We truly appreciate the comment submitted by Steven Thomas on May 6, 2015. Although we disagree with his critiques, we believe his comments will help the scientific community interested in scaling nutrient uptake better understand the limitations of current methods (which include those presented by Hall et al. (2013)).

Steven Thomas argues that our main criticism to the paper by Hall et al. (2013) fails to show that S_w vs. Q/w is a spurious correlation. For the sake of improving the readability and to ease the understanding of this reply, let us recall that in our collegial comment on the paper by Hall et al. (2013) we demonstrated that S_w vs. Q/w becomes u/K_c vs. $u \cdot h$. Due to the presence of a common factor in the scaling relationship proposed by Hall et al. (2013), we hypothesized that it is a spurious correlation (u influences both S_w and Q/w) that may be mechanistically irrelevant for scaling in-stream nutrient uptake. We later provided a numerical example demonstrating the nature of the spurious correlation. In his comment to our work, Steven Thomas argues that "S_w is effectively an empirical measurement that is independent (in the statistical sense) of any of the hydrological […] variables used to transform S_w into metrics like uptake velocity (v_f) and [areal] uptake rates (U)". We disagree with Steven Thomas' observation because the main reason why we estimate S_w as a function of longitudinal plateau concentrations is that there is a mechanistic, physically based model supporting that method. Such model is the advection-dispersion equation, with the addition of a first-order uptake rate coefficient (Stream Solute Workshop, 1990; Runkel, 2007):

$$\frac{dc}{dt} = -u\frac{dC}{dx} + D\frac{d^2C}{dx^2} - K_c c, \quad (1)$$

where c [M L^{-3}] is the concentration of the reactive solute; u [LT^{-1}] the mean flow velocity; D [LT^{-2}] the dispersion coefficient; K_c [T^{-1}] the first-order rate coefficient representing nutrient uptake; x [L] longitudinal distance; and t [T] time. Assuming that the effects of dispersion are negligible (i.e., effectively zero) at plateau concentrations (i.e., when $\mathrm{dc/dt} = 0$), equation (1) results in:

$$c = c_o \exp(-(K_c/u)\ x), \quad (2)$$

where c_o [M L^{-3}] represents the initial (or upstream) concentration. Equation (2), which constitutes the basis of the work by Hall et al. (2013), is a first-approximation model to estimate nutrient uptake in stream reaches where dispersion and transient storage do not play important roles (Runkel, 2007). Note that for using this model, an experimentalist would measure the plateau concentrations upstream (c_{up}) and downstream (c_{dn}) of a study reach of length L to estimate S_w ($S_w = u/K_c$), i.e.:

$$S_w = L/\ln{(C_{up}/C_{dn})}. \quad (3)$$

The model derived from equations (1-3) supports the utilization of plateau-like experiments to estimate nutrient uptake in stream reaches for a given, constant discharge. Therefore, the estimation of S_w using the above methods implies that such quantity is indeed a function of the ratio u/K_c. This supports our hypothesis that S_w vs. Q/w becomes u/K_c vs. $u \cdot h$, which is a spurious correlation. To summarize, S_w is not an isolated, "stream-specific" empirical variable, as Steven Thomas claimed. S_w is a complex variable that can be derived from different transport models assuming different levels of complexity (Runkel, 2007). Of course, the usefulness of S_w as a metric to compare nutrient uptake across a river continuum (and in general the use of any other metric) relies on the conviction that the model used to derive this metric correctly represents the system that it describes. The careful reader would note that in our comment we refer to the transport model used by Hall et al. (2013) (i.e., the advection-dispersion with first-order uptake), which would only represent nutrient uptake in a straight, impervious-channel where flow is uniform and mixing due to dispersion and transient storage does not occur (Runkel, 2007).

References

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