Reply to comment by R. González-Pinzón, J, Mortensen, and D. Van Horn

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González-Pinzón et al. (2015) wrote that the regression-based scaling approach used in Hall et al. (2013) contained a "spurious correlation" (sensu Pearson 1896, Kenney 1982). In Hall et al. (2013), we regressed *Q/w* (where *Q* is discharge and *w* is wetted width of a stream) with field measurements of uptake length (S_w). González-Pinzón et al. raised concern regarding this regression approach because both of the terms that we used can be reparameterized as a function of an unmeasured (i.e., derived) covariate which is stream velocity. Following their notation and assuming continuity,

$$\frac{Q}{w} = uh$$

where *u* is stream velocity and *h* is stream depth. Uptake length of nutrients, S_w (m) can also be reparameterized as

$$S_w = u/K_c$$

where K_c is the per time uptake rate of nutrients in a stream. González-Pinzón et al. (2015) suggest that our conclusions were compromised by the hidden role of *u* in the relationship between S_w and *Q/w*. While we appreciate their comments, we disagree, and explain our points below.

One goal of our paper was to assess the role of stream size on nutrient demand using literature data of field measurements of uptake as a function of stream and river size, as denoted by *Q/w* (also field-measured) in this case. Theoretical models conservatively assume that nutrient demand remains constant across a range of stream sizes, but this assumption had not been quantitatively investigated. Ironically, we designed our analysis to avoid exactly the mistake pointed out by Kenney (1982), González-Pinzón et al. (2015), and many others. For example, one approach to analyze the data would have been to regress a metric of nutrient demand, uptake velocity (v_f), as a function of river size. However, v_f is calculated as

$$v_f = \frac{Q}{w \cdot S_w}$$

Thus, had we performed this regression we would in fact have committed the statistical mistake of having a measured covariate on both sides of a regression (and we would likely have reported a spurious positive correlation between v_f and stream size); this error is the one described by González-Pinzón et al. (2015), and is well known in the literature (Pearson 1896). Studies of organismal scaling have grappled with this problem in that they seek to understand how e.g., specific metabolic rate varies with body size (Peters 1986), and there has been much discussion on the problems of using measured covariates (e.g., body size) on either side of the regression (Prairie and Bird 1989). Instead, we followed the approach used previously in organismal scaling studies (Warton 2006) in which we did not include a measured covariate on both axes of the regression, in that we regressed S_w as a function of Q/w. We recognize that these variables *covary* and that both are strongly affected by stream velocity; this pattern is expected and completely consistent with current theory (Stream Solute Workshop 1990). Rather, we were interested in identifying the functional form of the relationship between S_w and Q/w as the slope of this power-law relationship. Slopes > or < 1 indicate allometric scaling where there is some negative or positive effect of stream size on nutrient demand, which we felt was a novel analytical contribution.

Unlike v_f and *u,* we note, in our data set, that S_w and Q/w data were measured in the field, independently of one another. More specifically, Q was measured in the field via dilution gaging or the velocity-area method and w with measuring tape or a laser range-finder, while S_w was measured using nutrient addition approaches and calculated as the inverse of the per length uptake rate (K_m), where K_m is the slope of the decline in added nutrient concentration as a function of measured distance downstream. Velocity in these experiments may or may not have been measured in the field, and u was unknown to us when compiling our meta-analysis. If we had we measured a per-time decay rate of nutrients in a chamber, then used velocity to scale to S_w, and compared S_w to empirically measured uh, then González-Pinzón et al. (2015) would have been correct in noting that the relationships both contained u, and any correlation between the two variables would indeed be spurious.

What González-Pinzón et al. (2015) call a spurious correlation, we call mechanism. By deriving Q/w and S_w as a function of the unmeasured covariate u, González-Pinzón et al. (2015) have demonstrated a mechanistic basis for increasing S_w as a function of Q/w. The remedy suggested by González-Pinzón et al. (2015), as opposed to our approach, suffers from the common factor error that they discussed. Rather than removing the common factor (u), González-Pinzón et al. (2015) actually introduced it, multiplying both S_w and Q/w by $1/u$. We suspect that this misstep spuriously reduced their reported correlation between ($1/K_c$) and depth (h). (That h is often inferred from *Q, w, and u*, and thus is not independent of any of these, may also have been a factor). Additionally, velocity and depth play theoretically equivalent roles in the scaling of S_w. If one advocates removing u, from S_w, then one should also remove h. But, as noted above, multiplying S_w by ($1/uh$) produces $1/v_f$, which is mechanistically satisfying but, because it contains measures of stream size, cannot be rigorously regressed against any measure of stream size.

We fully agree with González-Pinzón et al. (2015), that having measured covariates on both sides of a regression equation causes inferential problems. However, the problem of spurious correlation, as presented in the heuristic example of González-Pinzón et al. (2015), as well is in the many examples (including lake depth, shrimp size, soil moisture) given by Kenney (1982), involves variables formed by the investigator as a combination of measured variables, but does not extend to unmeasured variables that may be mutually causative. The latter case, where the correlation may not reflect causality, is also sometimes referred to as "spurious" (e.g., Sugihara et al. 2012); however, this use is not relevant here because the "cause" of variation in S_w is not at issue. We would lose the ability to deduce mechanism, or test the functional form of relationships if we assume that unmeasured covariates negate the utility of the regression approach. For example, autotrophic metabolism is a likely explanation why S_w is shorter in streams with higher gross primary production (Hall et al. 2009). Moreover, this metabolism is implicitly on both sides of the regression equation as assimilatory N uptake and C fixation. Do these linked biochemical reactions negate the relationship, or is it mechanism? We strongly disagree with the assertion that unmeasured covariates negate the inference from our study or the many scaling studies that preceded it (Peters1986, Brown et al. 2004 etc).

Finally, we thank González-Pinzón et al. (2015) for initiating this productive discussion. Theoreticians can lose sight of mechanism without data, while at the same time, empiricists may not know the underlying theory in their data collection efforts. This debate has served to bridge the all-too-common divide between theoretical arguments and empirical data.

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