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Interactive comment on "Dynamics of global atmospheric CO_2 concentration from 1850 to 2010: a linear approximation" *by* W. Wang and R. Nemani

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The main criticism to our paper (i.e., W&N2014) in the second-around comments by Dr. Enting and the comments by another anonymous reviewer focuses on the mathematical simplicity of the used two-box model and the scientific significance of the results. We certainly agree with the reviewers on the use of simple models in our analysis. However, as we explained in our previous communication with Dr. Enting, the mathematical presentation of our modeling framework in W&N2014 was indeed purposely made simple in order to keep a clear physical picture of the methodology. We also argued that the simple presentation of our methodology does not compromise the scientific rigor and significance of our findings. Here we would like to extend our discussions on this matter in more details. The outline of our arguments is as follows:

1. The scheme to quantify global temperature's effects on atmospheric CO_2 concentration across interannual to centennial time scales in W&N2014 is the first time in the literature.
2. The correct representation of temperature's effects on atmospheric CO_2 significantly improves the linear models to achieve high accuracy in approximating the dynamic characteristics of the global carbon cycle under the investigation. Unlike previous studies, for instance, W&N2014 does not need to introduce additional nonlinearity in our model to explain the observed variations/changes of atmospheric CO_2 concentration in the past 160 or more years.
3. The results of W&N2014 provide independent evidence and/or new clues to address some of current research foci in the field, including the questions regarding the declining efficiency of natural carbon sinks as well as the intensification of the global carbon cycle.
4. As claimed in W&N2014, our modeling results (e.g., those regarding the characteristic disturbance responses of atmospheric CO_2) are not restricted to the two-box case but applicable to general situations. Here we give the mathematical proof.

1 The scientific significance of W&N2014

1.1 Quantification of temperature's effect on atmospheric CO_2 concentration

The literature has long recognized that temperature's effect on atmospheric CO_2 concentration varies at different time scales (Woodwell et al. 1998), but to quantify such effect by a simple scheme is thought to be difficult (Sheffer et al. 2006). Therefore, previous studies used to evaluate the effect at long-term (e.g., Rafelski et al. 2009) and short-term (e.g., Adams and Piovesan 2005; Wang et al. 2013) scales separately.

To the limit of our knowledge, W&N2014 is the first study in the literature to show that the effects of surface temperature on the atmospheric CO_2 from interannual to centennial time scales can be consistently quantified by a single sensitivity parameter (β_T) and its interactions with the e-folding time constants of the atmosphere and the surface carbon reservoirs. As shown in W&N2014, this scheme is highly consistent with the observed co-varying relationships between temperature and atmospheric CO_2 across various time scales in the past 160 years. In addition, the scheme can also successfully explain the depression of atmospheric CO_2 during the Little Ice Age (see our previous responses to Dr. Enting's comments).

1.2 The accuracy of linear approximation of global carbon cycle

Linear approximation is an old art, which (as the reviewers correctly pointed out) has been used in many previous studies in our field. On the other hand, it remains a challenging question regarding the accuracy of linear methods in general (i.e. how well can linear models approximate the global carbon cycle), for our understanding of the climate-carbon system is only limited. Because many individual processes (e.g., ocean carbonate chemistry) of the carbon system are nonlinear in nature, not surprisingly, as the reviewers indicated, recent studies on the subject tend to seek nonlinear representation in their models (e.g., Joos et al. 1996). However, this new research trend by no means indicates that the published literature has exploited all the possibilities of linear approximation.

Indeed, a unique contribution of W&N2014 on the subject is to show that, once the effect of temperature on the carbon cycle is appropriately accounted for, the dynamics of the atmospheric CO2 concentration in the past 160 (or more) years exhibit linear characteristics to a surprisingly high extent ($r^2 > 0.9$, Figs. 2 and 4 of W&N2014). No previous linear approximation in the literature (to our knowledge) has achieved this accuracy before. This finding also significantly simplifies the modeling efforts required to analyze the dynamic characteristics of the atmospheric CO_2 concentration

under the investigation scope of W&N2014, rendering important results to some of the current research foci of carbon-cycle science (see below).

1.3 The decreasing net efficiency of global carbon sinks and the intensification of the global carbon cycle

It has been a research focus in the past a few years to determine whether the strength of natural carbon sinks has declined since the late 1950 (i.e., the "Keeling era"). For instance, there was a famous debate over whether there is a detectable increasing trend in the airborne fraction (AF) of anthropogenic CO_2 emissions (Raupach et al. 2008; Knorr 2009). More recent studies have realized that AF can be influenced by other factors (e.g., Gloor et al. 2010; Raupach et al. 2013) and therefore may not be an ideal indicator for the efficiency of global carbon sinks. In particular, by calculating the so-called "CO_2 sink rate", for the first time Raupach et al. (2013) suggests that the efficiency of global carbon sinks has decreased by about 1/3 between 1959-2014. Here, as a totally independent research, W&N2014 also found that the "net" efficiency of global carbon sinks may have decreased by 30% during the same time period, consistent with the results of Raupach et al. (2013). We believe that W&N2014 is second only to Raupach et al. (2013) on this important finding. We noticed that the two studies came to the same conclusion from different approaches. For instance, Raupach et al. 2013 reasons that changes in climate only explain 20% of the deduction of the declining efficiency of the surface carbon sinks, but W&N2014 suggests the changing climate (the warming temperatures) is the dominant reason. However, to comprehensively compare between Raupach et al. 2013 and W&N2014 is beyond the scope of this study and needs to be done in the future.

Although the "net" efficiency of the global carbon sinks is decreasing, the absolute size of the surface sinks is still increasing (Ballantyne et al. 2012). This fact, along with other latest observational evidence (e.g., Graven et al. 2013), indicates the intensification of the global carbon cycle: on one hand, atmospheric CO_2 is sequestered by surface

carbon pools at a faster rate; on the other hand, carbon is increasingly released by warmer temperature from the surface carbon pools back to the atmosphere. A recent study by Zeng et al. (2014) suggests that human activities (the "Green Revolution") may also significantly contribute to the phenomenon. However, to separate natural and anthropogenic influence on the global carbon cycle requires the quantification of their mutual uncertainties. We believe that the findings of W&N2014 provide new clues to re-evaluate the temperature's effect on atmospheric CO_2 as well as the (fertilization) effect of atmospheric CO2 on natural carbon sinks.

2 Generalization of the Modeling Framework of W&N2014

2.1. Generalization of the basic equation

A generalized version of Eq. (1) in W&N2014 that describes an N-component ("N-box") carbon-cycle system is represented by:

$$\dot{\mathbf{s}}' = \mathbf{X} \cdot \mathbf{s}' + \beta_T T' \cdot \mathbf{y} + \dot{E}' \cdot \mathbf{z} \qquad \text{(G1)}$$

where $\mathbf{s}'$, $\mathbf{y}$, and $\mathbf{z}$ represent $N \times 1$ vectors, and $\mathbf{X}$ is an $N \times N$ matrix. Specifically,

- $\mathbf{s}'$ represents all the anomalous carbon state variables (e.g., carbon in atmosphere, land, ocean, interior ocean, etc.). In particular, we assume the first element of $\mathbf{s}'$ to be the atmospheric carbon anomalies, that is, $\mathbf{s}'_1 = A'$ in Eq. (1) of W&N2014.
- $\mathbf{y}$ describes the distribution weights of the carbon impacts of temperature anomalies ($\beta_T T'$) on different carbon pools. Per the reasons explained in W&N2014, the elements of $\mathbf{y}$ are subject to the constraint $\sum_{i=1}^{N} \mathbf{y}_i = 0$. Without the loss of generality we set $\mathbf{y}_1 = 1$, reflecting that positive temperature anomalies release more carbon into the atmosphere.

- $\mathbf{z}$ describes the distribution weights of the CO_2 anthropogenically emitted into the system. It is clear that $\mathbf{z}_1 = 1$ and $\mathbf{z}_i = 0$ $(i = 2, \ldots, N)$.
- $\mathbf{X}$ describes dynamics of and interactions among all the carbon reservoirs. In particular, the diagonal elements of $\mathbf{X}$ represent the decaying rates of the carbon reservoirs, i.e., $\mathbf{X}_{ii} = -\alpha_i = -1/\tau_i$. The off-diagonal elements $\mathbf{X}_{ij} \geq 0$ $(i \neq j)$ represent the rates of carbon flow from the j-th reservoir to the i-th reservoir.

2.2 Generalization of the short-term responses of atmospheric CO2 to an impulse disturbance of CO2 emissions – Eq. (5b) in W&N2014

1. Note that the characteristic equation of Eq. (G1) is given by

$$\det(\mathbf{X} - \lambda \cdot \mathbf{I}) = 0, \qquad \text{(G2)}$$

   where λ represents the eigenvalue, $\mathbf{I}$ is the identity matrix, and "$\det$" stands for the determinant of the matrix.

2. By the binomial theorem, it is easy to see that the sum of the eigenvalues equals the trace of the state matrix $\mathbf{X}$, i.e.,

$$\sum_{i=1}^{N} \lambda_i = tr(\mathbf{X}) = \sum_{i=1}^{N} \mathbf{X}_{ii} = \sum_{i=1}^{N} -\alpha_i. \qquad \text{(G3)}$$

3. Because of the conservation of mass, the rank of $\mathbf{X}$ is $N-1$ (see the explanations in W&N2014). Therefore, one of the eigenvalues is zero. We denote this zero-valued eigenvalue to be λ_N. For simplicity of discussion, we also assume that the characteristic equation (G2) does not have multiple roots (i.e., the state matrix $\mathbf{X}$ is not degenerated). This simplification does not affect our discussions on the system's short-term responses (i.e., when $t \approx 0$).

4. The response function of atmospheric CO_2 to a unit impulse emission disturbances is thus determined by

$$A'(t) = \mathbf{s}'_1 = \sum_{i=1}^{N-1} \phi_i \exp(\lambda_i t) + \omega_1, \tag{G4}$$

where ω_1 is the steady-state (i.e., long-term) response of atmospheric CO_2 to the disturbance and ϕ_i are some constant coefficients subject to $\sum_{i=1}^{N-1} \phi_i + \omega_1 = 1$ so that $A'(0) = 1$ (i.e., initial conditions).

5. Using the approximation that $\exp(\lambda_i t) = 1 + \lambda_i t$ for small t (i.e., $t \approx 0$) in Eq. (G4) and rearranging the items on the right-hand side we arrive at:

$$A'(t) \approx 1 + (\sum_{i=1}^{N-1} \phi_i \lambda_i) t = \exp(-\alpha_1^* t), \tag{G5}$$

where $\alpha_1^* = -\sum_{i=1}^{N-1} \phi_i \lambda_i$.

6. Using similar procedures as above we can derive that the response function of the i-th ($i = 2, \ldots, N$) carbon reservoir as

$$\mathbf{s}'_i \approx \omega_i [1 - \exp(-\alpha_i^* t)], \text{ for } t \approx 0. \tag{G6}$$

Here ω_i is the steady-state (i.e., long-term) response of the specific reservoir. For mass conservation it is apparent that $\sum_{i=1}^{N} \omega_i = 1$. Also note that $\mathbf{s}'_i(0) = 0$ (i.e., initial conditions).

7. According to Eqs. (G5) and (G6), when t is small ($t \approx 0$) , the first row of the original state equation (G1) becomes

$$\dot{A}' = \dot{\mathbf{s}}'_1 \approx \mathbf{X}_{11} \cdot \mathbf{s}'_1 = -\alpha_1 A', \tag{G7a}$$

which simply means that

$$\alpha_1^* = \alpha_1, A'(t) \approx \exp(-\alpha_1 t), \tag{G7b}$$

the same conclusion we stated in Eq. (5b) of W&N2014.

8. Finally, we emphasize that, because the eigenvalues of a linear system are preserved by any unitary transformation, the above derivation is independent of a particular structure of surface carbon reservoirs, providing further support to the generality of the corresponding conclusion in W&N2014.

2.3 Generalization of the long-term responses of atmospheric CO2 to an impulse disturbance of CO2 emissions – Eq. (3b) in W&N2014

1. At the steady state for an impulse disturbance of emissions, Eq. (G1) becomes

$$\mathbf{X} \cdot \mathbf{s}' = 0, \tag{G8}$$

with $\sum_{i=1}^{N} \mathbf{s}'_i = 1$ (mass conservation). This condition indicates that Eq. (G8) has a non-trivial solution, which can be found by well-known procedures (e.g., Gaussian elimination) of linear algebra.

2. Once the solution $\mathbf{s}'$ is found, by the first row of Eq. (G8) it is clear that

$$A'/\tau_A = -\mathbf{X}_{11}\mathbf{s}'_1 = \sum_{i=2}^{N} \mathbf{X}_{1i}\mathbf{s}'_i = S'/\tau_S, \tag{G9}$$

where $S' = \sum_{i=2}^{N} \mathbf{s}'_i$, denoting the total responses of all the surface carbon reservoirs; and $\tau_S = S'/\sum_{i=2}^{N} \mathbf{X}_{1i}\mathbf{s}'_i$, representing the "bulk" e-folding time constant of the surface carbon reservoirs. Also note that Eq. (G9) is the same as Eq. (3b) in W&N2014. The relationship is accurate for a general carbon dynamic system *if* we have complete knowledge of $\mathbf{X}$.

3. Because in reality we do not have accurate information of $\mathbf{X}$, the estimates of τ_S (and τ_A) are associated with uncertainties. To illustrate the difficulty of the problem, we consider a particular N-box system that includes only the atmosphere and the global oceans at different layers. Let $\mathrm{s}'_1 = A'$ and s'_i $(i = 2, \ldots, N)$ denote different ocean layers with increasing "i" indicating increasing ocean depth. We further assume that each ocean layer only interacts with its neighbors and the atmosphere only interacts with the surface ocean layer. The state matrix $\mathbf{X}$ of such a sysmte can be written as:

$$\mathbf{X} = \begin{pmatrix} -\alpha_1 & \beta_2\alpha_2 & 0 & 0 & \ldots & 0 \\ \alpha_1 & -\alpha_2 & \beta_3\alpha_3 & 0 & \ldots & 0 \\ 0 & (1-\beta_2)\alpha_2 & -\alpha_3 & \beta_4\alpha_4 & \ldots & \vdots \\ 0 & 0 & (1-\beta_3)\alpha_3 & \ddots & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & -\alpha_{N-1} & \alpha_N \\ 0 & 0 & \ldots & 0 & (1-\beta_{N-1})\alpha_{N-1} & -\alpha_N \end{pmatrix}, \quad \text{(G10)}$$

where $\alpha_i = 1/\tau_i$ and β_i are constant numbers in the range of [0, 1]. Thus β_i and $(1-\beta_i)$ reflect the relative weights of the carbon efflux of the i-th reservoir to its two neighbors. Also note that $\beta_1 = 0$ and $\beta_N = 1$.

4. Solve the steady-state equation (G8) with $\mathbf{X}$ of Eq. (G10) in terms of A', the results are

$$\mathrm{s}'_i = (\prod_{j=2}^{i} \frac{1-\beta_{j-1}}{\beta_j}) \cdot \frac{\alpha_1 A'}{\alpha_i} = (\prod_{j=2}^{i} \frac{1-\beta_{j-1}}{\beta_j}) \cdot \frac{\tau_i A'}{\tau_1}, \quad \text{(G11a)}$$

and by Eq. (G9) we have

$$\tau_S = \sum_{i=2}^{N} (\prod_{j=2}^{i} \frac{1-\beta_{j-1}}{\beta_j}) \cdot \tau_i. \quad \text{(G11b)}$$

5. Eq. (G11b) shows that τ_S is not only a function of τ_i but also of β_j. In particular, the relative sensitivity of τ_S to β_2 is

$$\frac{1}{\tau_S} \cdot \frac{\mathrm{d}\tau_S}{\mathrm{d}\beta_2} = \frac{-1}{\beta_2(1-\beta_2)}. \quad \text{(G12)}$$

Because of the characteristic buffering effect of the ocean carbonate chemistry, the anomalous carbon exchange (induced by anthropogenic disturbances) between ocean surface and the atmosphere is much (about 10 times) more effective than between ocean surface and deep oceans (Gruber and Sarmiento 2002). This means that the value of β_2 is close to 0.9 or $\frac{1}{\tau_S} \cdot \frac{\mathrm{d}\tau_S}{\mathrm{d}\beta_2} \approx 10$. Therefore, a 1% uncertainty in β_2 alone could induce a 10% uncertainty in τ_S (or S')!

6. The above example highlights the challenge in estimating τ_S and thus the long-term response of the atmospheric CO_2 to anthropogenic emission disturbances. This problem is particularly emphasized in W&N2014 (Line 10-26, Page 13968). It is clear that the problem is mainly induced by the limited observations of the global climate-carbon system, such that our knowledge of the state matrix $\mathbf{X}$ is incomplete.

2.4 Generalization of the long-term responses of atmospheric CO2 to a unit step change of global surface temperature – Eq. (6b) in W&N2014

1. The steady state Eq. (G1) for an step temperature disturbance is

$$-\mathbf{X} \cdot \mathbf{s}' = \beta_T T' \cdot \mathbf{y}. \quad \text{(G13)}$$

2. Using the notions developed in Eq. (G9), we can represent the first row of (G14) as

$$\alpha_A A' - \alpha_S S' = \beta_T T', \quad \text{(G14)}$$

where $\alpha_A = 1/\tau_A$ and $\alpha_S = 1/\tau_S$.

3. Note that changes in temperature do not induce changes in the total carbon of the system, which simply means that

$$A' + S' = 0 \text{ or } S' = -A'. \tag{G15}$$

4. Substituting Eq. (G15) in Eq. (G14) and rearranging the items, we obtain

$$A' = \frac{\beta_T}{\alpha_A + \alpha_S} \cdot T'. \tag{G16a}$$

Because the estimate of α_S is generally uncertain (see the discussions of Section 2.3) and note that approximately $\alpha_A >> \alpha_S$, for quick estimation we can also use

$$A' = \frac{\beta_T}{\alpha_A} \cdot T'. \tag{G16b}$$

Both Eqs. (G16a) and (G16b) are the same as reported in W&N2014.

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