ .

593 Because there is only one maximum in the growth rates of phytoplankton which  
 594 occurs at the depth  $z_0 = z_m - \frac{w\sigma^2}{2K_{v2}}$ , and  $z_{c1} < z_0 < z_{c2}$  (Eq. 11), we arrive at

$$595 \quad \min(f(I), g(N)) = \begin{cases} g(N) & z_{c1} \leq z \leq z_0 \\ f(I) & z_0 \leq z \leq z_{c2} \end{cases} \quad (\text{A2})$$

596 and

$$597 \quad \max(\mu_m \min(f(I), g(N))) = \mu_m f(I) \Big|_{z=z_0}. \quad (\text{A3})$$

598 That is, the maximum growth rate occurs at the depth  $z_0$  where is the transition

599 from nutrients limitation to light limitation, and the growth of phytoplankton is  
600 light-limited below the depth  $z_0$ .

601 **Appendix B**

602 The dead phytoplankton is entirely recycled ( $\alpha = 1$ ), and thus the system is closed. In  
603 this case, at steady state Eq. (19) reduces to

604 
$$w \frac{dP}{dz} = \frac{d^2}{dz^2} (K_v (P + N)) \quad (\text{B1})$$

605 Integrating this equation twice from the surface to bottom boundary ( $z_b$ ) and using  
606 boundary conditions (Eq. 6) gives

607 
$$w \int_0^{z_b} P(z) dz = K_{v1} (P + N) \Big|_0^{z_s} + K_{v2} (P + N) \Big|_{z_s+0}^{z_b} \quad (\text{B2})$$

608 Note that we have known that the SCML occurs only when the growth of  
609 phytoplankton within the surface mixed layer is nutrient-limited, then we further  
610 assume the surface nutrients value is negligible. Using the assumption of small Chl a  
611 at the top and the bottom boundaries of the model domain, we obtain

612 
$$h = \frac{K_{v2}}{w} N(z_b) \quad (\text{B3})$$

613 and the intensity of SCML is

614 
$$P_{\max} = \frac{K_{v2}}{\sqrt{2\pi\sigma w}} N(z_b) \quad (\text{B4})$$

615 where  $N(z_b)$  is the nutrients concentration at depth  $z_b$ . Therefore, with  $\alpha = 1$ , the  
616 intensity of SCML is affected by the ambient nutrients concentration below the  
617 surface mixed layer. The total Chl a in the water column depends on the sinking  
618 velocity of phytoplankton and the diffusivity, but it is independent on the growth rate  
619 and loss rate of phytoplankton. Analogous results have been obtained by Liccardo et  
620 al. (2013). Beckmann and Hense (2007) also found similar result by introducing an  
621 explicit compartment for the detritus in their models.

622 **Acknowledgements.** We gratefully acknowledge E. Boss, J. Pitarch, and two  
623 anonymous reviewers for constructive and insightful reviews. We also thank  
624 particularly X. H. Liu and Z. Y. Cai for programming assistance and precious advice.

625 This work is funded in part by the National Key Basic Research Program of China  
626 under the contract 2014CB953700, the National Nature Science Foundation of China  
627 under the contract (41406010, 41210008, 41106007), and the China Postdoctoral  
628 Science Foundation under the contract 2013M541958.



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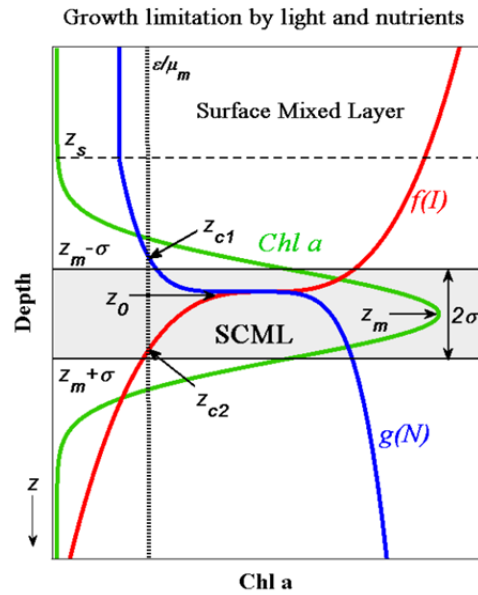
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772 List of figures and tables

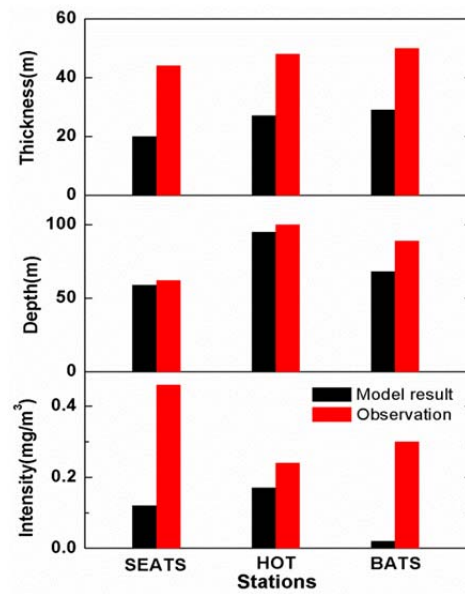
773 Figure 1



774

775 Fig. 1 Schematic picture of Chl a distribution under the limitation by light and nutrient in  
776 stratified water column (red solid line is Chl a concentration as a function of depth; black dashed  
777 line is the growth limiting term with respect to light,  $f(I)$ ; blue dashed line is the growth limiting  
778 term with respect to nutrients,  $g(N)$ ; horizontal dashed line represents the depth of surface mixed  
779 layer,  $z_s$ ; horizontal solid lines indicate the locations of the upper- and lower-SCML,  $z_m - \sigma$ ,  $z_m + \sigma$ ,  
780 respectively; vertical dotted line is the ratio of loss rate to maximum growth rate,  $\varepsilon/\mu_m$ ;  $z_{c1}$  and  $z_{c2}$   
781 refer to the two compensation depths where  $\mu_m g(N) = \varepsilon$  and  $\mu_m f(I) = \varepsilon$ , respectively;  $z_0$  and  $z_m$   
782 indicate the depths of maximum in growth rates and in Chl a concentrations, respectively; double  
783 arrow represents the thickness of the SCML,  $2\sigma$ )

784 **Figure 2**



785

786 Fig. 2 Comparisons of the model results and observations (in terms of thickness, depth, and  
787 intensity of SCML) at SEATS, HOT, and BATS (black columns represent the model results, red  
788 columns are the observations at the three stations which were fitted by Gaussian function using  
789 annually averaged data obtained from <http://www.odn.ntu.edu.tw/>,  
790 <http://hahana.soest.hawaii.edu/hot/hot-dogs/cextraction.html>, and <http://bats.bios.edu/>,  
791 respectively)

792 Table 1 Influences of dynamic model parameters on the steady-state SCML thickness ( $2\sigma$ ), depth  
 793 ( $z_m$ ), intensity ( $P_{max}$ ), and the total Chl a in the water column ( $h$ ).

Model parameters ( $\uparrow$ )	$2\sigma$	$z_m$	$P_{max}$	$h$
$I_0$ (Surface light intensity)	-	$\uparrow$	-	-
$K_I$ (Half-saturation constant of light limited growth)	-	$\downarrow$	-	-
$K_{v2}$ (Vertical diffusivity below surface mixed layer)	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$
$w$ (Sinking velocity of phytoplankton)	$\downarrow$	$\downarrow$	$\uparrow$	-
$K_d$ (Light attenuation coefficient)	$\downarrow$	$\downarrow$	$\uparrow$	-
$\varepsilon$ (Loss rate of phytoplankton)	$\downarrow^*$	$\downarrow$	/	$\downarrow$
	$\uparrow^{**}$	$\downarrow$	$\downarrow$	$\downarrow$
$\alpha$ (Nutrient recycling coefficient)	-	-	$\uparrow$	$\uparrow$
$\frac{dN}{dz} \Big _{z=z_b}$ Nutrient gradient at the lower boundary of SCML	-	-	$\uparrow$	$\uparrow$
$K_N$ (Half-saturation constant of nutrient limited growth)	-	-	-	-
$K_{v1}$ (Vertical diffusivity in surface mixed layer)	-	-	-	-
$\mu_{max}$ (Maximum growth rate of phytoplankton)	/	/	/	/

794  $\uparrow$  indicates increase,  $\downarrow$  indicates decrease, - indicates no effect, / indicates no straightforward  
 795 result, \* indicates a result when  $\varepsilon < \mu_{max}/2$ , and \*\* indicates a result when  $\varepsilon > \mu_{max}/2$ .

796

Table 2 Parameter values at SEATS, HOT, and BATS

Parameters	Units	Values at Stations		
		SEATS	HOT	BATS
$I_0$	$\mu\text{mol photos m}^{-2} \text{s}^{-1}$	700 <sup>(1, 2)</sup>	550 <sup>(1, 3)</sup>	448 <sup>(1, 4)</sup>
$K_d$	$\text{m}^{-1}$	0.052 <sup>(1, 5)</sup>	0.04 <sup>(1, 3)</sup>	0.042 <sup>(1, 4)</sup>
$K_{v2}$	$\text{m}^2 \text{s}^{-1}$	$5 \cdot 10^{-5}$ <sup>(6)</sup>	$5 \cdot 10^{-5}$ <sup>(3)</sup>	$1 \cdot 10^{-4}$ <sup>(7, 8)</sup>
$\mu_{max}$	$\text{d}^{-1}$	1.2 <sup>(9, 10)</sup>	0.96 <sup>(3)</sup>	1 <sup>(11)</sup>
$K_I$	$\mu\text{mol photos m}^{-2} \text{s}^{-1}$	40 <sup>(12)</sup>	20 <sup>(3)</sup>	20 <sup>(3, 12, 13)</sup>
$\varepsilon$	$\text{d}^{-1}$	0.5 <sup>(9, 10)</sup>	0.24 <sup>(3)</sup>	0.5 <sup>(14)</sup>
$\alpha$	dimensionless	0.3 <sup>(10)</sup>	0.5 <sup>(3)</sup>	0.16 <sup>(8)</sup>
$w$	$\text{m d}^{-1}$	1 <sup>(15)</sup>	1 <sup>(3, 15)</sup>	2 <sup>(8)</sup>
$dN/dz$ at depth of $z_b$	$\text{mmol N m}^{-4}$	0.1 <sup>(16)</sup>	0.05 <sup>(17, 18)</sup>	0.02 <sup>(19, 20)</sup>
$z_b$	$\text{m}$	200	200	200

798 Superscripts refer to the references that provide the source for the parameter value and the  
799 citations are as follows: <sup>(1)</sup><http://oceandata.sci.gsfc.nasa.gov/SeaWiFS/Mapped/Annual/9km/>;  
800 <sup>(2)</sup>Wu and Gao, 2011; <sup>(3)</sup>Huisman et al., 2006; <sup>(4)</sup>Varela et al., 1994; <sup>(5)</sup>Lee Chen et al., 2005; <sup>(6)</sup>Lu  
801 et al., 2010; <sup>(7)</sup>Hood et al., 2001; <sup>(8)</sup>Salihoglu et al., 2008; <sup>(9)</sup>Cai et al., 2006; <sup>(10)</sup>Liu et al., 2007;  
802 <sup>(11)</sup>Ayata et al., 2013; <sup>(12)</sup>Raven and Richardson, 1986; <sup>(13)</sup>Mara On and Holligan, 1999;  
803 <sup>(14)</sup>Tjiputra et al., 2007; <sup>(15)</sup>Bienfang and Harrison, 1984; <sup>(16)</sup>Chen et al., 2006; <sup>(17)</sup>Fennel and Boss,  
804 2003; <sup>(18)</sup>Beckmann and Hense, 2007; <sup>(19)</sup>Cianca et al., 2007; <sup>(20)</sup>Cianca et al., 2012.

**List of what we changed in the revised version of manuscript:**

Original	Revised
Page 9512, line 15-17	<b>Line 22-24:</b> Change “The analytical solutions can be used to estimate environmental parameters which are difficult to measure on site.” to “ <b>In addition, the limitation and potential application of the analytical solutions were also presented.</b> ”
Page 9512, line 24	<b>Line 31-41:</b> Add the difference between SCMs and SBMs “ <b>The subsurface biomass maxima (SBMs) are also common in stratified water columns. The chlorophyll-to-biomass ratio generally increases with depth in the euphotic zone. Thus, SCMs may not necessarily represent SBMs (Cullen, 1982; Fennel and Boss, 2003) and are usually deeper than SBMs (Fennel and Boss, 2003; Hodges and Rudnick, 2004). However, both the subsurface maxima in chlorophyll and biomass are usually formed in certain regions of the water column where two opposing resource (light and nutrient) gradients combined with vertically heterogeneous turbulent mixing is amenable for survival of phytoplankton. Thus, SCMs are approximately equal to SBMs in many studies (Klausmeier and Litchman, 2001; Sharples et al., 2001; Huisman et al., 2006; Raybov et al., 2010).</b> ”
Page 9515, line 10	<b>Line 105-107:</b> Spell out the assumption of Chl a equal phytoplankton “Moreover, the photoacclimation of phytoplankton was not considered here and the Chl a distribution is supposed to represent the distribution of phytoplankton biomass. <b>This is a significant simplification. In fact, phytoplankton increases inter-cellular pigment concentration when light level decreases (Fennel and Boss, 2003).</b> ”
Page 9515, after line 16	<b>Line 111-114:</b> Clarify the issue on a conversion factor between the units of $\text{mg m}^{-3}$ and $\text{mmol N m}^{-3}$ “ <b>Usually, the unit of Chl a concentration is <math>\text{mg m}^{-3}</math>, the concentrations of phytoplankton and the limiting nutrients are in unit of <math>\text{mmol N m}^{-3}</math>. A ratio of 1.59 g chlorophyll per mol nitrogen (Cloern et al., 1995; Oschlies, 2001) is thereby used for unit conversion.</b> ”
Page 9515, line 19	<b>Line 116-118:</b> Explicitly mention “ <b>w is the sinking velocity of phytoplankton, which is non-negative in the chosen coordinate system and assumed to be constant with depths.</b> ”
Page 9516, line 1	<b>Line 122-124:</b> We add the other possible location of $z_b$ “ <b><math>z_b</math> is the bottom of water column or the location where the Chl a concentration reduces to nearly zero below the euphotic zone.</b> ”
Page 9516, after line 8	<b>Line 126-129:</b> Add a gradual transition of vertical diffusivity between the surface mixed layer and the deep one, to consist



	with the general Gaussian Chl a profile “A gradual transition from the surface mixed layer to the deep one written in terms of a generalized Fermi function is adopted (Ryabov et al., 2010), that is, $K_v(z) = K_{v2} + \frac{K_{v1}-K_{v2}}{1+e^{(z-z_s)/l}}$ , where parameter $l$ characterizes the width of the transient layer. In our study, we assumed this transient layer is finitely thin.”
Page 9516, line 9-10 and after line 13	Spell out the exclusive effect of self-shading on light attenuation coefficient, <b>Line 136-137</b> : “Light intensity is assumed to decrease exponentially with depth according to Lambert-Beer’s law, i.e.,” and <b>Line 140-141</b> : “Assuming a constant $K_d$ , we ignore the effects of the self-shading and the dissolved and particulate material on the attenuation coefficient.”
Page 9516, line 22-23	<b>Line 150-151</b> : Adopt a single amplitude $P_{\max}$ in Eq. (7), “ $P(z) = P_{\max} e^{-\frac{(z-z_m)^2}{2\sigma^2}} \quad 0 \leq z \leq z_b,$ where $P(z)$ is Chl a concentration as a function of depth $z$ , and $P_{\max} = \frac{h}{\sigma\sqrt{2\pi}}.$ ”
Page 9518, line 6-8	<b>Line 181-184</b> : Rewrite this sentence “That is, the boundary of SCML is located at the depth where there is the balance between phytoplankton growth and all losses (including the divergence of the sinking flux $w \frac{dP}{dz}$ and the loss $\varepsilon$ due to mortality, respiration, and grazing), named the community compensation depth (Ono et al., 2001).”
Page 9518, line 15-17	<b>Line 191-193</b> : Rewrite this sentence “Previous numerical studies (Huisman et al., 2006; Ryabov et al., 2010) showed that the ecosystem dynamical model (Eqs. 1 and 2) can approximately reproduce the bell-shape feature of the vertical Chl a profile (Fig. 1).”
Page 9518, line 23	<b>Line 200</b> : Modify the factor of the first term on the right hand side in Eq. (10) $-K_{v2}/\sigma^2$ as $-K_{v2}/\sigma^4$
Page 9519, after line 3	<b>Line 204-209</b> : Add the definition of the compensation depths “Clearly, the inequality $\mu_m \min(f(I), g(N)) - \varepsilon > 0$ is satisfied in the interval $(z_{c1}, z_{c2})$ . This indicates that the subsurface net production occurs only between the two compensation depths where the growth rate $\mu_m \min(f(I), g(N))$ equals the loss rate

	<p><math>\varepsilon</math>. Beckmann and Hense (2007) found similar results by numerical modeling and emphasized the often overlooked fact that an SCML has to have two compensation depths.”</p>
Page 9519, line 4-5	<p><b>Line 210-213:</b> Correct the wrong typo and add two special issues, “From Eq. (11), we obtain <math>z_{c1} \leq z_m - \sigma</math> and <math>z_m \leq z_{c2} \leq z_m + \sigma</math> (Fig. 1). Especially, <math>z_{c1} = z_m - \sigma</math>, and <math>z_{c2} = z_m + \sigma</math> when the sinking velocity of phytoplankton <math>w</math> is too small to be considered. This result is identical to that of Beckmann and Hense (2007) for neglecting sinking velocity of phytoplankton.”</p>
Page 9519, line 12-17	<p><b>Line 221-225:</b> Rewrite these three sentences “We define <math>T = \sigma^2 / K_{v2}</math> as the characteristic vertical mixing time scale in the SCML of thickness <math>\sigma</math> (Bowdon, 1985; Gabric and Parslow, 1989). Let the length scale be <math>L = 2K_{v2} / w</math>, which determines the scale height of the phytoplankton distribution (Ghosal and Mandre, 2003). Thus, the right hand terms of Eq. (13) can be rewritten as <math>1/T + w/(2L)</math>.”</p>
Page 9521, after line 13	<p><b>Line 265-266:</b> Add a proof for this condition “That is, the necessary condition for the existence of SCM is <math>\mu_m f(I_0) &gt; \varepsilon</math>, which is identical with the result of Fennel and Boss (2003) when vertical sinking is constant as a function of depth in their model.”</p>
Page 9524, line 1	<p><b>Line 304-305:</b> Add the equality of Taylor expanding “To simplify, by Taylor expanding <math>e^{K_d \sigma}</math> at <math>\sigma = 0</math> and truncating the Taylor series after the linear term, i.e., <math>e^{K_d \sigma} = 1 + K_d \sigma + o(\sigma^2)</math>”</p>
Page 9525, line 17	<p><b>Line 345:</b> Change “the denominator of the last term on the right hand side in Eq. (25), <math>(\mu_m / K_d - w) / \sigma^2 + \varepsilon \sigma</math>” to “<math>\mu_m / K_d - w + \varepsilon \sigma</math>”</p>
Page 9526, after line 5	<p><b>Line 361-365:</b> Add “Considering the two compartment system (nutrients and Chl a) in steady state and a general Gaussian function for vertical Chl a concentration, we derived the analytical solution for the fundamental relationships between SCM characteristics and various parameters. Three special sceneries, limitation and implications of this study were discussed below.”</p>
Page 9526, line 25-26	<p><b>Line 386-388:</b> change the sentence to “Clearly, Eq. (26) is identical to that of Fennel and Boss (2003) for constant sinking velocity of phytoplankton.”</p>

Page 9529, after line 11	<b>Line 448-540:</b> Add the new <b>Section 4.2 Limitation and implication</b>
Page 9530, line 21	<b>Line 584:</b> Add the integral symbol to the first term on the left hand in Eq. (A1), i.e., “ $\int (\mu_m \min(f(I), g(N)) - \alpha \varepsilon) P(z) dz \approx K_{v2} \frac{dN(z)}{dz} \Big _z$ ”
Page 9531, line 4-6	<b>Line 593-595:</b> Rewrite this sentence “Note that we have known that the stable SCML occurs in stratified water column only when the growth of phytoplankton in the surface mixed layer is nutrient-limited (Mellard et al., 2011; Ryabov et al., 2010).”
Acknowledgements	<b>Line 622-655:</b> Add many thanks to the three referees and other two friends “We gratefully acknowledge E. Boss, J. Pitarch and two anonymous reviewers for constructive and insightful reviews. We also thank particularly X. H. Liu and Z. Y. Zhong for programming assistance and precious advice.” <b>Line 627:</b> Add 41416010 before “41210008, 41106007”
References	Add 18 references in <b>line 636-638, 641-648, 653-661, 680, 688-689, 712-713, 716-717, 721-722, 729-730, 735-736, 744-746, 758-760, 766-769.</b>
Figures	Add $z_{c1}, z_{c2}, z_m, z_m - \sigma, z_m + \sigma, 2\sigma, z_0, z_s, \varepsilon/\mu_m, f(I)$ , and $g(N)$ in Fig. 1, and change the colors of profiles and the corresponding captions. Add Fig. 2.
Tables	Add Table 2.

## **Reply to Anonymous Referee #1**

### **1 General comments:**

*The study presented in this manuscript considers the parameter dependencies of subsurface chlorophyll maximum layers (SCML) in aquatic systems based on an analytical approach. Assuming a Gaussian shape of the Chl a profile leads to a set of analytical expressions that link the three parameters thickness, amplitude and depth of the SCML to phytoplankton growth and losses, surface irradiance and light attenuation, phytoplankton sinking speed and subsurface vertical mixing.*

*This new approach allows us to integrate previous results from a variety of studies. Some of the results confirm existing knowledge, others go beyond. Having analytical expressions for the functional relationship between SCMLs and various parameters is certainly very helpful. There are, however, a few points that need clarification and/or improvement, before I can recommend publication:*

**Response:** We thank the helpful comments and revise our manuscript accordingly.

### **2 Specific comments**

**Comments---** *p. 9513: The discussion of the difference between SCM and SBM (subsurface biomass maximum layer) is weak. The text uses phytoplankton concentration and chlorophyll concentration as synonyms, an assumption which is not necessarily valid. This also leads to another point:*

**Comments---** *p. 9515: The model currency seems to be  $\text{mg m}^{-3}$  (according to Figure 1) but then the limiting nutrient N needs to be given in the same units as well (or a conversion factor needs to be introduced). In my view, the prognostic model variables P and N should be given in  $\text{mmol N m}^{-3}$ , in which case it is more appropriate to talk about an SBML instead of an SCML.*

**Response:** In the revised version, we rewrite the part to clarify the difference between SCM and SBM in Introduction, i.e., ‘The subsurface biomass maxima (SBMs) are also common in stratified water columns. The chlorophyll-to-biomass ratio generally increases with depth in the euphotic zone. Thus, SCMs may not necessarily represent SBMs (Cullen, 1982; Fennel and Boss, 2003) and are usually deeper than SBMs (Fennel and Boss, 2003; Hodges and Rudnick, 2004). However, both the subsurface maxima in chlorophyll and biomass are usually formed in certain regions of the water column where two opposing resource (light and nutrient) gradients combined with vertically heterogeneous turbulent mixing is amenable for survival of phytoplankton. Thus, SCMs are approximately equal to SBMs in many studies (Klausmeier and Litchman, 2001; Sharples et al., 2001; Huisman et al., 2006; Raybov et al., 2010).’

In the revised version, we will clarify the issue on a conversion factor between the units of  $\text{mg/m}^3$  and  $\text{mmol N/m}^3$  in the Method Section, i.e., Usually, the unit of Chl a concentration is  $\text{mg/m}^3$ , the concentration of phytoplankton and the limiting nutrient is in unit of  $\text{mmol N/m}^3$ . A ratio of 1.59 g chlorophyll per mol nitrogen (Cloern et al., 1995; Oschlies, 2001) was adopted to convert to the same unit ( $\text{mmol N/m}^3$ ) in the

following equations.

**Comments---** *I suggest the authors point out (e.g., in the discussion section), that several effects have been neglected: self-shading (p. 9516(09): self-shading is only included in this formulation, if the vertical concentration of P is constant, clearly not the case for the assumed Gaussian profile.), sinking of detritus as a separate compartment, etc. I find it quite remarkable that the 2-equation model reproduces some of the results of a more complex model with three equations (e.g., Beckmann and Hense, 2007). This fact should be mentioned explicitly.*

**Response:** We will spell out the assumption and limitation in the newly added Section 4.2. Please see the revision.

**Comments---** *p. 9519(04): I am unable to confirm the statement that  $z_{c1} > z_m - \sigma$ . Instead, it seems to me that  $z_{c1} \leq z_m - \sigma$ . As a consequence,  $z_o$  is not generally within the SCML interval  $[z_m - \sigma, z_m + \sigma]$  (this is true only for  $K_{v2}/w\sigma \geq 1/2$ ). This also affects equation (A2) and the arguments connected to it.*

**Response:** Sorry for this typo, it is  $z_{c1} \leq z_m - \sigma$ . In the revision, we will correct it and the arguments related. In equation (A2), the interval is  $[z_{c1}, z_{c2}]$  and therefore it is not influenced by this error.

**Comments---** *p. 9538: in Figure 1, please indicate typical locations of  $z_o$ ,  $z_m$ ,  $z_{c1}$  and  $z_{c2}$  (rather than a depth in m – which is misleading anyway), as well as  $\sigma$ .*

**Response:** Thank you very much for this helpful suggestion. We will use  $\sigma$  and these four depths,  $z_o$ ,  $z_m$ ,  $z_{c1}$  and  $z_{c2}$ , instead of a depth in m in the revised Figure 1.

**Comments---** *In general, the text could be more explanatory. For example, 9519(12-19) is not easily understandable and should be rewritten.*

**Response:** We rewrite the part to make it more explanatory, *i.e.*, ‘We define  $T = \sigma^2 / K_{v2}$  as the characteristic vertical mixing time scale in the SCML of thickness  $\sigma$  (Gabric and Parslow, 1989; Bowdon, 1985). Let the length scale be  $L = 2K_{v2}/w$ , which determines the scale height of the phytoplankton distribution (Ghosal and Mandre, 2003). Thus, the right hand terms of Eq. (13) can be rewritten as  $1/T + w/(2L)$ . In other words, the maximum net growth rate of phytoplankton,  $\max(\mu_m \min(f(I), g(N)) - \epsilon)$ , is determined by the vertical mixing time scale ( $T$ ) and the time taken by a phytoplankton sinking ( $w$ ) through lengths ( $2L$ ). We also make revision throughout the whole manuscript to enhance our analysis. Please see our revised version.’.

### 3 Technical corrections

**Comments---** *9515(17): “etc.” refers to which processes and factors?*

**Response:** Sorry for the typo, we have removed “etc.”.

**Comments---** *9515(19): it should be explicitly mentioned that  $w$  is positive in the chosen coordinate system.*

**Response:** In the revision, we will spell out this, *i.e.*,  $w$  is the sinking velocity of phytoplankton, which is non-negative in the chosen coordinate system and assumed to

be constant with depth.

*Comments---9523(03): what does “etc.” include – the list not so long that it could not be given completely.*

**Response:** In the revision, we will list all the terms including sinking velocity of phytoplankton  $w$ , vertical diffusivity below the surface mixed layer  $K_{v2}$ , loss rate  $\varepsilon$ , maximum growth rate  $\mu_m$ , recycling rate  $\alpha$ , half-saturation constant for light  $K_I$ .

*Comments---9523(19): “environmental factors (... , ...)” the loss rate of phytoplankton  $\varepsilon$  is not really an environmental factor as it includes natural mortality.*

**Response:** We will replace environmental parameters with model parameters in the revision.

*Comments---9524(01): it should be added that the Taylor series is truncated after the linear term.*

**Response:** Agree. This sentence will be revised as ‘To simplify, by Taylor expanding  $e^{K_d\sigma}$  at  $\sigma = 0$  and truncating the Taylor series after the linear term, i.e.,  $e^{K_d\sigma} = 1 + K_d\sigma + o(\sigma^2)$ , Eq. (18) can thereby be rewritten as ...’.

*Comments---9530(21): it should be mentioned that this approximate equation is derived from equation (2). Furthermore, it seems to be dimensionally incorrect (even if we assume that  $P$  and  $N$  have the same unit): the left hand side has is in  $\text{mg m}^{-3} \text{s}^{-1}$  while the right hand side is in  $\text{mg m}^{-2} \text{s}^{-1}$ .*

**Response:** This approximate equation is derived from equation (2), and Equation (A1) has been modified as  $\int (\mu_m \min(f(I), g(N)) - \alpha\varepsilon) P(z) dz \approx K_{v2} \frac{dN(z)}{dz} |_z$ .

9512(12): “but independence of” should be “but independent of”.

9512(16): “parameters difficultly obtained from on-site observations” should probably better be “parameters which are difficult to observe on site”.

9512(24): “SCM is commonly believed” should be “An SCM is commonly believed” or “SCMs are commonly believed”.

9513(02): “reason forming” should be “reason for forming”.

9513(12): “SCM has been attracted” should be “The SCM has attracted”.

9513(25): “thickness of SCML” should be “thickness of the SCML” or “thickness of SCMLs”.

9514(03): “variations of environment parameters” should be “variations of environmental parameters”.

9518(20): “stead” should be “steady”.

9518(23): the factor of the first term on the right hand side should be  $-K_{v2}/\sigma^4$ , instead of  $-K_{v2}/\sigma^2$ .

9519(21): “the depth of the SCML must occur below or equal to the depth for phytoplankton having the maximum growth rate” should be something like “the depth of the SCML maximum lies at or below the depth of maximum phytoplankton maximum growth”.

9520(03): *“the numerical modelling can support” should be “numerical modeling results support”.*

9520(19): *“et al.” should only be used in the context of unnamed co-authors of a paper, not instead of “etc.”. Besides, it would be better to give a complete list here (as it is not too long).*

9522(18): *“provided” should be “provide”.*

9523(04): *“appendixes” should be “appendices”.*

9523(19): *“it is not surprised” should be “it is not surprising”.*

9524(14): *with “should be identified by  $\mu_m/2$ ” the authors probably mean “depends on  $\mu_m/2$ ”.*

9524(16): *“half-saturate constant” should be “half-saturation constant”.*

9524(19): *“In summarize” should be “In summary” or “To summarize”.*

9525(19): *“can be infered” should be “can be inferred”.*

9525(01): *“is constant with varying surface light intensity” simpler “does not depend on surface light intensity”.*

9528(15): *“the thickness of SCML thickens” should be “the thickness of the SCML increases” or “The SCML thickens”.*

**Response:** Many thanks for your detailed correction. The revised manuscript will be edited using the service provided by Elsevier WebShop English language editing. Please see the revision.

**Reply to Reviewer:** Emmanuel Boss, University of Maine

*This paper deals with an important problem, the distribution of phytoplankton in the upper ocean. A kinematic distribution is assumed which is forced on a dynamical set of equation so that parameters associated with the profile could be inferred.*

*I find the paper of interest, including novel results. The paper is, in general, clearly written, however it is ridden with English mistakes. I urge the writers to consult with an English native speaker before submitting a final version.*

*I am in favor of publishing this paper, but I have some major comments that I feel, if addressed, can clearly improve this paper's utility.*

**Response:** We are very grateful for the suggestions and comments. The revised manuscript will be edited using the service provided by Elsevier WebShop English language editing. Please see the revision.

**Comments---** *The kinematic solution assumed (a Gaussian, eq. 7) is not an exact solution of the dynamical equations (1-2) used (even at steady state). At best, it is an approximation. This needs to be clearly spelled out. For example, you should substitute the solution(s) you get into the ODE (1-2) and see how well the terms balance each other (or how small the residuals are relative to the sizes of each terms). Best to do it after appropriate non-dimensionalization of the equations.*

**Response:** Agree. The Gaussian function of vertical Chl *a* profile is, at best, an approximate solution for Eqs. (1-2), and we will spell out this point in the newly added Section 4.2, please see the revision. After nominating the values of model parameter (please see Table 2) and substituting the solutions we get into ODE (1-2), we find that at depth of SCML the dominant balance is between growth of phytoplankton and vertical eddy diffusion.

**Comments---** *The assumption should be clearly spelled out, including their limitation. As a start, the continuous profile of phytoplankton assumed is clearly not consistent with a piecewise eddy-diffusion coefficient. The surface concentration of chlorophyll is nowhere zero in the ocean, and if diffusion in the ML is indeed sufficient to homogenize it (as assumed) the phytoplankton function could not have a continuous derivative across the boundary between the two diffusivities (as assumed – the only case where it may work is if the vertical derivative of *P* is identically zero at the transition between the diffusivities). It is assumed that the maxima is significantly deeper than the base of the ML – it is therefore not surprising  $k_{v,1}$  plays no role in the solution, and in fact you will obtain the same kinematic solution if you simply used a single constant eddy diffusion coefficient for the whole water column. You neglect photo-acclimation and assume  $Chl\_a = \text{phytoplankton}$  – this is a significant simplification as it is well known that phytoplankton increase inter-cellular pigment concentration when light level decrease (e.g. Fennel and Boss, 2003).*

**Response:** We will spell out the assumption and limitation by adding a new Section 4.2 in the revision. Please see the revised version. We agree that  $K_{v,1}$  plays no role on SCM, and the corresponding results will be deleted in the revision.



*Comments---* The treatment of grazing loss, is, in the least, an over simplification (yes, Fennel and Boss, 2003, used a similar one). Grazing loss depends strongly on concentration (it is an encounter based process) and, given that zooplankton can move, or, in the least, grow faster where more food is available, are unlikely to have a constant concentration distribution (which is assumed for a constant epsilon). I realize that accounting for it will cause the equations to become nonlinear, and probably non-solvable, but mentioning this limitation is needed. This assumption is in the heart of the Sverdrup's critical depth model, which we now understand to have significant limitations.

**Response:** Agree. We will spell out the assumption and limitation by adding a new Section 4.2 in the revision. Please see the paper.

*Comments---* You claim (e.g. Appendix B) that nutrient limitation is required to get a SCM. In Fennel and Boss, 2003 we found, we similar equation, that we do get it with saturating Nutrient (in this case vertical velocity is required).

**Response:** Many thanks to Dr. Boss for noticing this question. Just as Fennel and Boss (2003), in nutrient-saturated case, to get a SCM the vertical velocity is required, because the primary importance of sinking in the formation of a SCM lies in its nutrient-depleting effect on the surface layer (Hodges and Rudnick, 2004). Ryabov et al. (2010) simulated the formation of a SCM by starting with an initial nutrient rich system. They first observed a rapid formation of a transient phytoplankton maximum close to the surface. This phytoplankton profile is, however, not stable. With the depletion of the nutrient in the surface layer the production layer, i.e., the layer where the growth rate is larger than the loss rate of phytoplankton, shifts downwards, until the system reaches a stable DCM configuration. Mellard et al. (2011) analytically derived that in equilibrium nutrient limitation in the surface mixed layer is required to get a SCM. Thus, the sentence in the revised Appendix B will be modified as 'Note that we have known that the stable SCML occurs only when the growth of phytoplankton within the surface mixed layer is nutrient-limited (Mellard et al., 2011; Ryabov et al., 2010), ...'.

*Comments---* We are still far from the days when we can use remote sensing to get a phytoplankton profile (unless using empirical parameterization such as in the works of Uitz or Westberry). To start, you could suggest field experiments (e.g. grazing, primary productivity, and measurements of turbulence) that could test if your results are consistent with reality (rather than assume that your model captures reality). Models are always approximations. Testing these approximations is required before we can assume they apply in the field.

**Response:** In the revision, we point out the requirement of field experiments for testing whether our results are consistent with reality. In addition, encouraging by Reviewer 3 and Jaime Pitarch we have tried to apply our theoretical results to three time-series stations in different regions, i.e., the South East Asia Time-series Station (SEATS) in the South China Sea, the Hawaii Ocean Time-series (HOT) station, and the Bermuda Atlantic Time-Series Study (BATS) site, please see the revision.

**Minor comments:**

p. 9515, l. 11: these equation do not include the 'fundamental physical and biological processes', as best they are judicious simplification created to make this extremely complex problem tractable. Upper ocean dynamics exhibit many processes (turbulence, internal waves, storms, slant-wise and vertical convection) which are not captured in the model (a constant eddy diffusion coefficient, basically assuming a constant  $N^2$ , e.g. Gargett, 1984). Similarly the biological representation is extremely limited (some may call it simplistic); the microbial loop is represented by  $\alpha$ , all the loss process, but sinking, are assumed to be linearly proportional to phytoplankton concentration. Constant sinking velocity. Temperature plays no role. Spelling it out does not diminish from your results but makes sure that the reader does take it with a grain of salt.

**Response:** In the revision, we will delete this sentence 'fundamental physical and biological processes', and will spell out the assumptions in the newly added Section 4.2.

p. 9516

Eq. 5 - This is another approximation (e.g. Morel, 1988, JGR). Even in a layer of constant optical properties,  $k_d$  varies with depth, particularly near the surface (due to sun angle and equilibration between losses to absorption and redistribution of light by scattering). You assume in your model that  $k_d$  is not a function of  $P$  hence you neglect 'self-shading' (another assumption).

**Response:** We will point out the approximation and the assumption in the revision.

Eq. 6 – you do not require continuous flux between your two layers, which you should (and which will not be consistent with your profile).

**Response:** Thank you for this suggestion. To consist with the Gaussian profile, a gradual transition from one area to another written in terms of a generalized Fermi function (Ryabov et al., 2010) will be added, that is,  $K_v(z) = K_{v2} + \frac{K_{v1} - K_{v2}}{1 + e^{(z - z_s)/l}}$ , where parameter  $l$  characterizes the width of the transient layer. In our study, we assumed this transient layer is infinitely thin.

Eq. 7 – suggestion: why not use a single amplitude ( $A$ , or  $P_{max} = h/\sigma \sqrt{2\pi}$ )? It will simplify the reading of the manuscript. In the least change  $h$  (often used to denote layer depth) with  $int\_P$  or something else which will make the reading of the paper easier.

**Response:** We will adopt a single amplitude, i.e.,  $P_{max} = h/\sigma \sqrt{2\pi}$  in the revision.

p. 9518, Eq. 10: checking units I find them inconsistent between the left and right side of the equation and hence this equation is wrong.

**Response:** Sorry for the typo. The factor of the first term on the right hand side should be  $-K_{v2}/\sigma^4$ , instead of  $-K_{v2}/\sigma^2$ .

*p. 9520, l. 27: 'the popular compensation depth' is only sound within the assumption of its model. Since, like you, Sverdrup assumed a constant epsilon, it is not surprising you find similar results. This does not validate your or Sverdrup approach wrt to ocean ecology. In particular the treatment of grazing (a constant epsilon throughout the water column which is independent of phytoplankton concentration) is lacking in your (and Sverdrup's) approach. See Behrenfeld and Boss, 2014, for a review of this issue.*

**Response:** According to the review paper on this issue (Behrenfeld and Boss, 2014), we will rewrite this paragraph, please see the revision.

*p. 9521, l. 13: This condition is identical with Eq. 4a of Fennel and Boss when vertical sinking is constant as function of depth.*

**Response:** Thank you for this suggestion. This will be added in the revision.

*p. 9523, l. 21-22: It is by design (having the SCM be much deeper than the surface ML) that  $k_{v1}$  has no influence on  $\sigma$ . No surprise there.*

**Response:** We will delete this sentence in the revision.

*p. 9525, l. 3: nothing 'dramatical' with logarithmic functions. They increase much slower than exponential or power-law functions.*

**Response:** We will delete this word in the revision.

*p. 9526, Eq. 26 is identical to that of Fennel and Boss, 2003, for constant settling velocity.*

**Response:** We will add this sentence in the revision.

*p. 9514*

*l. 3: environment -> environmental.*

*l. 15, 18: 'the infinite assumption' – not clear. I think you refer to the 'delta-function' layer.*

*l. 25: 'etc' does not belong there. Remove it.*

*p. 9518*

*l. 7: 'where is the balance...' should read 'where there is a balance...'*

*l. 8: delete 'carefully' and add and 's' to 'reflect'.*

*l. 11: not clear what you mean.*

*l. 20: should be 'steady'*

*p. 9520*

*l. 5: replace 'obviously' with 'identically'.*

*l. 24: no 'etc', and since you called epsilon the loss-rate and described it above, there is no need to, again, describe the processes causing loss.*

*p. 9521*

*l. 10: I think you mean 'requiring a positive solution' rather than 'According to the*

*property of the logarithm function*'.

**Response:** Many thanks for your detailed corrections. The grammatical errors have been corrected in the revision; meanwhile this paper will be edited using the service provided by Elsevier WebShop English language editing. Please see the revision.

### **Reply to Anonymous Referee #3**

#### **General Comments:**

*This study presents an interesting analysis of features of the sub-surface chlorophyll max and how they depend on environmental parameters. Given that the sub-surface chlorophyll max is a ubiquitous feature in the ocean and has implications for planktonic ecosystem processes, the results of this study are an important contribution to the field.*

*The authors do a good job in the introduction of highlighting what previous work has been done in this area theoretically, and what the specific contribution of this study is.*

*The results of the study are in general well presented and well organized, and many of the results provide important advancements in our conceptual understanding of what controls the sub-surface chlorophyll max. However, the two major weaknesses of this study, which should be addressed before I can recommend publication are: (1) a more explicit connection needs to be made between the theoretical results of this study and its applications in the field and (2) the entire paper suffers from grammatical errors. For the latter point, I have provided as many corrections as I could in the technical comments below, but the authors need to have a native English speaker carefully read this paper for more thorough editing. For the first point, I have made a few suggestions below for how the applications of this study for the field can be incorporated. With addressing these comments, I believe the paper will be much stronger and a great addition to the literature on this topic.*

**Response:** We thank the helpful suggestions and comments. We will add a new Section 4.2 to link the theoretical results of this study and its applications in the field. This revised manuscript will be edited using the service provided by Elsevier WebShop English language editing. Please see the revision.

#### **Specific Comments:**

*1. The part of this paper with the most potential for expanding the applicability beyond theory is in the results when it is discussed how this model can be coupled with satellite data (pg. 9522, line 18-19 of the Results). This is an interesting potential application of this type of model down the line (although as E. Boss points out, we are far from being able to obtain phytoplankton profiles from satellites.) Right now, this text is misplaced (in the Results) given that this analysis was not actually done. It would definitely strengthen the paper and make the applications of this model to the natural environment much more clear if the authors ran a quick analysis with some satellite data and some parameters from previously published field studies (to obtain  $w$ ,  $K_v$ , etc.). Even though assumptions would be made, this type of quick analysis would give some idea of how real-world data could be incorporated into the model and thus be applied to the field. A comparison of the model results (in terms of the thickness, depth, and intensity of SCML) could be shown for different regions of the ocean and displayed in a new figure.*

**Response:** Thank you for the suggestion. Combining some satellite data ( $K_d$ ,  $I_0$ ) and

parameters obtained from previously published field studies (to obtain  $w$ ,  $K_v$ , etc.), we will explore the applications of this model to three time-series stations in different regions, i.e., the South East Asia Time-series Station (SEATS) in the South China Sea, the Hawaii Ocean Time-series (HOT) station, and the Bermuda Atlantic Time-Series Study (BATS) site, please see the newly added Section 4.2 in the revision. Meanwhile, a comparison of the model results (in terms of the thickness, depth, and intensity of SCML) will be shown for the three different regions (Figure 2).

*2. Another way the message of this paper could be strengthened, particularly for less mathematically-inclined readers, is for some of the important results to be reiterated in more intuitive terms in the discussion. Right now the discussion is largely more analysis, but I think there is an opportunity to re-emphasize some of the important points that were only briefly mentioned in the results. For example, it would be great to describe in non-mathematical terms, the conditions necessary for the existence of SCM (from section 3.1 in Results), which is very interesting but could be missed by many readers. Another important result that should be highlighted is the derivation of  $h$  and  $P_{max}$  (as shown in section 3.1 in Results) is irrespective of the form of the growth limiting function. Since functional forms in phytoplankton models are still debated in the literature, this is an interesting finding and the implications of it should be described more in the discussion.*

**Response:** We will add a new Section 4.2 in the revised version to enhance discussion in terms of important points. For example, the conditions necessary for the existence of SCM will be re-emphasized in non-mathematical terms, i.e., ‘Our model suggests that the necessary condition for the existence of SCM is the growth rate under the limitation of sea surface light intensity is larger than the loss rate in stratified water columns. This condition is identical with the result given by Fennel and Boss (2003) when vertical sinking is constant as a function of depth. This result indicates that in stratified water columns whether or not the SCM occurring has no relation with the sinking velocity of phytoplankton and the vertical diffusivity. Many numerical studies have reproduced the SCM phenomenon, of which the condition of SCM occurrence were met with variable values of the sinking velocity of phytoplankton and the mixing diffusivity (Huisman et al., 2006; Klausmeier and Litchman, 2001; Mellard et al., 2011).’.

*3. I think the results could be better illustrated through some improvements to Figure 1. The concept behind Figure 1 I believe is very strong, but I think it would help tie the paper together more if some of the results were incorporated into the figure. For example, including the various depths in the figure ( $z_m$ ,  $z_0$ ,  $z_{c1}$ ,  $z_{c2}$ ) will help make these parameters more intuitive for the readers and showing where they are located in different situations (perhaps making three separate panels for the different scenarios considered?). The other note is that I believe “light-limitation” and “nutrient-limitation” are switched in the figure.*

**Response:** Agree. In the revision, we will incorporate the various depths ( $z_m$ ,  $z_0$ ,  $z_{c1}$ ,  $z_{c2}$ ), as well as  $\sigma$ , in Figure 1. The notes of light-limitation and nutrient-limitation will

be replaced with the marks of  $f(I)$  and  $g(N)$  in Figure 1 to avoid confusion.

*4. One last note is I think the authors should re-think about the placement of some of the text in different sections – right now it seems like some of the statements in the methods and results belong in the discussion and much of the discussion belongs in the results. For example, the paragraph (starting on line 4 of Pg. 9523 in the Results) belongs in the Discussion since it highlights the potential importance of this study, but no actual results are given. However, I think the Summary is very well-written and does a great job of emphasizing the importance of this work.*

**Response:** We will reorganize a few parts in Method, Results and Discussion, please see the revision. For example, the paragraph (starting on line 4 of Pg. 9523 in the Results) has been moved to the newly added Section 4.2.

***Technical (mostly grammatical) Comments:***

*Pg. 9519, line 4-6: it might help (particularly for less mathematically-inclined readers) to define the compensation depths in words so that the following argument about the location of maximum phytoplankton growth is clear:*

**Response:** Thank you for the suggestion. We will add the definition of the compensation depths in the revision, i.e., ‘Clearly,  $\mu_m \min(f(I), g(N)) - \varepsilon > 0$ , in the interval  $(z_{c1}, z_{c2})$ . This indicates that subsurface net production occurs only between the two compensation depths where the growth rate  $\mu_m \min(f(I), g(N))$  equals the loss rate  $\varepsilon$ . Beckmann and Hense (2007) found similar results by numerical model.’

*Pg. 9519, line 15-19: it is unclear how this discussion relates to the previous part of the paragraph.*

**Response:** This paragraph will be rewritten as: ‘We define  $T = \sigma^2 / K_{v2}$  as the characteristic vertical mixing time scale in half of the SCML thickness (Gabric and Parslow, 1989; Bowdon, 1985). Let the length scale be  $L = 2 K_{v2} / w$ , which determines the scale height of the phytoplankton distribution (Ghosal and Mandre, 2003). Thus, the right hand terms of Eq. (13) can be rewritten as  $1/T + w/(2L)$ . In other words, the maximum net growth rate of phytoplankton,  $\max(\mu_m \min(f(I), g(N)) - \varepsilon)$ , is determined by the vertical mixing time scale ( $T$ ) and the time taken by a phytoplankton sinking ( $w$ ) through lengths ( $2L$ ).’

*Note that “nutrients” should be plural throughout when used as a noun.*

*Abstract, line 1-2: should be “referred to”*

*Abstract, line 9: should be “phytoplankton located at”*

*Abstract, line 12: should be “but independent of”*

*Abstract, line 14: “shrunk”*

*Abstract, line 16: should be “parameters that are difficult to obtain from”*

*Pg. 9512, line 21: should be “conventionally referred to as”*

*Pg. 9512, line 24-25: “regions”*

*Pg. 9512, line 26: “with vertically”*

Pg. 9513, line 7: "and was thin"

Pg. 9513, line 10: "Chl a was relatively low"

Pg. 9513, line 12: "SCM has attracted"

Pg. 9514, line 3: "variations in environmental parameters"

Pg. 9514, line 11: "for limiting nutrients and light"

Pg. 9514, line 24: remove "etc." – too vague.

Pg. 9515, line 21: "Kv depends on depth in the following way"

Pg. 9516, line 6: "light and nutrients"

Pg. 9516, line 7: "if both the light limiting term"

Pg. 9516, line 9: "Because of absorption and self-shading"

Pg. 9516, line 12: "surface light intensity and Kd is the light"

Pg. 9516, line 15: "of the water column"

Pg. 9516, line 16: "white nutrients are replenished"

Pg. 9517, line 9: "between two locations"

Pg. 9517, line 10: "where Chl a is a certain fraction"

Pg. 9517, line 13: remove "respectively"

Pg. 9517, line 19: "which were located at the depths"

Pg. 9517, line 20-21: this sentence is unclear, remove.

Pg. 9518, line 6-8: sentence needs to be rewritten, very unclear

Pg. 9518, line 10-11: rewrite as: "As described in eq (7), the depth of the SCML is defined as  $z_m$ , that is, the location of the point-wise maximum value of Chl a."

Pg. 9518, line 15-17: rewrite, not correct grammatically and not clear

Pg. 9518, line 17-18: "Gaussian function of the vertical"

Pg. 9518, line 20: "with the steady-state version of Eq. (1)"

Pg. 9518, line 22 "follows"

Pg. 9519, line 1: "Letting"

Pg. 9519, line 5: "are located"

Pg. 9519, line 12-13: needs to be rewritten

Pg. 9519, line 13-14: "We define  $T = \sigma^2 / K_v^2$  as the characteristic.."

Pg. 9520, line 1: "have supported this"

Pg. 9520, line 3: remove "the" before "numerical modeling"

Pg. 9520, line 5: "used to solve for the"

Pg. 9520, line 8-9: "is at the location of equal limitation by nutrients and light"

Pg. 9520, line 11: "of SCML is located where phytoplankton growth is limited by light"

Pg. 9520, line 23: "equals the loss rate"

Pg. 9520, line 25: "equals the loss rate"

Pg. 9521, line 5: "into Eqs. (14).."

Pg. 9522, line 16: "the water column"



*Pg. 9522, line 17: “a similar result”*

*Pg. 9523, line 19: “it is not surprising”*

*Pg. 9523, line 23: “many numerical modeling studies”*

*Pg. 9526, line 6-8: I would reword to be more clear “Equation (18) indicates that the parameter sigma is affected by changes in the vertical diffusivity: : :”*

*Pg. 9526, line 24: “phytoplankton is equal to the loss rate”*

*Pg. 9527, line 4: “a similar result”*

*Pg. 9527, line 15: “The second special situation occurs when the term: : :”*

*Pg. 9528, line 5: “is located at”*

*Pg. 9528, line 9: “in the case of”*

*Pg. 9528, line 21: “The third special situation occurs when ..”*

*Pg. 9530, line 5: “regions dominated by non-sinking phytoplankton”*

*Pg. 9530, line 8: “potential risk of climate change”*

**Response:** Many thanks for your detailed corrections. The grammatical errors have been corrected in the revision; meanwhile the revised manuscript will be edited using the service provided by Elsevier WebShop English language editing. Please see the revision.

Reply to J. Pitarch Jaime. PitarchPortero@artov.isac.cnr.it

*Dear all: I see that the possibility (or not) to retrieve the vertical distribution of a water constituent has been mentioned in this paper and its following discussion. If I understand it well, the author finds analytical expressions relating the gaussian parameters. He suggests the link to the remotely-sensed chl<sub>a</sub>. Posteriorly, E. Boss replies that such goal is far from being achieved (in practice?) In a recently published paper, we show that the remote sensing reflectance is sensitive to the gaussian parameters in a way that the latter can be retrieved when the former is measured. It is a model study where we set the theoretical basis. I think it can add some light to this discussion:*

*Jaime Pitarch, Daniel Odermatt, Marcin Kawka, and Alfred Wüest, "Retrieval of vertical particle concentration profiles by optical remote sensing: a model study," Opt. Express 22, A947-A959 (2014)*

<http://www.opticsinfobase.org/oe/abstract.cfm?URI=oe-22-S3-A947>

**Response:** Thank you very much for your kind comments. In the revised version, we cite the paper and try to apply our theoretical results to retrieve the Gaussian profile of Chl *a* at three time-series stations in different regions, i.e., the South East Asia Time-series Station (SEATS) in the South China Sea, the Hawaii Ocean Time-series (HOT) station, and the Bermuda Atlantic Time-Series Study (BATS) site. Please see our revised manuscript.