Table 1. Key features of microbial decomposition models.

FWD Model

German et al., 2012

FWD Model with maintenance respiration

As FWD model but microbial respiration is partitioned into temperature insensitive growth and temperature sensitive maintenance respiration terms.

REV Model

Depolymerisation and uptake relative to microbial biomass decreases with increasing M (diminishing return mechanism).

REV Model with maintenance respiration

As REV model but maintenance respiration added.

REV Model with equilibrium microbes

As REV model but fast microbial adjustments.

OPT Model

Optimisation of microbial enzyme production to maximise microbial growth, and consideration of carbon costs associated with enzyme synthesis.

OPT Model with maintenance respiration

As OPT model but maintenance respiration added.

OPT Model with equilibrium microbes

As OPT model but fast microbial adjustments.

FOD Model

First order decomposition model, modified to account for temperature sensitive carbon use efficiency.

Table 2. Parameters used in microbial decomposition models (In subsequent models, we provide only those parameters where modifications have been made.)

Parameter	Unit	Value	Description	Source
FWD Model				
I	$mg\ cm^{-3}\ hr^{-1}$	0.001	Input of fresh litter	
λ_d	hr^{-1}	0.0005	Death rate of microbes	
$V_{max\ ,FWD,0}$	$mg\ cm^{-3}\ hr^{-1}$	0.0049	Maximum catalytic rate @ 15°C	
$Q_{10},\ _{Vmax\ ,FWD}$	-	1.9	Q_{10} of maximum catalytic rate	
K_E	$mg\ S\ cm^{-3}$	270	Half-saturation constant @ 15°C	German et al., 2012
ε_0	-	0.39	Microbial growth efficiency @ 15°C	
ε_{slope}	$°C^{-1}$	-0.016	Microbial growth efficiency temperature slope	
FWD Model with maintenance respiration				
$\lambda_{r,0}$	hr^{-1}	0.0006	Maintenance respiration @ 15°C	
$Q_{10,\lambda r}$	-	2.2	Q_{10} of maintenance respiration	This study
g	-	0.24	Growth respiration coefficient	
REV Model				
$V_{max,REV}$	$mg^{-1}\ M\ cm^{-3}\ hr^{-1}$	$2.61*10^{-5}$	Maximum catalytic rate @ 15°C	
K_M	$mg\ M\ cm^{-3}$	0.68	Half-saturation constant @ 15°C	This study
OPT Model				
$V_{max,OPT}$	$mg^{-1}\ M\ cm^{-3}\ hr^{-1}$	$1.71*10^{-5}$	Maximum catalytic rate @ 15°C	
μ		0 , 0.1, 0.5	Enz production cost (as % of decomposition)	This study
$K_P * c$	$mg\ M\ cm^{-3}$	0, $1.64*10^{-5}$ 0.0004	combined cost and the half saturation constant	
FOD Model				
K^*	hr^{-1}	$1.71*10^{-5}$	First order decay constant @ 15°C	This study

* K in FOD model is identical to $V_{max,OPT}$ in OPT model.

Table 3. Equilibrium solutions for microbial biomass, soil organic carbon, and CUE at short/fast timescale (if, S ≠Eq. S) and long timescale (if, S = Eq. S). Note, for simplicity, we did not substitute S in the long-term microbial equilibrium for OPT model.

Model	Short/Fast time scale		Long time scale	
	M	Decomposition	S	M
FWD	no solution *	no solution *	$\frac{\lambda_d K_E}{V_{max1}\ \varepsilon - \lambda_d}$	$\frac{I\ \varepsilon}{(1-\varepsilon)\ \lambda_d}$
REV	$\frac{V_{max,Rev}\ S\ \varepsilon\ - K_M\ \lambda_d}{\lambda_d}$	$V_{max,Rev}\ S\ \varepsilon - K_M\ \lambda_d$	$\frac{I}{V_{max,Rev}\ (1-\varepsilon)} + \frac{K_M\ \lambda_d}{V_{max,Rev}\ \varepsilon}$	$\frac{I\ \varepsilon}{\lambda_d\ (1-\varepsilon)}$
OPT	$\frac{(X-Y)^2\ \varepsilon}{\lambda_d}$	$X^2 - XY$	$\frac{1}{2\ V_{max,OPT}\ \ (1-\varepsilon)^2}\left[-Y\ (2\varepsilon-1)\sqrt{4IY\ (1-\varepsilon)+Y^2} + (1-\ \varepsilon)\ (2I - 2\varepsilon Y^2) +\ Y^2\right]$	$\frac{\varepsilon(X-Y)^2}{\lambda_d}$

$X = \sqrt{S\ V_{max4}}$, $Y = \sqrt{K_P c}$, $\varepsilon = \frac{(1-g)\ \lambda_d}{b}$

* requires $\lambda_d\ = \frac{V_{max,FWD} S\varepsilon}{S + K_E}$