Biogeosciences
Discussions

Interactive comment on "Dynamics of global atmospheric CO_2 concentration from 1850 to 2010: a linear approximation" *by* W. Wang and R. Nemani

I G Enting (Referee)

ian.g.enting@gmail.com

Overview: This paper by Wang and Nemani (denoted W&N) discusses: (i) linear representation of carbon cycle behaviour (in a superficial manner), (ii) gives an illustrative model from which misleading generalisations are drawn (iii) discusses the influence of temperature on CO_2 fluxes, using an analysis based on applying flawed logic to a model that exhibits a gross failure to agree with the observed CO_2-temperature relations.

Linear modelling Linear models of the carbon cycle go back to the earliest days of carbon cycle modelling. Discussions of the generic implications of being able to do such modelling also go back many decades. Some of the results are:

- Any such linear model can be represented in terms of an impulse response function $R(.)$ as $A'(t) = \int^t R(t - t') \, \dot{E}(t') \, dt'$ (Oeschger and Heimann, 1983).

- Consequently any model-derived 'carbon budget' (i.e. the partitioning of atmospheric fluxes) will depend on the past history of changes.

- Laplace transform analysis can provide a convenient way of discussing response function representations of the carbon cycle (Enting, 2007, and references therein), e.g. in describing how to get a combined response function by combining response functions for subsystems (Enting et al., 1994). The Laplace transform analysis can be related to the W&N by the noting that the airborne fraction γ for an emission growth rate p is given by $\gamma(p) = pR^*(p)$ where $R^*(p)$ is the Laplace transform of $R(t)$.

- In particular, for an exponential growth on E as $E(t) \propto \exp(pt)$ leading to $A(t) \propto \exp(pt)$ means that all that we know about $R(t)$ is the value of its Laplace transform, $R^*(p)$, at one particular value, $p \approx 0.02$. Many functions can be fitted to pass through this one point (see for example Enting, 2007, Fig 2). (In terms of $R(t)$, what we can say, without explicit reference to Laplace transforms, is that all we know about the function is one time-weighted moment.) Thus the data from ice cores giving $A(t)$ for t prior to 1958 has not greatly changed the earlier situation where most information about the dynamics of the carbon cycle came from isotopic studies (see for example Broecker et al., 1980). (A recent study, motived by the role of $R(t)$ in defining global warming potentials, aimed to quantify how well $R(t)$ is known (Joos et al., 2013). This paper is cited by W&N, but they ignore the significance.)

- The Laplace transform analysis has also been extended to give a generic description of the carbon cycle coupled to temperature changes Enting (2010).

(Note that these examples are chosen for convenience to illustrate a very large body of

work and do not neccessarily represent priority of publication. Other examples of linear modelling of CO_2 include studies, e.g. by P. Young and colleagues at the University of Lancaster, using time series approaches.)

The 'two-box' example The present paper describes a particular case. Notionally it is given as an illustration but:

- Since the mathematical result is simple and well known, there is really no justification for giving such an illustration in a research paper (as opposed to an introductory textbook);

- In practice, W&N try to use the model to make specific deductions about the carbon cycle.

An alternative way of thinking about this is that the relation $A'(t) = \int^t R(t - t') \, \dot{E}(t') \, dt'$ defines two 'inverse problems': (i) deduce $E(t)$ given $A(t)$ and $R(t)$ − this is often called deconvolution; and (ii) deduce $R(t)$ given $A(t)$ and $E(t)$ which corresponds to model calibration. Evans and Stark (2002) have noted that non-parametric statistical techniques are the appropriate way to analyse such inverse problems.

The effect of temperature: The influence of temperature on CO_2 is parameterised as a flux $\beta_T T'$ with an estimated $\beta_T \approx 1.64$ ppm yr^{-1} °C^{-1}, based on fitting interannual variations. The authors sole justification for their model is that 'the same temperature-CO_2 coupling **may** [IGE emphasis] also operate on longer time scales'. This claim seems highly implausible given the relation between CO_2 and temperature through the little ice age (see for example Scheffer et al., 2006). A depression of temperature by say $0.5°$ (or more) for a century or two did not lead to a CO_2 reduction of 40 to 80 ppm (assumimg $\gamma \approx 0.5$).

The inadequate model is then applied (see W&N fig 4) in a logically inconsistent way. The limitations of the W&N analysis can be seen if one takes the mass-balance relation,
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in terms of a set of fluxes, Φ_j

$$\dot{E}(t) = \dot{A}(t) + \sum_j \Phi_j(t)$$

and divides through by $\dot{A}(t)$

$$\frac{1}{\gamma_{\mathsf{obs}}} = 1 + \sum_j \Phi_j(t)/\dot{A}(t) = 1 + \sum_j \mu_j(t)$$

where the factors μ_j, which will be constant for the case of exponential emissions growth, describe the partitioning of fluxes, or in terms of Laplace transforms:

$$\frac{1}{\gamma_{\mathsf{obs}}(p)} == 1 + \sum_j \mu_j^*(p)$$

(This generalises the analysis where only oceanic and terrestrial fluxes are included, as given by Enting (2007) who noted that the result of constant μ_j in linear systems was obtained by Oeschger and co-workers in 1980).

Various models will differ in how the fluxes are combined (or conversely the degree of detail in which fluxes are described). However, regardless of the way in which fluxes are modelled, the sum has to include each flux once and only once.

- What is not valid is the analysis given by W&N which splits the fluxes into two sets (X and Y in generic terms)

$$\frac{1}{\gamma_{\mathsf{obs}}(p)} == 1 + \sum_{j \in X} \mu_j^*(p) + \sum_{j \in Y} \mu_j^*(p)$$

- then use γ to fit only the terms corresponding to $\sum_{j \in X} \mu_j^*(p)$
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- and consequently claim $\sum_{j \in Y} \mu_j^*(p) = 0$ (i.e. the 'fertilisation' balances the 'temperature feedback' term).

(As examples of how simplified models may end up lumping multiple processes, it has long been recognised that some of the uptake that is modeled as 'CO_2 fertilisation' **may** in fact be due to co-varying nitrogen deposition. More directly relevant to the W&N analysis is the observation by Enting (2010) that regardless of whether or not a model explicitly includes temperature to CO_2 feedback processes, calibrating models against 20th century CO_2, when these has been a co-varying temperature increase, will mean that the effects of temperature to CO_2 feedback processes will still be captured in the calibration.

Similarly the W&N extension of the Bern model seeks to attribute all the requisite net uptake to fertilisation, rather than query their enhanced respiration and/or include direct temperature enhancement of NPP.

As a final point, referring to $\dot{E}' - \dot{A}' + \beta_T T'$ as a 'gross flux' is a major departure from standard terminology. In the vast majority of carbon cycle studies, the term 'gross flux' refers to something like $(A(t_0) + A')\alpha_A$.

Other technical errors: It is simply false to claim that 'the full potential on anthropogenic CO_2 emissions for changing the climate has not yet been reached is because only 41–45% of the CO_2 emitted between 1850 and 2010 remains in the atmosphere ...'. The reason that 'the full potential on anthropogenic CO_2 emissions for changing the climate has not yet been reached' is that the oceans are not yet in thermal equilibrium with the atmosphere and so are acting as a net sink of heat. Of lesser importance is the erroneous claim that 'the only eigenvalue is $\lambda = \alpha_A + \alpha_S$ (4)' — it should read 'the only non-zero eigenvalue is ...'

Summary: This paper adds nothing to an extensive literature on linear analysis of the carbon cycle. This paper is not suitable for publication in *Biogeosciences*.

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Notation

Quantities used in the Wang and Nemani paper are denoted by *. Following W&N, the prime, as in A', denotes perturbations from equilibrium and the dot denotes time derivatives.

A' Perturbation in atmospheric CO_2. *

$E(t)$ Cumulative anthropogenic CO_2 emissions. *

p Inverse time variable used in the Laplace transform.

$R(t)$ Impulse response function describing the carbon cycle – the response of concentrations to emissions.

$R^*(p)$ Laplace transform of $R(t)$.

t Time. *

T' Temperature perturbation.

α_A Rate coefficient in two-box model. *

α_S Rate coefficient in two-box model. *

β_T Coefficient in two-box model. *

γ Airborne fraction of CO_2. *

$\gamma(p)$ Airborne fraction of CO_2 for growth rate p.

$\mu_j(t)$ Partition factor (a.k.a. dilution factor) for flux Φ_j relative to the atmospheric increase. Constant for linear system with exponential increase.

$\Phi_j(t)$ Generic (non-anthropogenic) flux from atmosphere.

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