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## ***Interactive comment on “Dynamics of global atmospheric CO<sub>2</sub> concentration from 1850 to 2010: a linear approximation” by W. Wang and R. Nemani***

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We would like to first express our sincere appreciation to all the three reviewers for their constructive comments. Through the open discussion on these comments, we believe the scientific significance of this paper has become clearer. In particular, we believe that this study has developed a simple but effective scheme to quantify the dominant effects of temperature changes on the atmospheric CO<sub>2</sub> concentration in a linear model, which helps improve the model's accuracy in approximating the dynamics of atmospheric CO<sub>2</sub> in the past 160 years as well as helps us diagnose important issues regarding recent changes (e.g., intensification) of the global carbon cycle.

The reviewers have also raised several questions about the writing style of the paper. For instance, the technical details of our methodology needed further explanations

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and the context of this study with reference to literature also required clarifications. Here we try to answer these unresolved questions to the best of our knowledge. We have thoroughly revised our paper by incorporating many of these discussions and clarifications, which have helped significantly improve the quality of the manuscript. These revisions will also be described below.

#### 1. On the Linearization of Temperature's Effects on Atmospheric CO<sub>2</sub> (Dr. Jarvis, C7420-7422)

Dr. Jarvis is correct in stating that effects of changes in temperature on the global carbon cycle are rarely approximated in linear model studies. Except for a few possible exceptions (e.g., Enting (2010) likely discussed the connection between warming and atmospheric CO<sub>2</sub> increasing in terms of linear response functions though we haven't read the paper in full), the literature we know considers only the IRFs of the global carbon cycle to disturbances of CO<sub>2</sub> emissions. According to Scheffer et al. (2006), one reason that the quantification of temperature's effects (by a simple scheme) was deemed hard because such effects vary across different timescales (Woodwell et al. 1998). However, timescale dependence is only a characteristic of dynamic systems and not necessarily associated with nonlinearity of the system. Nor does it indicate that such effects have to be represented by more than one parameters.

Mathematically speaking, because we can always linearize the behavior of a nonlinear system at one of its stable points (attractors) under small perturbations, in principle the linearization of temperature's effects on atmospheric CO<sub>2</sub> has little difference from the linearization for CO<sub>2</sub> emission disturbances. The questions of interest here are thus whether or not the experienced warming (and other disturbances) has pushed the global carbon system away from its linear resilience zone and, in either case, how we should understand/explain the obtained answer in a biogeophysically meaningful way. The good agreement between the observations and the linear approximation results in this study statistically answered the first question, indicating that the global carbon sys-

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tem is still within its resilience zone for the period between 1850 and 2010. The second question deals with the biogeophysical interpretation of the model parameters ( $\beta_T$ , in particular), which we tried to answer with the help of the Bern model in our analysis. The following paragraph in the revised manuscript provides further explanations of our understanding towards this question:

“The biogeophysical implication of the parameter  $\beta_T$  needs further discussion. Our previous analysis (Wang et al. 2013) suggests that this parameter mainly reflects the temperature sensitivity of respiration of land-surface carbon pools (biomass and soil carbon). This explanation is supported by the simulations of the Bern model in this study, in which terrestrial carbon sinks have much stronger responses to temperature changes than its ocean counterpart. Furthermore, both our simulations and those from the literature (e.g., Canadell et al. 2007; Le Quéré et al. 2009) indicate that the total carbon storage in the land-surface reservoirs remains largely stable between 1850 and 2010, a necessary condition for  $\beta_T$  to be constant. For instance, because terrestrial carbon uptake accounts for 50-60% of the global net sinks in our simulations, the accumulated terrestrial net carbon sinks are about 71-85 ppm in 2010, representing a 7-8% increase in the total terrestrial carbon storage (1040 ppm as of 1850). At the same time, the accumulated terrestrial carbon losses through land-use changes are about 74 ppm in 2010 based on the Houghton (2003) dataset. These results suggest that the net changes in the total terrestrial biomass and soil carbon are (relatively) small during the past 160 year, providing further justification for our linear modeling approach.”

Therefore, although simulations by the Bern model do not necessarily reflect the real-world conditions (as Dr. Enting mentioned in his comments, all modeling is conditional on assumptions), we believe that the linearization of temperature's effects on atmospheric CO<sub>2</sub> in our model is mathematically justifiable and biogeophysically plausible.

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## 2. On the Accuracy of Linear Approximation and Inverse Problems in General (Dr. Enting, C7625-7628)

We first answer Dr. Enting's question regarding the comparisons between simulations by the two-box model and by the Bern model. All the results labelled as "Bern" in Figs 2 and 3 are simulated by the revised version of the Bern model. We have revised the corresponding figure captions to eliminate this confusion. We must emphasize that the interannual variations of the atmospheric CO<sub>2</sub> shown in Fig. 2 can only be accurately simulated by the Bern model after we incorporated the effects of temperature on the respiration of land carbon pools (with a Q10 of 1.5; see the Appendix for details). This fact supports the argument in our previous responses that the appropriate representation of temperature's effects on the carbon-cycle help improve the approximation accuracy of our models. We have revised the manuscript (e.g., the abstract and the introduction, see Response #7) to emphasize this point.

Second, we agree with Dr. Enting that non-linear terms are necessary for carbon-cycle models for prognostic applications. On the other hand, the linear model we derived in W&N2014 is diagnostic in nature. We explicitly stated in the title of our paper that our analysis is only based on the time period between 1850 and 2010. We also emphasized this point in Conclusions (the last paragraph), where we stated that the carbon-cycle will likely exhibit more nonlinear dynamic characteristics in the future and thus "the simple linear model developed in this study may serve as a convenient tool to monitor the early signs when the natural carbon system is pushed away (by anthropogenic disturbances) from its linear zone."

We also concur with Dr. Enting' comments on the difficulties of the general inverse problem of inferring dynamic structures from the observations of the global carbon-cycle as well as his remarks regarding the two types of inverse problems (i.e., the estimation of linear response function  $R(t)$  and the representation of  $R(t)$  as a sum of exponentials). The ill-conditioned nature of the problem is actually demonstrated by the results presented in W&N2014. For instance, the IRFs of the two-box model

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and the Bern model (Fig. 3) show certain distinct features (over the long-term, in particular), but their differences in approximating the observed atmospheric CO<sub>2</sub> are very small (Fig. 2). As described in the manuscript (Section 5, last paragraph), we could have chosen a higher-order model so that its IRF follows that of Bern more closely. However, because the observed CO<sub>2</sub> records only allow us to reliably retrieve a limited number of parameters, choosing for a higher-order model does not reduce the retrieval uncertainties at all. As shown in the Appendix, the number of unconstrained system structural parameters grows at the order of N<sup>2</sup>, making the inverse problem worse to work out. The revised Introduction also includes a new paragraph to discuss our considerations on the use of a simple model in this study (see the Response #7 below).

Another key difference between the inverse problem of previous studies and this analysis is that, in previous model-based studies, the source of disturbance for the IRF is determined; while for observation of atmospheric CO<sub>2</sub>, we do not know the exact disturbances. In other words, in the observational case we face both the model complexity and the model-completeness problems. The results of this study highlighted the importance of the latter issue, which is further discussed below (#5).

3. On the Estimation of  $\beta_T$ : Ambiguity and Feedback (Dr. Enting C7629-7631; and Dr. Jarvis, C7420-7422)

Dr. Enting's concern about the estimation of  $\beta_T$  is valid and important. It is actually the main reason that we didn't directly estimate  $\beta_T$  from Eq. (2c),

$$\dot{E}' - \dot{A}' = [\alpha_A - (1/\gamma - 1)\alpha_S] \cdot A' - \beta_T \cdot T'$$

but instead from Eq. (2e)

$$\dot{A}'_{IAV} \approx \beta_T \cdot T'_{IAV}$$

where the subscript “*IAV*” denotes interannual variation.

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The rationale behind this special treatment is based on the assumption that the radiative forcing of atmospheric CO<sub>2</sub> on temperature operates mainly at long-term timescales (e.g., low-frequency components); at short-term timescales (e.g., inter-annual), the variations in temperature are dominated by the natural variability of the climate system. Using Dr. Enting's notation, this means that  $f(p)$  and  $\eta q(p)$  do not necessarily share the same response function  $u(p)$ , and in particular

$$u_q(p) \approx 0 \text{ for } Im(p) \gg \omega_0,$$

where  $Im(\cdot)$  means the imaginary part of the complex frequency  $p$  and  $\omega_0$  is some frequency threshold. It follows that

$$q(p) \approx r(p)[s(p) + h(p)u_f(p)f(p)] \text{ for } Im(p) \gg \omega_0,$$

that is, the feedback loop is impeded for high-frequency (e.g., interannual) variations. Further assuming  $s(p) \approx 0$  for  $Im(p) \gg \omega_0$  leads to our Eq. (2e) in the Laplace domain.

We have added the following new paragraph in the revised paper to further explain the above considerations:

“There is another practical reason that we use the  $\beta_T$  estimated from Wang et al. (2013) in this study. Because the long-term increases in global temperature ( $T'$ ) are mainly induced by the growing CO<sub>2</sub> concentrations in the atmosphere ( $A'$ ), the two variables are indeed significantly correlated ( $r \approx 0.9$ , with IAV in them removed). Therefore, estimating  $\beta_T$  directly from Eq. (2c) is inevitably subject to the influence of the collinearity between  $A'$  and  $T'$  (Enting 2010). On the other hand, the short-term variations (i.e., IAV) of global temperature are dominated by the natural variability of the climate system (e.g., the El Niño-Southern Oscillations). Therefore, we expect the  $\beta_T$  estimated with Eq. (2e) in Wang et al. (2013) to have less uncertainty.”

4. On the Estimation of  $\tau_A$  (Dr. Jarvis, C7420-7422)

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As explained in our manuscript, additional information is required to resolve  $\alpha_A$  and  $\alpha_S$  from the regression results of Eq. (2c). One source of such information comes from previous observation-based studies. For instance, by comparing the carbon isotope ratios in wood and in marine material, Revelle and Suess (1957) have long suggested that the response time ( $\tau_A$ ) of atmospheric CO<sub>2</sub> is on the order of 10 years. We also extracted information from process-based model studies. As we derived in our analysis (Section 5 and the Appendix), the initial decaying rate of the IRF of a global carbon-cycle model is mainly determined by  $\alpha_A$  (or  $\tau_A$ ). Applying this result to analyze the ensemble IRFs reported in Joos et al. (2013) suggests  $\tau_A$  to be 14 years. We choose  $\tau_A$  to be 12 years ( $\alpha_A \approx 0.083 \text{ yr}^{-1}$ ) so that the IRF of our linear model closely matches with the Bern model during the initial decaying stage (Fig. 2). We subsequently estimate  $\tau_S$  to be 34 years ( $\alpha_S \approx 0.029 \text{ yr}^{-1}$ ).

We have incorporated the above explanations into the revised paper.

#### 5. On the Comparison with Previous Diagnostic Studies, (Referee #2, C6950-6952)

Simple models (like the one derived in this study) are frequently used in previous studies to diagnose or explore dynamic characteristics of the global carbon cycle. The physics of these models are assumed to be self-evident and discussions on their development are usually neglected. However, as Drs. Enting and Jarvis suggested in their comments, inverse problems of inferring structural information from observed CO<sub>2</sub> records are ill-conditioned in nature and small negligence in model development can lead to large uncertainties in the retrieved results. Therefore, the detailed model derivation and analysis presented in our study may provide a basis to help us compare and clarify some results from previous studies.

For instance, Gloor et al. (2010) - referred to as G2010 hereafter - developed a fairly comprehensive analytic framework to examine the relationship between the airborne fraction of CO<sub>2</sub> and the efficiency of global carbon sinks. Their main conclusion, that

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the trend in AF does not necessarily indicate changes in the carbon sink efficiency, is scientifically valid and important. However, some of the quantitative results reported in G2010 have large uncertainties and, sometimes are incorrectly interpreted.

The basic model used in G2010, Eq. (2) in their paper, is as follows:

$$\frac{d\Delta C}{dt} = f(t) - \frac{\Delta C}{\tau_{sys}} \quad (\text{G2010})$$

where  $\Delta C$  stands for changes in atmospheric CO<sub>2</sub> concentration relative to the pre-industrial reference,  $f(t)$  is the anthropogenic CO<sub>2</sub> emissions, and  $\tau_{sys}$  stands for the system response time. Using the notations developed in W&N2014, therefore, we can translate this equation to

$$\dot{A}' = \dot{E}' - \alpha_{sys}A' \quad (\text{G2010, translated notation})$$

where  $\alpha_{sys}$  is just  $1/\tau_{sys}$ .

Comparing the G2010 model with the two-box model derived in this study (i.e., Eq. (2a) rearranged for easier comparison),

$$\dot{A}' = \dot{E}' - (\alpha_A + \alpha_S)A' + \alpha_S E' + \beta_T T', \quad (\text{W\&N2014})$$

it is easy to see that two terms,  $\beta_T T'$  and  $\alpha_S E'$ , are missing from the G2010 model. The absence of temperature's effects in G2010 is not a surprise and relatively less important (they can be thought of as additional forcing). However, the absence of  $\alpha_S E'$  indicates more serious structural incompleteness, meaning that in G2010 carbon exchange is "one-way only" from the atmosphere to the surface, so that all the CO<sub>2</sub> disturbances to the atmosphere will eventually be absorbed by the surface (See Eqs. (1a-c) in W&N2014 for details). This can be verified by the IRF of the G2010 model (i.e.,  $e^{-\alpha_{sys}t}$ , or more generally  $e^{-\int_0^t \alpha_{sys} ds}$ ), which decays to 0 as time progresses.

The above identified model incompleteness means that the G2010 model can only be used to simulate the AF when it is relatively stable. For instance, because the observed

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$E'$  and  $A'$  are related by a constant AF ( $\gamma$ ), the G2010 model still can reasonably approximate the observed variations in atmospheric  $\text{CO}_2$  or the observed AF. However, the retrieved system response time  $\tau_{sys}$  cannot be interpreted as intended. Indeed, by comparing the G2010 model with Eq. (2c) in our model, it is easy to see that

$$\alpha_{sys} = \alpha_A - (1/\gamma - 1)\alpha_S.$$

Therefore,  $\alpha_{sys}$  (and its later development in Raupach et al. (2014) as the "CO<sub>2</sub> uptake rate by land and ocean sinks") is not an intrinsic character of the system but influenced by the AF factor ( $\gamma$ ) itself! The metric can only be interpreted as the efficiency of global surface carbon sinks when the AF ( $\gamma$ ) is stable. Its estimation is also influenced by the absence of other disturbances (e.g., changes in temperature) in the G2010 model. The limitations associated with this metric are briefly discussed in W&N2014 (Section 6, under the discussions of the efficiency of surface carbon sinks).

## 6. On Specific/Minor Comments by the Referees

1. P13958, L26 (Dr. Enting, C7623-7624: the "committed warming" issue). We realize that the confusion was induced by the word "potential" used in the sentence, which is now changed to "strength" in the revised paper.
2. P13959, L7-12(Referee #2, C6950-6952: discussions on AF and the reference of Gloor et al. 2010). We have rewritten these sentences to introduce the study by Gloor et al. (2010).
3. P13960, L2-3 (Referee #2, C6950-6952: the pre-industrial equilibrium) The pre-industrial equilibrium is only an approximation that is often assumed in the literature (e.g., Enting and Mansbridge 1987). We agree that Earth's climate-carbon system always slowly evolves and therefore we used "quasi steady state" in this sentence.

4. P13858, L26 (Referee #2, C6950-6952: typo). This typo is now corrected.
5. P13965, L3 (Dr. Engting, C6416-6422, specifically on Page C6418: "may"). We recognize the use of soft language is not necessary here and at other places in the paper. We have tried to correct them in the revised paper.

## 7. On Other Revisions of the Paper

1. Abstract. We have also rewritten the abstract to emphasize the unique contribution of this paper to the research subject:

“Changes in Earth’s temperature have significant impacts on the global carbon cycle, yet the quantification of such impacts by linear schemes is traditionally deemed difficult. Here we show that, by incorporating a temperature sensitivity parameter into a simple linear model, we can satisfactorily characterize the timescale-dependent responses of atmospheric CO<sub>2</sub> concentration to temperature changes and carbon emissions while accurately reproducing the history of atmospheric CO<sub>2</sub> between 1850 to 2010. The linear modeling framework allows us to analytically examine the dynamic characteristics of the carbon system and associate them with the response times of the carbon reservoirs and the temperature sensitivity parameter. These results also have important biogeophysically implications that appear to highlight the intensification of the global carbon cycle. On one hand, they indicate that the elevated atmospheric CO<sub>2</sub> concentration have enhanced land carbon uptakes at a rate higher than traditionally thought. On the other hand, such enhanced gross carbon uptakes are partially offset by the increases in global surface temperatures, which accelerate the release of carbon from the surface reservoirs into the atmosphere. As a result, the net rate of atmospheric CO<sub>2</sub> sequestration by global land and oceans has slowed by 30% since 1960s. Therefore, the linear modeling framework outlined in this

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paper provides a convenient tool to diagnose the observed atmospheric CO<sub>2</sub> dynamics and monitor their future changes.”

2. Introduction, on the literature context. We have thoroughly revised the Introduction of the paper to bring in a more comprehensive review of the literature and to clarify the connections of this paper with previous studies. For instance, the following paragraphs are added to the revised paper:

“There is a rich literature on the application of linear methodology to study the global carbon cycle, either to approximate the system’s dynamics or diagnose its characteristics (e.g., Oeschger and Heimann 1983; Meier-Raimer and Hasselmann 1987; Enting and Mansbridge 1987; Wigley 1991; Jarvis et al. 2008; Gloor et al. 2010; Joos et al. 1996, 2013). At the heart of some of the most influential methods is the estimation of the system’s Impulse Response Function (IRF; or more generally the Green’s function), which describes the time-varying responses of atmospheric CO<sub>2</sub> to a pulse of external disturbances, usually anthropogenic carbon emissions. Because the analytical determination of IRFs is difficult for complex systems, they were often obtained by fitting exponential equations to the numerical experiment results with global carbon-cycle models or their sub-components (Meier-Raimer and Hasselmann 1987; Joos et al. 1996, 2013). Once the IRF is known, the state of atmospheric CO<sub>2</sub> can be conveniently calculated through linear convolution of the IRF and the records of CO<sub>2</sub> emissions. Results obtained by such linear approaches well agree with the simulations from the corresponding global carbon-cycle models unless the disturbances to the system are too large (Wigley 1991; Li et al. 2009).

“Although previous studies mostly use IRFs as convenient tools to substitute the corresponding “parent” models in calculation, the significance of IRFs in diagnosing the dynamic characteristics of the carbon-cycle system cannot be underestimated. The fact that IRFs can be represented by a few exponential functions

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(Meier-Raimer and Hasselmann 1987) indicates that the dynamic responses of their parent models are largely captured by a few dominant linear modes (Young 1999) – in other words, the fundamental dynamic characteristics of these global carbon-cycle models can be learned from suitable lower-order linear models. For instance, Li et al. (2009) were able to infer the response (e-folding) time constants of the major carbon reservoirs in the carbon-cycle model of Lenton (2000) by studying its IRF with a fifth-order linear model.

“Extending the line of thoughts from the literature, this study directly applies lower-order linear models to investigate the dynamic characteristics of the global carbon cycle based on observations. Because the IRF of the real-world system is unknown, we can only treat the global carbon cycle as a “black box” and use the observed forcing-response relationships to constrain our models. Nevertheless, the independence from a parent model also gives us more freedom to diagnose some important dynamic modes that have been less investigated in previous linear models ... ”

3. Introduction, on the use of a simple model. The following paragraph is totally re-written to explain our considerations in choosing a simple model (as a demonstration tool) in this study:

“A practical factor to decide in developing a diagnostic model for the global carbon cycle is the complexity of the linear tool itself. This may not represent a serious difficulty in the forward model construction and analysis, where well-established mathematical tools are at our disposal (see the example in the Appendix). For the inverse problem of model identification, on the other hand, it is the resolution of available observations that essentially determines the number of independent system parameters that can be reliably retrieved. In this study, we decided to demonstrate our analytical framework by a simple two-box model that represents carbon exchanges between the atmosphere and

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the surface (i.e. land and ocean) reservoirs. This decision is based on multiple considerations besides the constraints of model identification, which include that, for instance, the analysis of a two-box model involves only simple mathematical techniques but render clear physical pictures of the problem under investigation. Though such a “toy” model may sit at the lowest rank on the hierarchy of global carbon-cycle models (Enting 1987), new and important characteristics of the atmospheric CO<sub>2</sub> dynamics can still be learned from it. Furthermore, the use of a simple model by no means implies the compromise of scientific rigor of our findings, which are verified in a generalized linear model framework as described in the Appendix.”

4. Appendix. We included the analysis of the generalized N-Box model in our previous responses (C7237-7249, Section 2) into the revised Appendix.

(All the references to the literature can be found in the revised manuscript provided here as a supplemental PDF document, which shows the editorial markups to help the reviewers identify the revisions that have been made.)

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