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**Atmospheric CO₂
modelling: error
correlations**

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Structure of the transport uncertainty in mesoscale inversions of CO₂ sources and sinks using ensemble model simulations

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Abstract

We study the characteristics of a statistical ensemble of mesoscale simulations in order to estimate the model error in the simulation of CO₂ concentrations. The ensemble consists of ten members and the reference simulation using the operational short range forecast PEARP, perturbed by Singular Vector (SV) technic. We then used this ensemble of simulations as the initial and boundary conditions for the meso scale model simulations, here the atmospheric transport model Méso-NH, transporting CO₂ fluxes from the ISBA-A-gs land surface model. The final ensemble represents the model dependence to the boundary conditions, conserving the physical properties of the dynamical schemes.

First, the variance of our ensemble is estimated over the domain, with associated spatial and temporal correlations. Second, we extract the signal from noisy horizontal correlations, due to the limited size ensemble, using diffusion equation modelling. Finally, we compute the diagonal and non-diagonal terms of the observation error covariance matrix and introduced it into our CO₂ flux matrix inversion over 18 days of the 2005 intensive campaign CERES over the South West of France.

On the horizontal plane, variance of the ensemble follows the discontinuities of the mesoscale structures during the day, but remain locally driven during the night. On the vertical, surface layer variance shows large correlations with the upper levels in the boundary layer (>0.6), down to 0.4 with the low free troposphere. Large temporal correlations were found during the afternoon (>0.5 for several hours), reduced during the night. Diffusion equation model extracted relevant error covariance signals on the horizontal space, and shows reduced correlations over mountain area and during the night over the continent. The posterior error reduction on the inverted CO₂ fluxes accounting for the model error correlations illustrates finally the predominance of the temporal over the spatial correlations when using tower-based CO₂ concentration observations.

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1 Introduction

Atmospheric inversions are a widely-used tool for the quantification of surface sources (e.g. Gurney et al., 2002; Rayner et al., 2008) for CO₂, Bousquet et al. (2006) for CH₄ and Petron et al. (2002) for CO. The theory and applications are described in Enting (2002). In the most common approach, Bayesian Synthesis Inversion, one starts with a prior distribution of surface fluxes. These are used as inputs to an atmospheric transport model (possibly including chemical modification). The transport model simulates concentrations at a set of observing locations. The fluxes are adjusted to optimize consistency with both the observed concentrations and a priori flux information. Under the usual assumption of multivariate normal probabilities, the required degree of consistency is described by covariance functions for the prior fluxes and the model-data mismatch.

The proper specification of these covariances is as important as the prior and data values themselves. For the calculation of posterior best estimates the values always appear normalised by their standard deviations while, for posterior uncertainties, it is only the covariances and transport model that appear. For the prior flux covariance the case is relatively simple since the covariance should represent the statistics of the difference between the truth and our chosen prior. This requires some independent knowledge of the truth which should, in turn, not be used in our later inversion. See Chevallier et al. (2006) for an example of using eddy covariance flux measurements as truth estimations.

This paper concerns the description of the model-data mismatch covariance. In the full theory (see Tarantola, 2005, ch.1.) there are separate covariances describing both the model and measurement parts. The measurement covariance describes the statistics of the difference between a measurement and the true value. As with the prior this can often be assessed with independent measurement and one has access to a well-developed discipline of metrology. The model uncertainty describes the statistics of the difference between a simulation and the true value that would be observed if the

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real atmosphere was forced with the same fluxes as the model. We see immediately that this error is much harder to characterize since there are few cases in which we know what fluxes influenced the real atmosphere. In atmospheric transport inversions the model contribution usually dominates the measurement error and this model error is our focus here.

There are many contributions to the model error. Transport models are mesh-based. Thus there is likely to be a mismatch between a point measurement and the average value over a model grid box. The importance of this depends on the magnitude and structure of concentration variability in the atmosphere and we can attempt to quantify it with high-resolution models (e.g. Corbin and Denning, 2006; Corbin et al., 2008) or with spatially dense measurements (e.g. Gerbig et al., 2003). Finite resolution equally confounds accurate description of the input fluxes leading to so-called aggregation error (Kaminski et al., 2001). Next the formulation of the model can be incorrect. In the absence of further data to demonstrate this we are forced to treat the statistics of an ensemble of models as if they are the statistics of the difference between the model and the truth. This is the assumption underlying the series of Transcom studies (e.g. Gurney et al., 2002; Baker et al., 2007; Law et al., 2008). This approach can hide bias in the model ensemble and is commonly criticized for understating model error although Stephens et al. (2007) noted that screening models against available independent data can actually reduce the apparent dispersion of the ensemble.

Finally, even if the physical formulation of the model is close to truth, uncertainties in the analyses with which it is constrained coupled with the inevitable error growth of any forecast (e.g. Lorenz, 1982) means that our transport fields are only one realisation of an ensemble consistent with available meteorological information. The impact on atmospheric inversions of the dispersion of this ensemble, and particularly the strong likelihood of correlations in this dispersion has not been previously investigated. A direct estimate of the ensemble statistics is fraught with difficulty since a sufficiently large ensemble is prohibitively expensive to compute. Instead, where available, we use a physical model of error growth and propagation previously applied to the numerical

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weather prediction problem (short range ensemble PEARP).

We apply our approach to an inversion of CO₂ surface fluxes in a limited domain although we stress at the output that it is applicable to global problems. We calculate the ensemble statistics of transport error on the domain of the CarboEurope Regional Experiment Strategy (CERES) (Dolman et al., 2006; Lauvaux et al., 2008). At this scale we can compare the ensemble behaviour with other measures of the model-data mismatch such as the variability in measured concentrations (Gerbig et al., 2003). Also, mesoscale inversions are heavily reliant on the ability of transport models to simulate details of concentration variations usually ignored in global inversions. The improvement of atmospheric models now allows the use of high frequency data on limited domains (Sarrat et al., 2007). Although model errors should decrease with increasing spatial and temporal resolution, the complexity of dynamical processes at the mesoscale are likely to produce a complex uncertainty structure. Discontinuities in the dynamical flow at higher resolution induced by the surface properties also imply abrupt changes in model error.

In this study, we propose to assess the structure of the model error by perturbing the synoptic conditions, that affect the mesoscale dynamical structures, using an ensemble of simulations at a larger scale. The ensemble presented in this study remains physically consistent, thanks to the two-step perturbation method that allows us to keep the original dynamical schemes of the meso scale model, here Méso-NH (Lafore et al., 1998). Previous studies attempted to disturb the model stability by perturbing directly the modelled wind fields (Law et al., 2003), or by exploring the physical parameters of the model (Annan et al., 2005). It is impossible to tell whether a given perturbation of wind fields or parameters is physically realisable while the perturbations we use are generated from an ensemble forecast system designed to explore the plausible range of synoptic variations.

The outline of the paper is as follows: Starting from the mesoscale ensemble of simulations, (i) we estimated the variance with its spatio-temporal correlations; (ii) we modelled the horizontal correlations by using the diffusion equation to filter the noise of

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our limited size ensemble; and (iii) we tested the combined spatio-temporal observation covariance matrix in our CO₂ flux matrix inversion. Further implications are discussed from the model correlations for regional network design, inversion using layered data, and temporal inertia of the error.

2 Models and diffusion equation filtering

The major task of this work is to estimate the spatial and temporal correlations in the model-data mismatch using an ensemble of regional CO₂ simulations. In this section we first describe the underlying physical models, the generation of the ensemble and finally the calculation of the various correlation terms, focusing on horizontal correlations. The role of the resulting correlation matrix in the subsequent flux inversions is treated in Lauvaux et al. (2008) and only briefly reviewed here.

2.1 Models

The ensemble is based on the global spectral ARPEGE model (Courtier et al., 1991) used with the nominal spectral truncature T358 and a stretching coefficient of 2.4, corresponding roughly to 20 km resolution over France. On the vertical axis, 41 levels describe the atmosphere from the surface to 1 hPa. The model is coupled with a four dimensional variational assimilation system (4DVAR) including classical meteorological observations and satellite data. The 11 ARPEGE simulations run over 102 h, from 6 p.m. the 23 of May, to 12 p.m. the 27 of May 2005.

The non-hydrostatic atmospheric mesoscale model Méso-NH (Lafore et al., 1998) was used to simulate the atmospheric dynamics during the same period (23 to 27 of May 2005), over the limited domain of the CERES campaign, a 300×300 km² of south western France, (Dolman et al., 2006) at 8 km resolution. 65 vertical levels describe the atmospheric column up to 13 km. The boundary conditions from the ARPEGE ensemble simulations are coupled each 3 h to constrain the meso scale model. The

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reference simulation and the ten different members run over 102 h, on the same period as the ARPEGE simulations.

The land surface scheme ISBA-A-gs (Calvet et al., 1998) was coupled on-line to the atmospheric model Méso-NH to simulate the surface fluxes, ie water, energy, and CO₂ (Sarrat et al., 2007, 2008). The land cover map describes the vegetation cover at 250 m resolution from the ECOCLIMAP database to calculate the biogenic CO₂ surface fluxes. The anthropogenic emissions are prescribed by the 10 km resolution inventory from the University of Stuttgart (IER), and the sea fluxes by Takahashi et al. (1997). The three-week simulation was run over the CERES domain of 61 by 51 grid points (respectively for x and y axis) at 8 km resolution. The interaction at each timestep between the atmospheric model and the surface scheme implies that the surface energy fluxes are slightly different in each member, amplifying the boundary layer dynamics differences. Nevertheless the initial ground water content remains identical for all the different simulations limiting the surface flux differences for the different members.

The mesoscale inversion we used to estimate the impact of the observation error structure requires the computation of the adjoint transport. This tracer backward transport was simulated here by the Lagrangian Particle Dispersion Model (LPDM) described by Uliasz (1994). Particles are released from the receptors in a “backward in time” mode with the wind fields generated by the eulerian model Méso-NH. The dynamical fields in LPDM are forced by mean winds (u, v, w), potential temperature, and turbulent kinetic energy (TKE).

2.2 Generation of the ensemble

The perturbations represented here by singular vectors are added to the large scale simulations with the global spectral model ARPEGE (Courtier et al., 1991) over the whole period and the resulting ARPEGE fields used as boundary conditions for MESONH. The optimal perturbations are defined by the maximisation of the follow-

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ing ratio

$$R_{\text{opt}} = \sqrt{\frac{\langle PM^* M_x(t_0), P_x(t_0) \rangle}{\langle x(t_0), x(t_0) \rangle}} \quad (1)$$

with M the tangent-linear propagator, $\langle \rangle$ the scalar product of the different energy terms, and P the projection operator to select the zones of interest. The solutions of the maximisation are the singular vectors of the operator, which consist of an orthonormal basis describing the most unstable directions of the initial state. The projection operator used in our study is limited to the eastern Atlantic Ocean and western Europe to extract the maximum growth rate of the errors around the final nested domain (Western Europe). In the actual ensemble prediction system, 16 singular vectors are computed and added gradually during the first 48 h of simulation.

Our approach to generating the ensemble maintains the physical and thermodynamic consistency of the coupled higher resolution model Méso-NH. Even though the parameters of the numerical scheme play a certain role in the mismatch between modeled and observed concentrations at fine scale, the atmosphere shows relevant spatial patterns depending on the meso scale structures rather than the length-scales of the perturbations. The perturbation approach has been demonstrated for the ozone air quality models (Carvalho et al., 2007). We modelled the spatial correlations on the horizontal plane based on the local structure of the variance, to filter at each grid point the noise and extract the significant correlations. This method allows then anisotropic structures on the plane.

2.3 Observation error covariance estimation and modelling

We separately estimate the horizontal, vertical and temporal components of the error correlation. For the horizontal component we follow the methodology of Pannekoucke and Massart (2008). The structural properties of the ensemble variability are estimated by using a diffusion operator based on a local diffusion tensor.

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Our starting point is the 11 (1 reference and 10 perturbations) CO₂ fields from MESONH at 1 h resolution. The estimation of the correlation tensor \mathbf{C} is computed in the model space from the ensemble, then mapped to the observation space by the operator \mathbf{P} , and weighted by the variances $\mathbf{\Sigma}$, estimated independently by data comparison, as follows:

$$\mathbf{R} = \mathbf{\Sigma} \mathbf{P} \mathbf{C} \mathbf{P}^T \mathbf{\Sigma}^T \quad (2)$$

As the ensemble is of finite size, the estimation is affected by sampling noise (Lorenz, 2003). Thus, the estimation of the local observation error has to be done carefully by extracting the signal and filtering the noise.

A possible way to filter the noise is to benefit from some local ergodic properties through some local spatial averaging of local correlation functions (Berre et al., 2007; Pannekoucke et al., 2007). Another way is to model the observation error correlation from the information contained in the vicinity of each geographical position $\tilde{\mathbf{x}}=(x, y)$. This is done following the work of Pannekoucke and Massart (2008): the local correlation function is derived through the estimation of the local anisotropy tensor $\mathbf{v}(\mathbf{x})$ under a local Gaussian approximation, and then the correlation function is modelled around each \mathbf{x} with the formulation based on the diffusion equation (Weaver and Courtier, 2001). In that framework, the anisotropy tensor is no more than the local diffusion tensor. The idea of such an error model derives from the particular solution of the homogeneous diffusion equation: if the local diffusion tensor $\mathbf{v}(\mathbf{x})$ is constant over the domain (which is assumed to be the plane), then the solution of the diffusion equation

$$\partial_t \eta = \nabla \cdot (\mathbf{v} \nabla \eta), \quad (3)$$

with initial state $\eta(\mathbf{x}, t=0) = \delta_{\mathbf{x}'}(\mathbf{x})$ where δ , the Dirac distribution, is

$$\eta(\mathbf{x}, t) = \frac{1}{2\pi|\mathbf{\Gamma}|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{x} - \mathbf{x}')^T \mathbf{\Gamma}^{-1}(\mathbf{x} - \mathbf{x}')\right), \quad (4)$$

with \mathbf{x}' the surrounding points of \mathbf{x} , and with

$$\mathbf{\Gamma} = 2t\mathbf{v}, \quad (5)$$

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and $|\Gamma|$ is the determinant of Γ . The inverse of Γ can be written as

$$\Gamma^{-1} = \begin{pmatrix} \frac{1}{L_x^2} & \frac{1}{L_{xy}} \\ \frac{1}{L_{xy}} & \frac{1}{L_y^2} \end{pmatrix}. \quad (6)$$

The scales L_x and L_y correspond to the one-dimensional differential length-scale along the direction x and y (Daley, 1991; Pannekoucke et al., 2008).

This particular heat kernel Eq. (4) can be seen as a Gaussian correlation function on the plane (except for the normalization term). Thus, it is possible to construct quasi-Gaussian correlation functions as the result of time integration of the diffusion equation. In the more general case, the local diffusion tensor $\mathbf{v}(\mathbf{x})$ is non constant, and it can be approximated locally, under some local homogeneous assumption. This approximation is based on the estimation of the local matrix $\Gamma^{-1}(\mathbf{x})$ from the estimation of the local length-scales $L_x(\mathbf{x})$, $L_y(\mathbf{x})$ and $1/L_{xy}(\mathbf{x})$. From our ensemble of simulations, we estimate the length-scales $L_x(\mathbf{x})$ and $L_y(\mathbf{x})$ at each pixel based on the gaussian approximation of the correlations for each surrounding pixel on the two axis x and y , and the terms $1/L_{xy}(\mathbf{x})$ of the local tensor with a gaussian function, averaged on the four diagonal directions. Then the local diffusion tensor $\mathbf{v}(\mathbf{x})$ is computed from (5) as half the local tensor Γ (for the particular time integration $t=1$).

The Gaussian assumption implies that the estimated correlations are positive. However, with the dynamic and the sampling noise, negative correlation can occur. In such a case, the points associated to negative correlations are not taken into account. The original domain was duplicated two times to avoid boundary issues.

The construction of the correlation is then achieved through a randomization method (Weaver and Courtier, 2001; Fisher and Courtier, 1995): 400 independent realizations of a Gaussian vector are diffused from time $t=0$ to time $t=1/2$, using the heterogeneous diffusion equation with the local diffusion tensor.

Note that the covariance tensor $\tilde{\mathbf{C}}$ modelled with the diffusion equation has to be normalized in order to obtain a correlation tensor $\mathbf{C}=\mathbf{\Lambda}\tilde{\mathbf{C}}\mathbf{\Lambda}^T$, where $\mathbf{\Lambda}$ is the diagonal

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matrix of inverse standard deviation of $\tilde{\mathbf{C}}$. This is a reappearance of the normalization term $2\pi|\mathbf{\Gamma}|^{1/2}$ that occurs in Eq. (4). In the heterogeneous case, the normalization is not constant but it can be either estimated through the randomization method, or it can be approximated by the local normalization as $\mathbf{\Lambda}^2(\mathbf{x})=2\pi|\mathbf{\Gamma}(\mathbf{x})|^{1/2}$ (when this estimation is accurate enough). Finally, the above development generates the horizontal correlation for all points in the domain. We need simply to choose points corresponding to the locations of measurements to generate the observation correlation matrix we need.

2.4 Vertical and temporal correlations

Unfortunately, there is no theory for vertical and temporal error correlations as well developed as that used in the previous section for horizontal correlation. We therefore calculate vertical and temporal correlations directly using the 11 ensemble members at our disposal. These correlations are hence more subject to sampling noise than the horizontal.

2.5 Pseudo-flux inversion and spatial error correlations

The models used in this study and the inversion framework that illustrates the impact of the filtered correlation lengths are described in Lauvaux et al. (2008). Empirical analysis will be based on the error reduction Err that depends only on the observation and the prior flux uncertainties, but not on the prior CO_2 flux nor concentration data:

$$Err = 1 - \frac{\sigma_{\mathbf{A}}}{\sigma_{\mathbf{B}}} \quad (7)$$

with σ the error associated with \mathbf{B} the background error covariance matrix, and to \mathbf{A} the posterior flux error covariance matrix estimated by the following equation:

$$\mathbf{A}^{-1} = \mathbf{B}^{-1} + \mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} \quad (8)$$

with \mathbf{H} the jacobian of the transport matrix, and \mathbf{R} the observation error covariance matrix. The error reduction presented here corresponds to the optimization of the 6-day

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flux averages over 18 days for nighttime and daytime. Due to the limited number of observations (two towers observing hourly CO₂ concentrations, ie 2×144 observations) and the large number of unknowns (2×61×51), the 6-day average inversion ameliorates the problem of under-constraint but implies complete temporal correlation of the fluxes over the period, consistent with the study of Chevallier et al. (2006).

The covariance matrix of model-data mismatch R combines spatial and temporal correlations that we extract separately from the ensemble simulations. The spatial correlation between the two tower locations is computed after the diffusion modelling step described in the Sect. 2.3, through the 400 independent realizations.

2.6 Temporal correlations combined to the spatial dimension

The temporal correlations correspond to the first three days of the simulation period, averaged over the continental part of the domain. At each hour h , the temporal correlation is estimated with each following hours $t+n$ with $1 < n < 24$. For each hour, results are averaged on the three days (fourth has no following hours).

Associated correlation tensors, C_t for the temporal component and C_s for the spatial component, are combined into C' the correlation matrix in the observation space via:

$$C' = \left(C_t^{1/2} C_s^{1/2} \right) \left(C_t^{1/2} C_s^{1/2} \right)^T \quad (9)$$

that implies similar weights of the two components, reasonable without complementary information. The final R observation error covariance matrix is then constructed as defined in the Eq. (2) by $R = \Sigma C' \Sigma^T$ with Σ the diagonal matrix of standard deviations estimated from the model-data mismatch.

3 Results

In this section we first study the structure and magnitude of the covariances derived from our ensemble. Then we investigate the effect of these, separately and in combi-

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nation, on an inversion.

3.1 Atmospheric CO₂ variability over the domain

Considering the 11 simulations at the two measurement locations of the 2005 CERES campaign (Biscarosse and Marmande towers), we estimated the variability of the different members compared to the observed CO₂ concentrations. In Fig. 1b, at the Marmande site, the ensemble spread shows a large diurnal cycle corresponding to different stability conditions during the nights between the different members of the ensemble, and similar modelled CO₂ concentrations due to well-developped convective boundary layers during the day. Maximum values of CO₂ during the night correspond to larger spread of the ensemble, but the largest modeled concentrations are smaller than the observed peaks.

At the Biscarosse location (Fig. 1a), the first two days show little spread in diurnal variability, but the last two days correspond to a large range of simulated concentrations, without any diurnal cycle. Figure 2a shows the spatial pattern of the larger CO₂ variability region including the location of the Biscarosse tower. This well-defined structure mirrors the location of the sea breeze which occurs during the afternoons of the campaign along the Atlantic coast and over the ocean. The Biscarosse tower is located close to the front of the breeze, influenced by air masses coming from the ocean or from the continent depending on the ensemble member.

The CO₂ variance spatial structures are induced by meso scale processes like the changing of the direction of the “Autan wind” over the Mediterranean sea. During the night (Fig. 2b), the CO₂ spatial patterns reflect the topography of the river valleys or mountain ranges in addition to the local flux variability. The nighttime standard deviations are larger compared to the daytime, up to 20 ppm over the continent (9 ppm during the day).

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3.2 Vertical structure of the error covariances

We computed the averaged correlation between the first model level variability and the upper levels over the continent. On the vertical axis, the spatial correlation and the CO₂ ensemble variance show distinct profiles during the night and the day (Fig. 3). During daytime, the ensemble standard deviation of CO₂ decreases linearly up to 1500 m from 2 ppm to 1 ppm, then remains constant until 3000 m, and decreases again in the mid troposphere. The associated correlation coefficients show a similar vertical profile, decreasing linearly up to 1000 m from 1. to 0.5–0.6, then constant until 3000 m about 0.5, and finally decrease to insignificant values in the upper troposphere. At night, the vertical profile of the standard deviation shows a different shape, decreasing rapidly in the first 500 m from 5 ppm to 1.5 ppm. Between 1000 m to 3000 m, the profile is constant around 1 ppm, and then decreases in the mid troposphere. Contrary to the daytime profiles, the vertical correlations decrease from 1. to 0.6 in less than 200 m, and remain constant (0.4–0.5) up to 3000 m. Compared to the nighttime standard deviation, the shape of the correlation profile suggests some residual variability in the lower troposphere from the daytime, but the surface layer is mostly influenced by the CO₂ surface flux variability and not by the lower troposphere, which is more correlated during daytime. The implication of such correlations for future inversions will be discussed in Sect. 4.

3.3 Temporal error correlations

We estimated the spatially averaged temporal correlation with the next 24 h over the whole domain. Figure 4 illustrates the temporal correlations between different reference hours with the next 24 h, starting from midday to 9 p.m. in Fig. 4a, and from midnight to 9 a.m. in Fig. 4b. The two figures show basically similar 24-h variations, with negative correlation values between the day and the night. The case starting at midday (Fig. 4a) shows non-significant correlation with the rest of the afternoon (about 0.2) whereas the later starting cases show correlation values greater than 0.5. For corre-

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lations with the next day, the strongest correlation reached 0.4, for the 9 p.m. starting case. The nighttime cases (Fig. 4b) decrease to negative values faster, in less than five hours for midnight and 1 a.m. starting cases. The nocturnal stability is reached later, from 4 a.m. to 9 a.m. cases showing larger values during the first hours (more than 0.4). As with the daytime cases, the next night shows small values of correlations (maximum 0.4).

3.4 Estimation of the correlation lengths over the domain

The diurnal variability of the ensemble variance gives rise to two separate sub-ensembles corresponding to the diurnal length-scales (from midday to 9 p.m.), and the nocturnal length-scales (from 10 p.m. to 7 a.m.). We modelled the spatial length-scales of the two sub-ensembles using the diffusion equation described in Sect. 2.3. The raw diurnal length-scales (Fig. 5a) show high values over the Mediterranean sea and the Atlantic ocean, up to 150 km, but no clear signature over the continent. After the diffusion modelling (Fig. 5b), the low length scale values map the mountain areas (Pyrenees, and Massif Central) more clearly. Higher values over the sea and the ocean remain but low values along the sea shores were filtered. In the Golfe du Lyon (between the eastern Pyrenees to the Rhone river Estuary), the length-scales originally about 50 km decrease to 30 km. On the average, the highest values over the continent are smaller after the diffusion modelling, decreasing from 80 km to 60 km. For the nocturnal length-scales, averaged initial values are smaller than nocturnal values (Fig. 5c), about 20 to 30 km over the continent, whereas oceanic length-scales are similar to diurnal values (Fig. 5d). We notice similar effects concerning mountain areas with lower values after diffusion modelling (Pyrenees). The noise over the continent is reduced after the diffusion modelling, between 0 to 10 km. On the contrary, to the North of Bordeaux, length-scales are higher than initially. We compute the correlation coefficients between Marmande and Biscarosse towers during the day and the night. The diurnal correlation is about 0.14 and the nocturnal correlation 0.05, corresponding to the length-scales of 40 to 50 km during the day, and 20 to 30 km during the night in

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Fig. 5.

3.5 Impact of the modelled covariances on the inversion

After the diffusion modelling of the length-scales and the estimation of the temporal correlations, we build-up the error covariance matrix \mathbf{R} in two steps. The spatial component called C_S (Sect. 2.5) is a tri-diagonal matrix formed by ones on the diagonal (full auto-correlation) and the two correlation coefficients for nighttime and daytime periods. These coefficients appear on two diagonals corresponding to the correlation between Biscarosse (index i) and Marmande ($i+n$) for each time step. The temporal component called C_T correlates one observation with the following ones. The temporal correlations are averaged over the four days for each hour, resulting in 24 correlation functions. Depending on the reference hour, the function correlates the next five to ten following hours, always decreasing from 1 to 0. No negative correlation is taken into account, and the correlation functions are strictly decreasing. We estimated the variances independently to the covariances, by comparison of the reference simulation to the aircraft and tower data, showing about 5 ppm during the day, up to 50 ppm for Marmande tower during the night. The standard diagonal matrix (i.e. no correlation) is compared to the “spatial only” and the “temporal only” correlation cases. The complete correlation (spatial and temporal) is finally applied to the three inversion segments. On the first period, the three different matrices are applied, and compared to the diagonal matrix case. Figure 6 shows the error reduction for daytime fluxes in the period (from 24 May to 30 May) (Fig. 6a) compared to the three different correlation cases (Fig. 6b, c, d). The “spatial only” set-up weakens the error reduction from 0% to 10%, proportionally to the original value of the error reduction (Fig. 6b). For the “temporal only” set-up (Fig. 6c), we observe the decrease of the error reduction around Marmande tower, but also increases and decreases over the rest of the domain. We discuss in Sect. 4 the origin of the positive and negative impacts. Finally, the impact of the full spatio-temporal covariance matrix is shown on Fig. 6d, highly similar to the “temporal only” case. This similarity is explained by the larger temporal correlations

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(up to 0.9) compared to the spatial correlations (0.05 to 0.15).

The combined correlation matrix is then used for the two other periods. Figure 7 shows the error reduction for the two periods (from 30 May to 5 June, and 5 to 10 June). As a preliminary comment, one can notice that the third period shows a different surface response compared to the two first periods, dominated by northern winds. We suppose here that spatial correlations established over the first four day period are the same for the three periods (18 days in total), even though atmospheric dynamics changes. This assumption has limited impact considering the small spatial correlations compared to the temporal component. The two periods show the increase of the error reduction over the Atlantic Ocean, corresponding to the sea breeze circulation of the afternoons and the northern coastal winds. On the contrary, local influence around the towers shows smaller error reductions for the two later periods.

4 Discussion

It is likely that our estimates of by the model variability of the ensemble are too low. Our perturbations were limited to the boundary conditions for MESONH. Our realisations used the same internal dynamical schemes. Larger variability is expected if we explored the full space of input parameters and formulations. The existing biases peculiar to our meso scale model affect the whole ensemble of simulations. First of all, perturbing the internal parameters means exploring different transport models. Generating such an ensemble, while maintaining the physical realism of the simulation, is a difficult task indeed. Second, modelled CO₂ concentrations are affected by flux model errors. Variance is under-estimated in this context. However our chief aim in the ensemble modelling is the correlation structure and this seems more robust than the modelled variances. As an illustration, after 18 h of simulation, the length-scales are about 50 km and remain constant during the next three afternoons. At the same time, the initial perturbations included in the boundary conditions are growing throughout the first 48 h. The identified length-scales are oriented differently depending on the

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meteorological situation but the norm of the error is constant in time.

The limited size of the ensemble could also act as a reduction of the true variance. The study of Talagrand et al. (1997) showed the linear decrease of the Brier Skill Score due to the finite size of the ensemble. Considering the correlation space, the number of singular vectors determines the estimation of the sensitivity. In our study, the 16 singular vectors selected over Western Europe are designed for short range forecasts over France. Even though we used a limited number of simulations, the operational ensemble forecast perturbs the critical regions controlling the synoptic conditions. The under estimation of the variance remains but the covariances are well-structured as shown in Figure 5 after diffusion modelling.

The error correlation length-scale used in the paper does not correspond to the classical definition used in trace gas inversions, i.e. the distance at which the correlation reaches $1/e$. Here we use the differential length-scale (Daley, 1991, p. 110), related to the curvature of the correlation function at the origin. The estimation of this length-scale is simple in the isotropic case as it is related to the power spectrum of the auto-correlation (with the help of the Wiener-Kintchine theorem). Directional approximations of this length-scale, with emphasis on there sampling distributions, exist (Pannekoucke et al., 2008).

Two sub-ensembles were derived from the temporal correlations, corresponding roughly to nocturnal and diurnal periods (each of them limited to a few hours). The length-scales (L_x and L_y) of the correlations are then associated with elliptic functions on the horizontal plane. The ellipse is defined by its longer axis denoted L , the smaller axis denoted l , with the anisotropy $1 - \frac{l}{L}$, and the angle of the anisotropy θ . As the model error is flow dependent, preferential directions of anisotropy are expected over the domain. For example, diurnal circulation dominated by advection or local thermal gradients constrains the orientation of the largest correlations.

We have calculated the correlations using the whole period. This represents a trade-off. We need a correlation model which is stable and represents the internal dynamics of the model (suggesting longer periods of integration) but on the other hand the cor-

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relation structure really should depend on flow regime. Our approach averages over several regimes and hence averages several directions of anisotropy (Fig. 8a), and makes the magnitude of anisotropy spatially homogeneous (Fig. 8b). When comparing the length-scales L_y (north-south) at the tower locations, the values are smaller (less than 30 km during the day), which corresponds to an east-west orientation of the anisotropy. In this region, the Atlantic ocean forces the error structure at Biscarosse, and the Garonne river valley does likewise at Marmande.

The above results have a range of impacts on current and future inversions. Recall that the errors we assign to an observation in an inversion are implicitly dependent on our ability to model that observation. We have shown that a part of such model errors shows a complex structure of correlations. In general, the existence of positive correlations in the model-data mismatch reduces the amount of information available from those observations (Sect. 3.5). This is not universal however. A correlation in the model-data mismatch implies a set of preferred directions for this mismatch. Mismatches in that direction will be deweighted (reducing available information) however those in orthogonal directions will be highlighted. This explains the mixed patterns in Fig. 8b.

The spatial correlation lengths we obtained do not suggest a large impact on most current observing networks since it is rare to have observations spaced less than 100 km apart. An immediate exception is measurements from aircraft. It appears that, even if the transport model we use for the inversion allows sampling at the resolution of a few km, spatial error correlation may reduce the effective sampling density. We cannot know the true import of this until we repeat the study of horizontal correlations for various lengths in the atmosphere.

A less immediate but stronger consequence is the inversion of data retrieved from upcoming satellite missions (e.g. Crisp et al., 2004; Hamazaki et al., 2004). The spatial density of these measurements is likely to impinge on the correlation length-scales of model error. Again, we need to understand how the ensemble variability projects into the column-integral that will be measured by these instruments.

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Considering tall tower based measurements with several levels, the vertical covariances we described here (see Sect. 3.2) show significant correlation between the first model level with the higher levels, up to 0.5 during the day and about 0.4 during the night. The use of several observation levels implies then model covariances that reduce the independence of the data, and so the constraint on the fluxes, but which we need to defined properly to avoid unrealistic corrections. Incorrect representation of the boundary layer height, for example, could lead to opposite increments between two levels. The same arguments hold for airborne profile data (Lauvaux et al., 2008).

For the limited observing network used here, temporal error correlations dominate spatial correlations. With the trend towards increasing numbers of continuous measurement sites this will remain the case for the surface network. It is already clear that models at coarse resolution perform poorly when simulating high-frequency observations over continents (e.g. Geels et al., 2007; Law et al., 2008). High resolution models (e.g. Lauvaux et al., 2008; Law et al., 2008) perform better. Even here, however, it appears that in such models errors are highly correlated in time.

Finally the totality of these results has serious implications for quantitative network design such as that described by Kaminski and Rayner (2008). The model-data mismatch covariance matrix is a key input into such studies (see, e.g. Gloor et al., 2000). Our results suggest a minimum separation of surface measurements of about 50 km before correlations reduce the marginal benefit of new observations. This is neither a function of model resolution nor of the footprint of measurements but of limitations in the simulation of transport.

This work hints at a promising partnership between studies in Numerical Weather Prediction and in atmospheric inversion. In general the characterisation of model error in NWP is much more advanced than in the field of inversions. It would be interesting to see how these errors appear at larger scales, both in space and time. We also need, as already noted, analogous developments for vertical and temporal correlations.

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5 Conclusions

We have used an ensemble prediction system coupled to a mesoscale transport model to estimate spatial and temporal correlations in the model-data mismatch for CO₂ inversions. Horizontal correlation lengths are of order 50 km. There are strong vertical correlations in the boundary layer, particularly at night. Temporal correlations are stronger than spatial and can last for most of a day. Taking account of these correlations reduces the effective information content of the mesoscale observations we use. The correlations also imply limits on the useful density of future observations.

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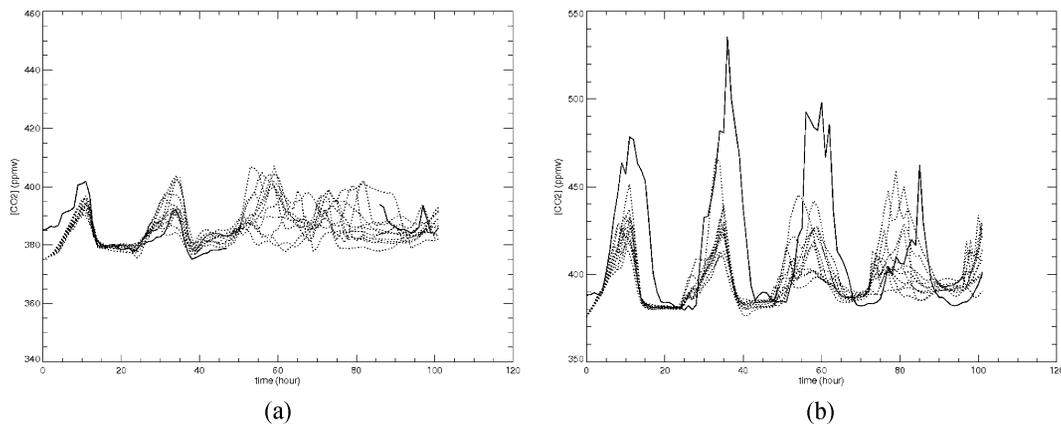


Fig. 1. Simulated CO₂ (dotted lines) from the 11 simulations over the four days at Biscarosse **(a)** and Marmande **(b)** towers, compared to the observed concentration (solid lines).

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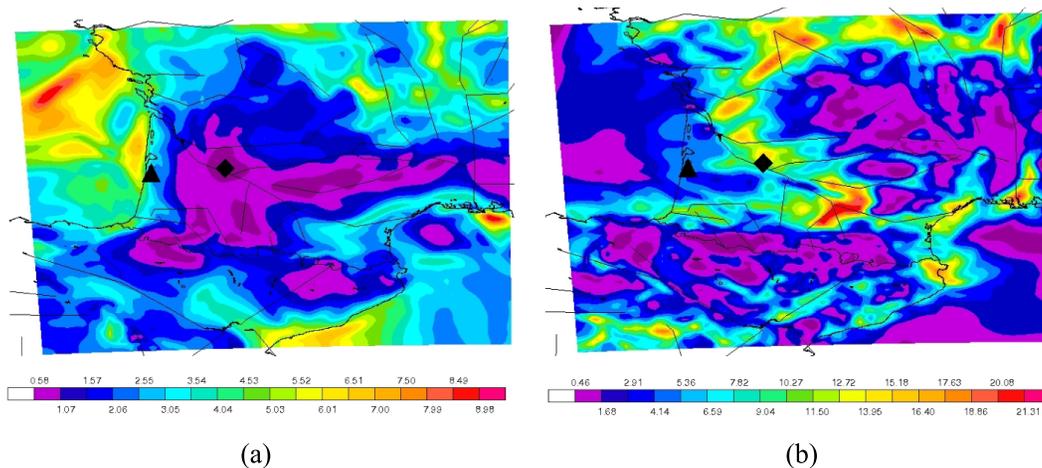


Fig. 2. CO₂ variability (standard deviation of the ensemble, in ppm) over the domain during the afternoon of 26 May 2005 **(a)** and during the night between the 26 to 27 May **(b)** (Marmande: diamond, Biscarrosse: triangle).

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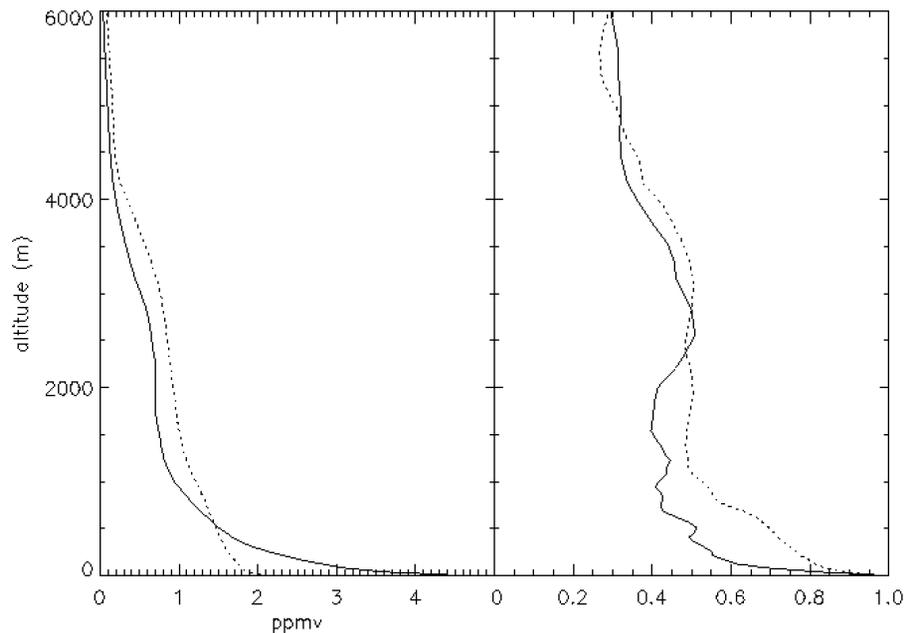


Fig. 3. Vertical CO₂ variances (left side) and associated vertical correlations (right side) over the domain during nighttime (solid line) and daytime (dashed line).

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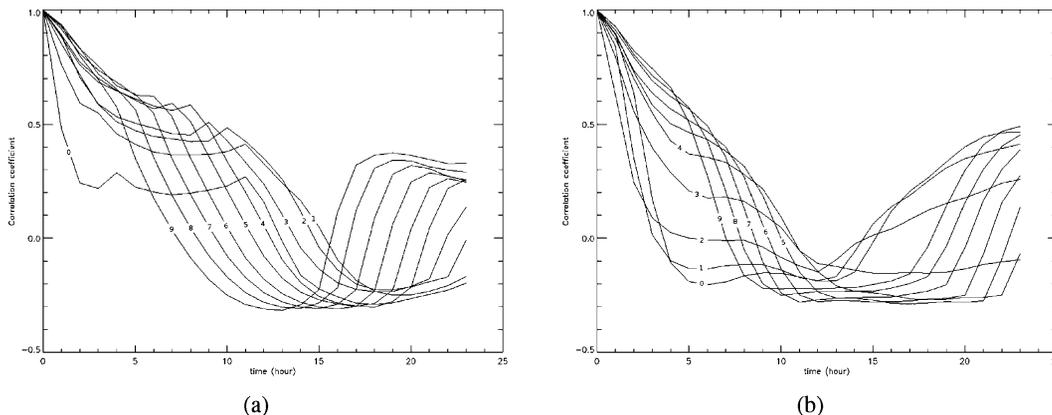


Fig. 4. Temporal correlation over the whole domain between midday to 9 p.m. with the next 24 h **(a)** and between midnight to 9 a.m. with the next 24 h **(b)**. Starting hours are indicated on the different lines, describing the temporal correlation averages on the first three days for each 24 different hours.

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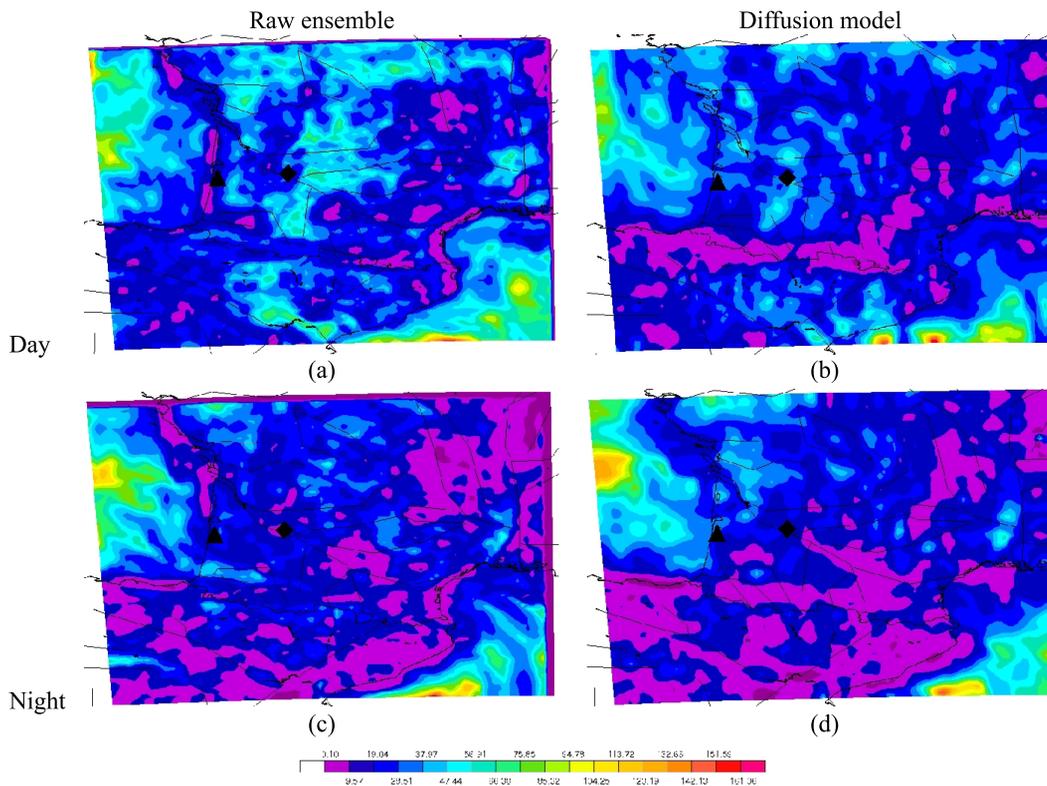


Fig. 5. Spatial length-scales (in km) over the domain during the day and night, computed from the raw ensemble ((a) and (c) respectively)) and modelled using the diffusion equation ((b) and (d) respectively).

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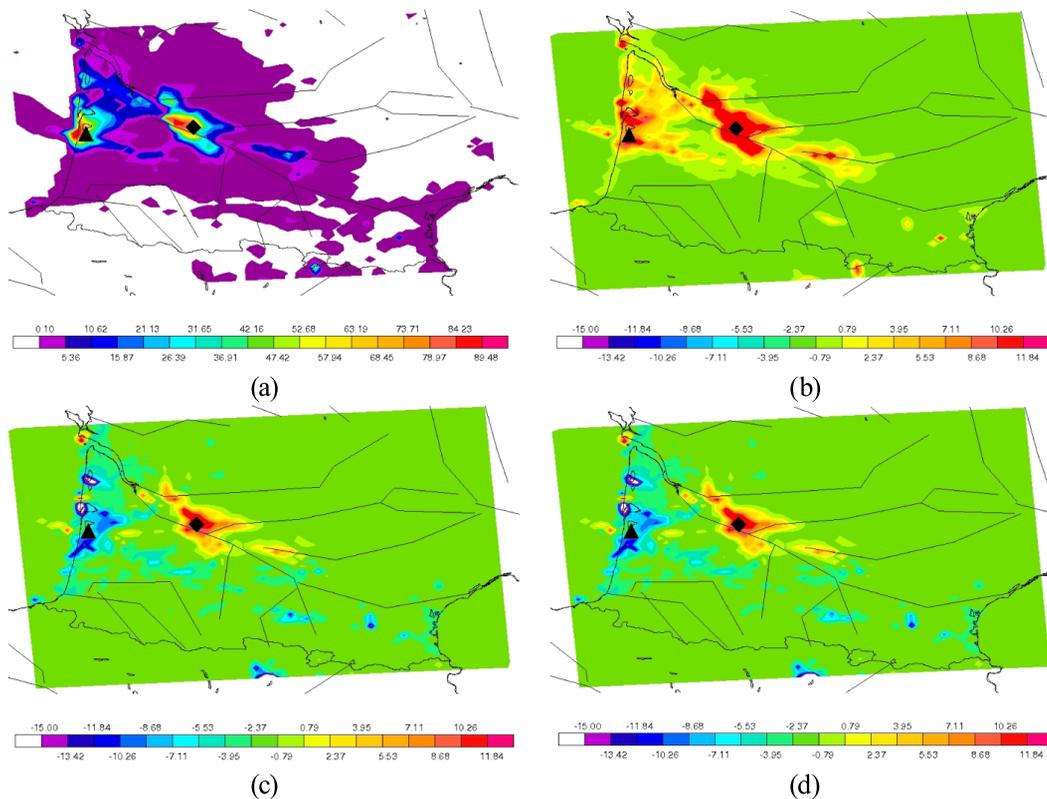


Fig. 6. Error reduction (in %) for the first 6-day inversion period (24 May to 30 May) using the uncorrelated covariance matrix (a) and the difference with the “spatial only” (b), “temporal only” (c), and fully correlated (d) cases.

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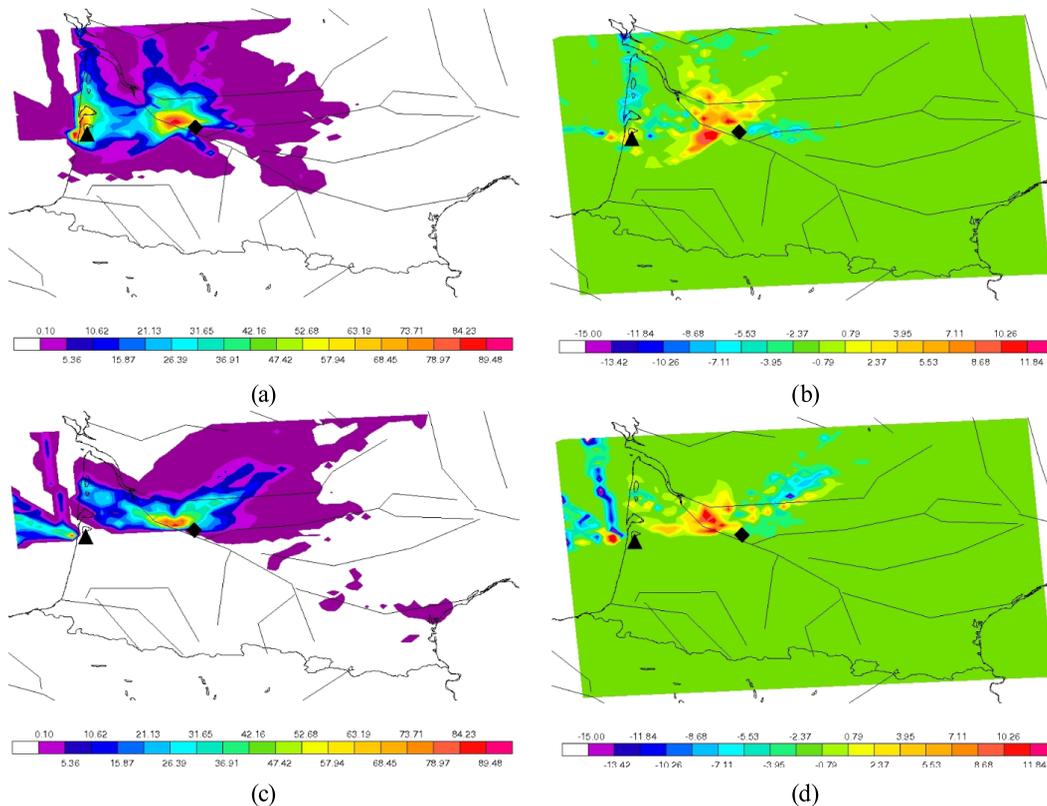


Fig. 7. Error reduction (in %) for the periods (30 May to 5 June, and 5 June to 10 June) using the uncorrelated covariance matrix (a) and (c), and their differences compared to the fully correlated matrix (b) and (d).

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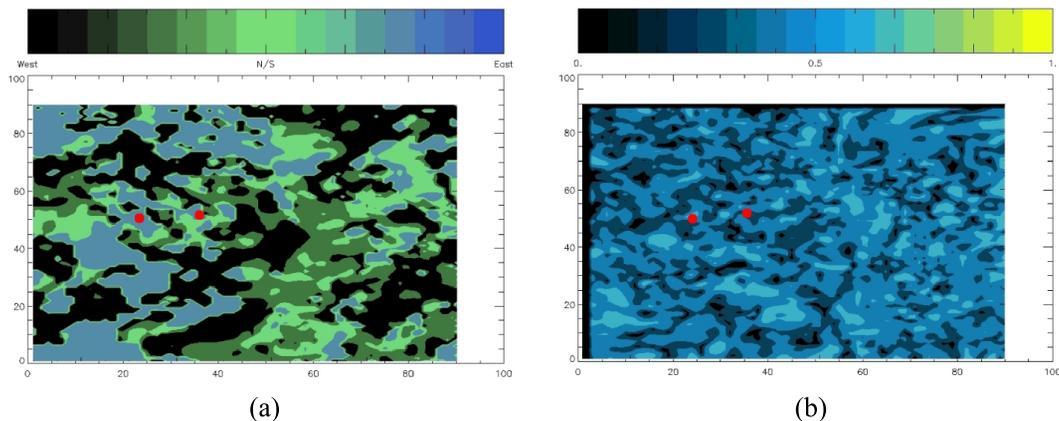


Fig. 8. Angle (θ) showing the main direction of the error correlation **(a)** and the norm representing the ratio between the long and the short axis of the ellipse ($1 - \frac{1}{L}$) **(b)** of the anisotropy modelled with the diffusion operator. The two towers are the red circles on the maps.

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