1 Supplimentary Material

 $\mathbf{2}$

Simultaneous assimilation of satellite and eddy covariance data for improving
 terrestrial water and carbon simulations at a semi-arid woodland site in Botswana

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9 Appendix A: Gap-filling procedure for LHF

10 The gap-filing procedure is done in the following steps (Appendix A of Reichstein et al., 11 2005): (1) In cases where only data of direct interest are missing, but all meteorological 12data are available, the missing values are replaced by the average values under similar 13meteorological conditions within a time-window of ± 7 days. If no similar meteorological 14conditions are present within the time window, the averaging window is increased to \pm 1514 days. (2) In cases where auxilary data such as air temperature or VPD are missing, 16 but radiation is available, the same approach is taken, but similar meteorological 17conditions can only be defined via shortwave radiation deviation less than 50 Wm⁻² and 18 the window size is not further increased. (3) In cases where radiation data are also 19missing, the missing values are replaced by the average values at the same time of the 20day $(\pm 1 h)$, i.e. by the mean diurnal course. In this case, the window size starts with \pm 210.5 days, i.e. adjacent hours. If after these steps the values could not be filled, the 22procedure is repeated with increased window sizes until the values can be filled.

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1 Appendix B: Model description of BETHY

2 Overview

3 BETHY is a process-based model of the terrestrial biosphere (Knorr, 1997, 2000). It 4 simulates carbon assimilation and plant and soil respiration embedded within a full $\mathbf{5}$ energy and water balance. Hence it is structured into four compartments: (1) energy 6 and water balance, (2) photosynthesis, (3) phenology, and (4) carbon balance. BETHY is 7driven by observed climate, and is run on a specific site, or on a regional to global spatial 8 scales. Each grid cell is designed to contain up to three different plant functional types 9 (PFTs) out of a total of 13 PFTs, with the amount specified by their fractional coverage. 10 Time steps are 1 hour for the energy balance, photosynthesis and autotrophic 11 respiration part and 1 day for the phenology, hydrology and heterotrophic respiration 12part. Control parameters in the set-up used here affect the photosynthesis, phenology 13and hydrology scheme. The actual parameters, which are optimized in this study, are 14listed with their a priori values and uncertainties in Table 3.

15

16 Photosynthesis

17 At each model grid cell, photosynthesis is simulated following the formulations of 18 Farquhar et al. (1980) or Collatz et al. (1992) for C_3 or C_4 metabolism, respectively. In 19 the case of C_3 photosynthesis, gross primary productivity (GPP) is calculated as the 20 minimum of an electron transport limited rate, J_E , and a rate, J_C , limited by the 21 carboxylation enzyme Rubisco from which the leaf or dark respiration, R_d , is subtracted,

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23 GPP = min
$$[J_c; J_E] - R_d$$
, [A1]



15 parameter,

16

17
$$J_{\max}(T_{\nu}) = J_{\max}^{25} \times a_{J,T} \times T_{\nu},$$
 [A5]
18

19 with

20

1
$$J_{\max}^{25} = a_{J,V} \times V_{\max}^{25}$$
 [A6]

 $\mathbf{2}$

Equation (A6) is a simple recasting of the usual photosynthesis equations to use the parameters $a_{J,V}$ and $V_{max}(25^{\circ}C)$ rather than the usual $J_{max}(25^{\circ}C)$ and $V_{max}(25^{\circ}C)$, and thus taking into account their correlation for the optimization. Initial values of $a_{J,V}$ are calculated from the a priori values of J_{max} and V_{max} at 25°C. The CO₂ compensation point, Γ^* , depends linearly on the vegetation temperature with the parameter $a_{\Gamma,T}$ being the dependency factor (Farguhar, 1988),

9

$$10 \qquad \Gamma^* = a_{J,T} \times T_V \tag{A7}$$

11

12 The temperature dependency of further enzyme kinetic rates such as the 13 Michaelis-Menten constants K_0 and K_c and the maximum carboxylation rate, V_{max} , are 14 computed from the following equation with E the respective activation energy (with T_v 15 in °C):

16

17
$$r(T_v) = r^{25} \exp\left\{\frac{(T_v - 25^\circ C)E}{298 KR(T_v - 273^\circ C)}\right\},$$
 [A8]

18

19 R is the general gas constant in JK⁻¹mol⁻¹, r stands for the rate in question, and r^{25} the 20 respective rates at 25°C, i.e., Ko^{25} , Kc^{25} and V_{max}^{25} (the values of Ko, Kc, and V_{max} at 21 25°C). These rates and the respective activation energies (E_{KO} , E_{KC} , and E_{Vmax}) are 22 taken as parameters in this study.

1 For C₄ photosynthesis, there are three limiting rates (Collatz et al., 1992), $\mathbf{2}$ $GPP = \min[J_C; J_F; J_i] - R_d,$ 3 [A9] $J_C = k \times Ci$, [A10] 4 $J_E = V_{\text{max}}$, [A11] $\mathbf{5}$ $J_i = \alpha_i I$, 6 [A12] 78 where in the case of C₄ photosynthesis, 9 $k^{25} = a_{IV} \times V_{\text{max}}^{25}$, [A13] 10 11 with k^{25} being the PEPcase (the initial CO₂ fixating enzyme in C₄ plants) CO₂ specificity 12at 25°C. The temperature dependency of k is then also calculated by equation (9) with 1314the activation energy E_k as a parameter. Here a is the integrated C4 quantum efficiency, 15and $a_{J,V}$ and α_i are both parameters. The dark respiration R_d is calculated as (Farquhar 16et al., 1980; Knorr, 1997) 17 $R_d(25^{\circ}C) = 0.011V_{\max}^{25}(C_3)$ or $0.042V_{\max}^{25}(C_4)$, 18 [A14] 1920The temperature dependency of R_d is also given by equation (A8) with the 21activation energy $E_{\rm R}$ as a controlling parameter. 2223Carbon balance

4

1	The net primary productivity (NPP) is calculated as gross uptake of CO_2 by the
2	leaves (GPP) minus total autotrophic respiration which includes plant maintenance
3	respiration $R_{\rm M}$ and growth respiration $R_{ m G}$. Following Knorr (2000), $R_{ m M}$ is calculated
4	from the leaf respiration as
5	
6	$R_M = R_d / f_{R,leaf} , \qquad [A15]$
7	
8	with $f_{R,leaf}$ the leaf fraction of the maintenance respiration. Growth respiration is itself
9	proportional to NPP and calculated as follows:
10	
11	$R_G = (f_{R,growth} - 1)NPP = (f_{R,growth} - 1)(GPP - R_M - R_G), $ [A16]
12	
13	where $f_{R,growth}$ is the amount of carbon to be produced for a unit gain in vegetation
14	biomass.
15	The net ecosystem productivity (NEP) is now defined as
16	
17	$NEP = NPP - R_s = NPP - R_{s,f} - R_{s,s}, \qquad [A17]$
18	
19	where the heterotrophic soil respiration ($R_{\rm S}$) is composed of respiration from a
20	short-lived litter pool with time-varying size and a long-lived soil carbon pool (Knorr,
21	2000). Input to the litter pool is parameterized by the annual course of LAI in the case of
22	deciduous PFTs or, in the case of evergreen PFTs, as a constant fraction of the leaf
23	carbon pool. Soil respiration is assumed to be temperature and soil moisture dependent

and calculated from the following equations:

$$R_{s} = (1 - f_{s})k_{f}C_{f} + k_{s}C_{s}, \qquad [A18]$$
with the sizes of the fast or litter pool, *G*, and the slow pool, *C*. *L* is the fraction of
decomposition from the fast pool that goes to the long-lived soil carbon pool. The rate
constants k_{f} and k_{s} for fast and slow pools are

$$k_{f} = \alpha^{x}Q_{10f}^{T_{0}/0}/\tau_{f}, \qquad [A19]$$

$$k_{s} = \alpha^{x}Q_{10f}^{T_{0}/0}/\tau_{s}, \qquad [A20]$$
where T_{s} is the air temperature, a^{s} the soil moisture dependence parameter, Q_{00f} and
 $Q_{00,s}$ temperature dependence parameters, and v and v_{s} the pool turnover times at 25°C.
Stomatal control
The model of stomatal control follows the assumption that, in the absence of water
stress, leaf-level photosynthesis operates at a standard ratio between the leaf internal
CO₂ concentration, C_{s} , and the CO₂ concentration of free air, C_{a} . This value is given by

$$C_{i,0} = f_{C}C_{a}, \qquad [A21]$$
with two values for f_{2} , one for C3 and one for C4 vegetation. In order to determine the
demand for CO₂ uptake, A_{n} is first calculated as A_{n} , 0 for $C_{i} = C_{i,0}$, and $T_{i} = T_{a}$. Inversion

of the diffusion equation for CO₂ at the stomatal boundary is then used to compute
stomatal conductance in the absence of water stress at each canopy layer (in ms⁻¹):

4
$$g_{s,0} = \frac{1.6A_{n,0}}{C_a - C_{i,0}} \frac{RT_k}{p},$$
 [A22]

 $\mathbf{5}$

p is air pressure (in Pa). If at the time of highest demand, *D*, transpiration rates exceed
a root water supply rate, *g*_s, stomatal conductance at each canopy layer is reduced
according to

9

10
$$g_s = \frac{g_{s,0}}{1 + b_e D_a},$$
 [A23]

11

12 The factor b_e is assumed to change with soil water status in such a way that during the 13 course of a day, the transpiration rate, E_t , does not exceed a root supply rate, S, 14 described by Federer (1982),

15

16
$$S = C_W \frac{W}{W_{\text{max}}}$$
, [A24]

17

18 W is the soil water content, adjusted to take soil freezing into account (Knorr, 1997), 19 and C_w an empirical parameter representing root density. W_{max} is the maximum plant 20 available soil water and used as a parameter here.

1	Next, the canopy temperature, $T_{\rm c}$, is computed consistent with the energy balance
2	after integrating g_{s} over the canopy to obtain the canopy conductance used in the
3	Penman–Monteith equation. Then, the photosynthesis model is run again, but at a fixed
4	stomatal conductance, $g_{\rm s}$, which yields the final GPP.
5	
6	Latent heat flux
7	Latent heat flux (λE) is the sum of two terms,
8	
9	$\lambda E = \lambda E_{v} + \lambda E_{s}, \qquad [A25]$
10	
11	where λ is latent heat, $E_{\rm v}$ is transpiration from vegetation and $E_{\rm s}$ is evaporation from
12	soil. E_v is primarily driven by the net radiative balance of the vegetation $(R_{n,v})$ and is
13	limited by the available amount of soil (W_s) and skin or intercepted water (W_i). If the
14	vegetation surfaces are wet ($W_i > 0$), the canopy conductance is infinite ($G_c \to \infty$) so that
15	evaporation follows its maximum rate, $E_{\rm v,max}$, with the evaporated water coming from
16	the skin or intercepted reservoir:

17

18
$$E_v = E_i = E_{v,\max} = \frac{sR_{n,v} + \rho C_p(e_s(T) - e_a)G_a}{s + \gamma}$$
 (Wi>0), [A26]

19

Here, it is allowed to assume negative values during dew formation. When the vegetation is dry ($W_i = 0$), evapotranspiration is determined by G_c , which is the combined conductance of all stomata within the plant canopy,:

1

2
$$G_c = \int_0^\Lambda g_s(l) dl, \qquad [A27]$$

3

4 Λ denote the leaf are index (LAI) of the canopy, and *dl* its differential element. The
5 transpiration rate is then calculated from the Penman-Monteith formula (Monteith
6 1965):

7

8
$$E_v = E_t = \frac{sR_{n,v} + \rho C_p(e_s(T) - e_a)G_a}{s + \gamma (1 + G_a/G_c)}$$
 (W_i=0), [A28]

9

10 An additional condition is given by $E_t \ge 0$, i.e. transpiration is only allowed from the 11 vegetation to the atmosphere.

The daily integral of the evaporation rate *E*_i also depends on the size of the skin
reservoir, *W*_i, and the rain input:

14

15
$$E_{i}(t)\Delta t = \min\left\{\int_{1day} E_{\nu,\max}(t')dt'; W_{i}(t-\Delta t) + (P_{i}(t)-P_{\nu}(t))\Delta t\right\},$$
 [A29]

16

17 with a one-day time step Δt . In the model, this and all other daily values are 18 approximated by integrating over the instantaneous rates given at the hourly time 19 steps.

According to equation 29, transpiration can only happen when the vegetation is dry. To account for this fact when calculating the daily rate $E_{\rm v}\Delta t$, a time average wetness fraction is defined: $F_i = \frac{E_i(t)\Delta t}{\int E_{\nu,\max}(t')dt'},$ [A30] and daily transpiration is reduced accordingly: $E_i(t)\Delta t = (1 - F_i) \int_{1day} \frac{sR_{n,v} + \rho C_p \Delta eG_a}{s + \gamma (1 + G_a/G_c)},$ [A31] The aerodynamic exchange between the canopy and the free air is described as $G_a = g_{a,v} u$, [A32] with wind speed, u, and a proportionality factor serving as a free model parameter.

14 with wind speed, u, and a proportionality factor serving as a free model parameter. 15 Wind speeds below 1 ms⁻¹ are uniformly set to 1ms⁻¹ to avoid unrealistically high canopy 16 temperature under conditions of extremely still air and high incoming radiation. The 17 prior value of $g_{a,v}$ is determined from the following formula:

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$$g_{a,v} = \frac{k^2}{\left[\ln(h_{ref}/r_z h_c + a_z)\right]^2}$$
 [A33]

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1 h_{ref} is the reference height above canopy (10 m), h_{c} the canopy height, k = 0.41, $r_{\text{z}} = 0.1$ 2 and $a_{\text{z}} = 1$.

3

4 Air humidity

 $\mathbf{5}$ Since no reliable data of near-surface air humidity exist for purposes of global modelling, 6 this quantity has to be estimated. It is often assumed that the daily mean of the vapour 7pressure is equal to the saturation vapour pressure at the daily minimum temperature. 8 Friend (1998) has checked this assumption with climate data by Müller (1982) and has 9 found a good agreement for Europe and North America. For weather stations in arid 10 regions, however, agreement is much less satisfactory, resulting in an overall 11 correlation coefficient of = 0.87 for 805 stations. An overestimate of the vapour pressure 12occurs, when the air is not saturated at the minimum temperature, T_{\min} , as under 13severe drought (Running et al. 1987), and an underestimate, when the vapour pressure 14rises during the day because of evapotranspiration (Rosenberg, 1974).

In order to account for such findings, the daily course of the vapour pressure, $e_s(t)$, is calculated from instantaneous saturation vapour pressure, $e_s(T)$, saturation vapour pressure at sunrise, $e_s(T_{\min})$, and the ratio of daily mean evapotranspiration and daily mean evaporative demand (cf. subsection "soil water balance"). Variation of the dependence is achieved through the parameters h_0 (relative humidity at sunrise, when $T=T_{\min}$, and total drought, i.e. $f_e = 0$) and \hat{h} (daily amplitude of the vapour pressure under moist conditions, i.e. = 1, as a fraction of the amplitude at constant saturation):

22

23
$$e_a = e_{a0} + f_e \hat{h}(e_s(T) - e_{a0}),$$
 [A34]

1 where

2
$$e_{a0} = (h_0 + (1 - h_0)f_e)e_s(T_{\min}),$$
 [A35]

3 and

4
$$f_e(t) = \frac{E(t - \Delta t)}{E_{t,\max}(t - \Delta t) + E_{s,\max}(t - \Delta t)},$$
 [A36]

 $\mathbf{5}$

6 f_{e} is defined as the ratio of actual evapotranspiration to potential evapotranspiration 7 from vegetation and soil. For the computation of e_{a} , the value of the preceding time step 8 Δt of one day is taken. The saturation vapour pressure over water or ice, $e_{s}(T)$, is 9 calculated from Murray (1967):

11
$$e_s(T) = \begin{cases} 610.78 \exp(17.269T/(237.3+T)) & \text{for } T > 0\\ 610.78 \exp(17.269T/(271.15+T)) & \text{for } T < 0 \end{cases}$$
 [A37]

12

13 Energy and radiation balance

PAR absorption is calculated according to the two-flux scheme by Sellers (1985) with three vertical layers of equal LAI. The diffuse fraction of PAR is calculated according to a procedure by Weiss and Norman (1985). Leaf-angle distribution is assumed to be uniform, and the only free parameters for this scheme is ω , the leaf single-scattering albedo. To determine evapotranspiration rates from the Penman–Monteith formula, BETHY computes net radiation balance of the canopy, $R_{n,c}$, according to the following equation:

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1
$$R_{n,c} = (1 - t_{1,V}) [(\varepsilon_a - \varepsilon_{sfc}) \sigma T_{K,a}^4 - G] - (1 - a_c - a_s) f_{PAR} R_s,$$
 [A38]

 $\mathbf{2}$

3 ε_{a} and $\varepsilon_{sfc} = 0.97$ are sky and surface emissivity, respectively, $T_{K,a}$ air temperature in Kelvin, $\sigma = 5.6703 \text{ x } 10^8 \text{ Wm}^{-2}\text{K}^{-4}$ the Stefan–Boltzmann constant, and t_{v} the longwave 4 transmissivity of the vegetation, assumed $t_{l,v} = f_c \exp(-0.5A/f_c) + (1 - f_c)$. f_c is the fraction $\mathbf{5}$ 6 of soil covered by vegetation. For the shortwave part, Rs is incoming solar radiation 7 (Wm^{-2}) , f_{PAR} is the fraction of PAR absorbed by the vegetation and computed by the 8 two-flux scheme, a_v the albedo of the vegetation at the limit of high LAI and closed 9 canopy, and as the amount of solar radiation absorbed by the soil under the canopy at 10 the same limit. G is the ground heat flux, assumed to be a fixed fraction of total net 11 radiation. (Contrary to $R_{n,c}$, total net radiation does not depend on G, so there is no 12implicit equation to be solved.) The sky emissivity is computed from

13

14
$$\varepsilon_a = \varepsilon s \left(\frac{e_a}{T_{K,a}} \right)^{1/7} \left(1 + 0.22 n_c^2 \right),$$
 [A39]

15

16 with the cloud cover fraction n_c . If no separate radiation data for PAR and solar 17 radiation are available, R_s is calculated from PAR according to Weiss and Norman 18 (1985).

19

20 Phenology

We assume that spatial variability within a grid cell is entirely the result of differences in the response among the existing PFTs, which are defined by threshold parameter for 1 each PFT as the trigger. This parameter is assumed to have a Gaussian probability 2 distribution in space. There are two of those threshold parameters: \tilde{T}_{ϕ} and \tilde{t}_{c} . It is 3 important to note that the transition to the active state requires both $\tilde{T}_{\phi} > T$ and $\tilde{t}_{c} > t_{d}$, 4 where T is a temperature and t_{d} length of day. The tilde ($\tilde{}$) denotes that these are 5 parameters. As shown next, these parameters are integrated over their probability 6 distribution, replacing the integration across the space of the grid cell.

7 Before proceeding to the spatial integration, we define a generic differential 8 equation in time for the LAI of individual plants, $\tilde{\Lambda}(t)$:

9

10
$$\frac{d\widetilde{\Lambda}(t)}{dt} = \begin{cases} f_1, \text{ if } T \ge \widetilde{T}_{\phi} \text{ and } t_d \ge \widetilde{T}_e, \\ f_2, else, \end{cases}$$
[A39]

11

12 where f_1 and f_2 are some arbitrary functions of the state of the vegetation. In this 13 discrete formulation, the response of LAI to changes in \tilde{T}_{ϕ} or \tilde{t}_c is usually 14 nondifferentiable at the threshold. The continuous version of equation (A39), which is 15 valid for the spatially integrated LAI, $\Lambda(t)$, resolves to an integral over the Gaussian 16 probability density functions (PDF), p and q, of the two trigger variables.

17

18
$$\frac{d\Lambda(t)}{dt} = f_1 \int_{-\infty-\infty}^{T} \int_{-\infty-\infty}^{t_d} p(\widetilde{T}_{\phi}) q(\widetilde{t}_c) d\widetilde{T}_{\phi} d\widetilde{t}_c + f_2 \left(1 - \int_{-\infty-\infty}^{T} \int_{-\infty-\infty}^{t_d} p(\widetilde{T}_{\phi}) q(\widetilde{t}_{\phi}) d\widetilde{T}_{\phi} d\widetilde{t}_c \right)$$
[A40]

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20 The spatial PDF p is characterized by a mean T_{ϕ} and its standard deviation $T_{\rm r}$, while 21 the mean of q is $t_{\rm c}$ and the standard deviation tr. All four are CCDAS control

parameters. Note the distinction between these two spatial PDFs and the fact that their
four parameters have again PDFs in the Bayesian sense in the same way as all other
control parameters.
The previous expression simplifies to

5
$$\frac{d\Lambda(t)}{dt} = ff_1 + f_2(1-f)$$
[A41]

6

7 With

8

9
$$f = \int_{-\infty}^{T} p(\widetilde{T}_{\phi}) d\widetilde{T}_{\phi} \int_{-\infty}^{t_d} q(\widetilde{t}_c) d\widetilde{t}_c = \Phi\left(\frac{T - T_{\phi}}{T_r}\right) \Phi\left(\frac{t_d - t_c}{t_r}\right)$$
[A42]

10

11 where Φ is the cumulative normal distribution. *f* is the fraction of plants within the 12 proportion of a grid cell occupied by each PFT that are actively growing or maintaining 13 leaves.

14

15 4.3. Time evolution of LAI

16 Describing the generic formulation for the time evolution of LAI of a single plant – 17 which is then integrated spatially via equation (A41) – requires definition of f_1 (for 18 plants in their growing season) and f_2 (for senescent plants). For f_1 , we assume the 19 simplest formulation that satisfies the following two conditions: leaf growth starts 20 immediately and is not limited by substrate availability, such as LAI itself; and growth 1 stops if a target LAI is reached that is in balance with the environmental limitations, 2 described as Λ_{max} . These conditions are met by the following formulation:

3

$$f_1 = \xi (\Lambda_{\max} - \Lambda), \tag{A43}$$

 $\mathbf{5}$

4

6 where ξ is a linear growth constant describing the increase in LAI per time unit shortly 7after bud burst. This rate is chosen to be independent of carbon gains (NPP), because 8 initial leaf development relies on buds and reserves from the previous year (Kaduk and 9 Heimann, 1996). This formulation differs from those used in similar applications, such 10 as the logro-P phenology model (C. Reick, personal communication, 2010, implemented 11 by Raddatz et al., 2007) or the one by Liu et al. (2008), where the initial growth is 12exponential resulting in a logistic function for the time integral under constant conditions. Equation (A43) results in a time dependence described by $\Lambda(t)/\Lambda_{max} = 1 - 1$ 1314 $\exp(-\xi_t)$ for $\Lambda(0) = 0$, which is linear for small t. The advantage of this approach is that it 15does not require setting a minimum LAI to set off growth, which was 0.1 in the work by 16 Liu et al. (2008). This would not work here, because $\Lambda_{\rm max}$ might be less than such a 17minimum value. The difference in approach can be explained by the fact that the work 18 just cited is restricted only to temperature controlled phenology and does not include 19situations where either the temperature or the water balance only allows small values 20of LAI. For those plants that are outside their growth stage, we again chose the simplest 21formulation that allows accommodating both deciduous and evergreen phenology:

22

23
$$f_2 = \Lambda/\tau_L$$
, [A44]

 $\mathbf{2}$ The new parameter τ_L , which is related to leaf longevity, describes how quickly leaves 3 are shed, or whether they stay inactive until the next growing season. Deciduous 4 vegetation will normally shed leaves (which includes leaves turning brown, see above) $\mathbf{5}$ within days to weeks. Evergreen vegetation on the other side should have values at the 6 order of a year or more. We now consider evolution of the spatially integrated grid-cell 7average LAI. Inserting equation (A43) and (A44) into equation (A41) yields: 8 $\frac{d\Lambda(t)}{dt} = \xi [\Lambda \max - \Lambda(t)] f - \frac{\Lambda(t)}{\tau_t} (1 - f),$ 9 [A45] 10 11 In order to find a convenient form for integrating this expression, we define 12 $r = \xi f + (1 - f)/\tau_{I},$ [A46] 131415and 16 $\Lambda_{\rm lim} = \xi \Lambda_{\rm max} f / r \,,$ [A47] 1718so that equation (A41) takes the form: 1920 $\frac{d\Lambda(t)}{dt} = \xi \Lambda_{\max} f - r\Lambda(t),$ 21[A48]

1
$$= r[\Lambda_{\rm im} - \Lambda(t)], \qquad [A49]$$

 $\mathbf{2}$

3 As long as f and Λ_{max} (and therefore r and Λ_{min}) do not depend on t, the equation above 4 has the following solution:

 $\mathbf{5}$

6
$$\Lambda(t + \Delta t) = \Lambda \lim_{t \to \infty} - [\Lambda \lim_{t \to \infty} - \Lambda(t)]e^{-r\Delta t}$$
, [A50]
7

8 Here it is sufficient to state that Λ_{max} depends on quantities that are updated either 9 daily or every few days, while *f* depends on daily values of temperature and day length. 10 Therefore, the last equation can be used to integrate over a single daily time step of the 11 phenology scheme, Δt . This mixture of analytical and numerical integration is not only 12 highly efficient, it also ensures stability, i.e. it avoids negative Λ as long as Λ_{lim} and $\Lambda(0)$ 13 are non-negative.

14

15 4.5. Water and structural limitations

16On a global scale, the main limiting factor on terrestrial plant growth is not 17temperature, but water (Woodward, 1987). Whenever photosynthesizing, plants loose 18water by transpiration through pore openings ("stomata") in their leaves. This 19limitation together with any other limitation on leaf growth, here considered 20"structural", is described by a single state variable, Λ_{max} . If soil water is limiting, an 21increasingly negative soil water potential leads to a falling leaf potential in a 22complicated process involving root water uptake, xylem resistance to flow, and transpiration through the leaf stomata. If stomata close, leaves can retain water, but 23

1 only to a degree that depends on its cuticular resistance, which in itself is dependent on $\mathbf{2}$ the plant functional type (PFT). However, it is not possible to represent those complex 3 mechanisms in a model designed for global-scale applications. The scheme chosen here goes back to Woodward (1987), who used annual potential evapotranspiration and 4 $\mathbf{5}$ precipitation to derive water limited LAI on a global scale. To accommodate the shorter 6 time scale of our model, we have modified his scheme by using daily actual 7transpiration and soil moisture instead of annual potential evapotranspiration and 8 precipitation. However, to have LAI react not to rapidly changing daily conditions but to 9 the longer-term climatic state, the water limited LAI is averaged back in time using the 10 same approach as for T. Generally speaking, leaf development will stop and leaves will 11 be shed if there is insufficient soil water for transpiration. At which level this happens 12exactly will be a function of various drought adaptations of the PFT concerned. 13Independent of the details, however, adaptation will determine how long the plant at a 14given LAI Λ can survive with a given amount of soil moisture without rain. This time scale, τ w, can serve as a universal parameter of water limitation. This defines a 1516water-limited LAI, Aw through

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18
$$E(\Lambda_W)\tau_W = W$$
, [A51]

19

where W is plant-available soil moisture. What is needed then is the total water loss after time τ_W as a function of leaf area.

To compute this water loss, we linearize the potential rate of transpiration, *E*, as a
function of the LAI, *Λ*:

1

2
$$E(\Lambda) \approx \frac{\widetilde{E}}{\widetilde{\Lambda}} \Lambda$$
, [A52]

3

4 \tilde{E} is the daily mean potential rate of transpiration last computed by the model at a LAI 5 of $\tilde{\Lambda}$. This approximation is most accurate at low values of Λ and $\tilde{\Lambda}$, where net 6 radiation of the leaf canopy, which drives evapotranspiration (Jarvis and McNaughton, 7 1986), can be assumed to scale linearly with LAI.

- 8 Combining Equ. (A51) and Equ. (A52) yields
- 9

10
$$\Lambda_W = \frac{W\Lambda}{\widetilde{E}\,\tau_W}$$
. [A53]

11

12The parameter τ_W represents the expected length of drought periods. For $\tau_W \to 0$ the 13plant "expects" its water reserves to always be sufficient for continuing survival. In this case, $\Lambda w \rightarrow \infty$, meaning the plant has no explicit drought adaptation in its phenology. 1415This is assumed for the cold-deciduous and cold-evergreen PFTs. For warm-evergreen 16plants, we expect the value for τw in the region of 1 year, and for grasses and 17warm-deciduous plants between one and two months. Water limitation is implemented 18separately for each PFT to reflect differences in the water use strategy, defined mainly 19by τw.

20 Observe also that for $\tilde{E} \to 0$ we have $\Lambda w \to \infty$, since without evaporative demand 21 the leaf area is not water limited, as it is the case for $\tau w \to 0$. Since the LAI cannot grow indefinitely, it must be limited by other factors, such as light availability, nutrients and
 structure. These additional limitations are summarized into a single universal
 parameter (cf., Knorr (2000)) and incorporated into the model via:

5
$$\tilde{\Lambda}_{max} = \upsilon (\hat{\Lambda}, \Lambda_w).$$
 [A54]
6
7 $\nu(\mathbf{x}, \mathbf{y})$ is a smoothed minimum function defined by

8

4

9
$$v(x,y) = \frac{x+y-\sqrt{(x+y)^2-4xy}}{2\eta}$$
 [A55]

10

11 with $\eta = 0.99$. $\tilde{\Lambda}_{max}$ is recomputed daily with daily values of the soil moisture, W, 12 whereas \tilde{E} may be recomputed at longer intervals. This allows avoiding re-computing 13 the diurnal cycle of photosynthesis and energy balance for every simulated day, while 14 keeping a daily time step of phenology and water balance, to save computing time with 15 the full BETHY model (Knorr, 2000).

16 Instead of $\tilde{\Lambda}_{max}$, equation (A43) uses Λ_{max} , its weighted time integration 17 computed in the same way as T from T_{2m} (equation (A42)), with the analogous 18 definition:

19

20
$$\Lambda_{\max}(t) = \frac{1}{\tau_s} e^{-t/\tau_s} \int_{-\infty}^{t} \widetilde{\Lambda}_{\max}(t') e^{t'/\tau_s} dt' .$$
 [A56]

21

1 It is computationally favourable to bring equation (A56) to an incremental form:

 $\mathbf{2}$

$$\Lambda_{\max}(t + \Delta t) = \frac{1}{\tau_s} e^{-(t + \Delta t)/\tau_s} \int_{-\infty}^{t + \Delta t} \widetilde{\Lambda}_{\max}(t') e^{t'/\tau_s} dt'$$

$$3 = \frac{1}{\tau_s} e^{-\Delta t/\tau_s} e^{-t/\tau_s} \left(\int_{-\infty}^t \widetilde{\Lambda}_{\max}(t') e^{t'/\tau_s} dt' + \int_t^{t + \Delta t} \widetilde{\Lambda}_{\max}(t') e^{t'/\tau_s} dt' \right).$$

$$= e^{-\Delta t/\tau_s} \Lambda_{\max}(t) + \frac{1}{\tau_s} e^{-(t + \Delta t)/\tau_s} \int_t^{t + \Delta t} \widetilde{\Lambda}_{\max}(t') e^{t'/\tau_s} dt'$$
[A57]

4

5 If Δt is very short and thus the time incremental of Λ_{\max} is assumed to be constant, and 6 then is expressed simply:

7

8
$$\Lambda_{\max}(t + \Delta t) = e^{-\Delta t/\tau_s} \Lambda_{\max}(t) + \hat{\Lambda}_{\max}(t) (1 - e^{-\Delta t/\tau_s}), \qquad [A58]$$

9

10 The advantage of this scheme is that it has only one free parameter, τ_{W} , in addition to τ_{S} . 11 Since changing τ_{S} might lead to instabilities of the optimization in a way similar to τ_{m} , 12 this parameter is also held constant at value of 30 days. Detailed information for this 13 phenology part is to be referred to Knorr et al. (2010).

- 14
- 15

16 References

17 Collatz, G. J., Ribas-Carbo, M., and Berry, J. A.: Coupled photosynthesis-stomatal

18 conductance model for leaves of C₄ plants, Australian Journal of Plant Physiology, 19,

19 519-538, 1992.

1	Farquhar, G. D., von Caemmerer, S., and Berry, J. A.: A Biochemical Model of
2	Photosynthetic CO_2 Assimilation in Leaves of C_3 species, Planta, 149, 78-90, 1980.
3	Farquhar, G. D.: Models relating subcellular effects of temperature to whole plant
4	response, Symp. Soc. Exper. Biol., 42, 395–409, 1988.
5	Federer, C. A.: Transpirational supply and demand: plant, soil, and atmospheric effects
6	evaluated by simulation, Water Resources Research, 18, 355–362, 1982.
7	Friend, A. D.: Parameterisation of a global daily weather generator for terrestrial
8	ecosystem and biogeochemical modelling, Ecological Modelling, 109, 121-140, 1998.
9	Jarvis, P. G., and McNaughton K. G.: Stomatal control of transpiration: scaling up from
10	leaf to region, Advances in Ecological Research, 15, 1-49, 1986.
11	Kaduk, J., and Heimann, M.: A prognostic phenology scheme for global terrestrial cabon
12	cycle models, Clim. Res., 6, 1–19, 1996.
13	Knorr, W.: Satellitengestu [¨] tzte Fernerkundung und Modellierung des globalen
14	CO ₂ -Austauschs der Landvegetation: Eine Synthese, Ph.D. thesis, Max-Planck-Inst.
15	fu" r Meteorol., Hamburg, Germany, 1997.
16	Knorr, W.: Annual and interannual CO ₂ exchanges of the terrestrial biosphere: Process
17	based simulations and uncertainties, Glob. Ecol. Biogeogr., 9(3), 225–252, 2000.
18	Knorr, W., Kaminski, T., Scholze, M., Gobron, N., Pinty, B., Giering, R., and Mathieu,
19	PP.: Carbon Cycle Data Assimilation with a Generic Phenology Model, J. Geophys.
20	Res. Atmos, 115, G04017, doi:10.1029/2009JG001119, 2010.
21	Liu, Q., Gu, L., Dickinson, R. E., Tian, Y., Zhou, L., and Post, W. M.: Assimilation of
22	satellite reflectance data into a dynamical leaf model to infer seasonally varying

leaf areas for climate and carbon models, J. Geophys. Res., 113, D19113,

1

2	doi:10.1029/2007JD009645, 2008.
3	Müller, M. J.: Selected Climatic Data for a Global Set of Standard Stations for
4	Vegetation Science. 306 pp. Dr. W. Junk Publishers, The Hague, 1982.
5	Murray, F. W.: On the computation of saturation vapour pressure, J. Applied
6	Meteorology, 6, 203-204, 1967.
7	Raddatz, T. J., C. Reick, W. Knorr, J. Kattge, E. Roeckner, R. Schnur, KG. Schnitzler,
8	P. Wetzel, and J. Jungclaus (2007), Will the tropical land biosphere dominate the
9	climate carbon cycle feedback during the 21st century?, Clim. Dyn.,
10	doi:10.1007/s00382-00007-00247-00388.
11	Reichstein, M., Falge, E., Baldocchi, D., Papale, D., Aubinet, M., Berbigier, P.,
12	Bernhofer, C., Buchmann, N., Gilmanov, T., Granier, A., Grunwald, T., Havrankova,
13	K., Ilvesniemi, H., Janous, D., Knohl, A., Laurila, T., Lohila, A., Loustau, D.,
14	Matteucci, G., Meyers, T., Miglietta, F., Ourcival. J.M., Pumpanen, J., Rambal, S.,
15	Rotenberg, E., Sanz, M., Tenhunen, J., Seufert, G., Vaccari, F., Vesala, T., Yakir, D.,
16	and Valentini, R.: On the separation of net ecosystem exchange into assimilation
17	and ecosystem respiration: review and improved algorithm, Global Change Biology
18	11, 1424-1439, 2005.
19	Rosenberg, N. J.: Microclimate. The biological environment. Ed. John Wiley sons, New
20	York, 315 p, 1974.
21	Running, S., Nemani, R., and Hungerford R.: Extrapolation of synoptic meteorological
22	data in mountainous terrain, and its use for simulating forest evapotranspiration
23	and photosynthesis, Canadian Journal of Forest Research, 17, 472-483, 1987.

1	Sellers, P. J.: Canopy reflectance, photosynthesis and transpiration, International
2	Journal of Remote Sensing, <i>6</i> , 1335–1372, 1985.
3	Weiss, A., and Norman, J.A.: Partitioning solar radiation into direct and diffuse, visible
4	and near-infrared components, Agricultural and Forestry Meteorology, 34,
5	205–213, 1985.
6	Woodward, F. I.: Climate and Plant Distribution, Cambridge Univ. Press, Cambridge, U.
7	K., 1987.
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2 3 4 6 8 9 Num. 5 7 10 11 12 13 14 1 16 17 18 19 20 21 22 23 24 1 PFT 2 2 10 2 2 2 2 All 2 10 2 All 10 All All 10 2 10 2 All 10 All All All $V_{\rm max}^{25}$ $V_{\rm max}^{~~25}$ $K_{\rm C}^{25}$ K_{0}^{25} $\tau_{\rm W}^{*}$ k^{25} $\tau_{\rm W}^{*}$ ξ fci_{C3} Parameter $a_{J,V}$ $E_{\rm Rd}$ E_{Vmax} $E_{\rm KO}$ $E_{\rm KC}$ $E_{\rm k}$ α_{q} α_{i} $\alpha_{\Gamma, T}$ Λ fci_{C4} $C_{
m W0}$ f_{RD0} $f_{\rm RH0}$ $W_{\rm max}$ $V_{\rm max}^{25}$ 2 0.95 1 10 $V_{\rm max}^{25}$ 0.65 2 0.01 0.00 1.00 3 2 0.00 $a_{J,V}$ 10 k^{25} 1.00 4 0.00 0.0 0.00 5 All $E_{\rm Rd}$ 0.00 0.00 0.00 0.00 1.00 -0.02 -0.02 0.00 0.00 0.00 0.98 All 6 $E_{\rm Vmax}$ 0.00 0.00 0.00 0.00 0.00 1.00 2 $E_{\rm KO}$ 0.00 7 2 0.00 0.00 0.00 0.99 8 $E_{\rm KC}$ 0.01 0.01 0.00 0.02 10 $E_{\rm k}$ 0.00 0.00 9 0.00 0.00 0.00 0.00 0.00 0.00 1.00 10 2 α_{q} 0.00 -0.02 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.99 10 0.00 -0.05 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.99 11 $\alpha_{\rm i}$ $K_{\rm C}^{25}$ 12 2 0.00 0.00 0.00 0.00 0.00 -0.01 0.00 0.00 0.00 1.00 0.01 0.01 K_0^{25} 13 2 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 2 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 14 $\alpha_{\Gamma, T}$ 0.00 0.01 0.00 0.00 0.00 0.00 15 All $\Lambda_{\rm max}$ 0.06 0.00 0.01 -0.05 -0.02 -0.02 0.00 0.01 0.75 -0.090.00 0.00 0.06 0.00 -0.01All ξ 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 16 0.00 0.01 0.00 0.00 0.00 0.00 0.00 0.00 2 0.00 17 $\tau_{\rm W}$ -0.14 0.09 -0.01 0.00 0.00 0.01 0.00 0.00 0.00 -0.03 0.02 0.02 -0.01 0.03 0.06 0.33 18 10 τw -0.01 -0.17 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.01 0.00 0.00 0.00 -0.01 0.03 0.02 -0.05 0.41 fci_{C3} 19 2 -0.07 -0.09 -0.01 0.00 0.00 -0.02 0.00 0.01 0.00 -0.02 -0.02 0.01 0.00 0.02 0.01 0.00 -0.27 0.04 0.81 fci_{C4} 0.00 -0.03 -0.05 20 10 0.00 -0.16 0.00 0.00 0.00 -0.01 0.01 0.00 -0.01-0.02 0.00 0.00 0.01 -0.03 0.00 0.04 0.93 $C_{\rm W}$ 21 All 0.00 -0.01 0.00 0.00 0.00 0.00 0.000.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.000.00 0.00 0.00 1.00 1.00 22 All h_0 0.00 0.01 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00ĥ 23 All 0.00 0.03 0.00 0.00 0.00 0.00 0.00 0.00 0.000.000.00 0.00 0.00 0.00 0.00 0.00 0.01 0.00 0.02 0.01 0.00 0.00 1.00 All $W_{\rm max}$ 0.00 0.02 0.00 0.00 0.00 -0.01 0.00 0.00 0.00 0.01 0.00 0.00 0.00 0.00 0.00 0.00 0.02 0.00 0.02 0.01 0.02 0.00 0.00 0.01 24

Table A1. Matrix of error co-variance in parameters in posterior runs assimilating the LHF (Experiment 1). Top rows: physiology, middle: phenology, bottom: energy and water budgets. Units of physiology parameters are V_{max} : $\mu \text{mol}(\text{CO}_2)\text{m}^{-2}\text{s}^{-1}$, k: $\mu \text{mol}(\text{air})\text{m}^{-2}\text{s}^{-1}$, $\alpha_{\Gamma, T}$: $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, K_{C} in $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, k: $\mu \text{mol}(\text{air})\text{m}^{-2}\text{s}^{-1}$, k: $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, K_{C} in $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, k: $\mu \text{mol}(\text{air})$, $\sigma_{\Gamma, T}$: $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, K_{C} in $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, k: $\mu \text{mol}(\text{air})$, $\sigma_{\Gamma, T}$: $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, K_{C} in $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, k: $\mu \text{mol}(\text{air})$, $\sigma_{\Gamma, T}$: $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, K_{C} in $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, k: $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, k: $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, K_{C} in $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, k: $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, K_{C} in $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, K_{C} in $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, k: $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, K_{C} in $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, K_{C} in $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, k: $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, K_{C} in $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{mol}(\text{CO}_2)\text{mol}(\text$

Italic:absolute number of error co-variance, except for diagonal values, > 0.1

Data assimilation for semi-arid woodland

-																										
Nur	n.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	1	16	17	18	19	20	21	22	23	24
	PFT		2	10	2	10	All	All	2	2	10	2	10	2	2	2	All	All	2	10	2	10	All	All	All	All
		Parameter	$V_{\rm max}^{25}$	$V_{\rm max}^{25}$	$a_{\rm J,V}$	k^{25}	$E_{ m Rd}$	$E_{\rm Vmax}$	$E_{\rm KO}$	$E_{\rm KC}$	$E_{ m k}$	$lpha_{ m q}$	$lpha_{ m i}$	$K_{\rm C}^{25}$	K_0^{25}	$\alpha_{\Gamma, T}$	$\tilde{\Lambda}$	ξ	$\tau_{ m W}{}^*$	$\tau_{ m W}{}^{*}$	fci _{C3}	fci _{C4}	$C_{ m W0}$	$f_{\rm RD0}$	$f_{\rm RH0}$	$W_{\rm max}$
1	2	$V_{ m max}^{25}$	0.94																							
2	10	$V_{ m max}^{25}$	-0.03	0.96																						
3	2	$a_{\rm J,V}$	0.00	0.00	1.00																					
4	10	k^{25}	0.00	0.00	0.00	1.00																				
5	All	$E_{ m Rd}$	0.00	0.00	0.00	0.00	1.00																			
6	All	$E_{\rm Vmax}$	0.00	-0.01	0.00	0.00	0.00	1.00																		
7	2	$E_{\rm KO}$	0.00	0.00	0.00	0.00	0.00	0.00	1.00																	
8	2	$E_{\rm KC}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00																
9	10	$E_{ m k}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00															
10	2	$\alpha_{ m q}$	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00														
11	10	$lpha_{ m i}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00													
12	2	$K_{\rm C}^{25}$	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00												
13	2	K_{0}^{25}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00											
14	2	$lpha_{\Gamma, T}$	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00										
15	All	Λ_{\max}	-0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.01	0.98									
16	All	ξ	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	-0.01	0.00	-0.01	0.03	0.94								
17	2	$\tau_{ m W}{}^{*}$	-0.04	0.06	-0.01	0.00	0.00	-0.01	0.00	0.01	0.00	-0.02	0.01	0.00	0.00	0.02	-0.01	0.01	0.32							
18	10	$\tau_{ m W}{}^{*}$	-0.05	-0.13	0.00	0.00	0.00	-0.03	0.00	0.02	0.00	0.01	-0.02	0.01	0.00	0.00	0.06	-0.04	0.12	0.40						
19	2	fci _{C3}	-0.15	-0.05	-0.01	0.00	0.00	-0.01	0.00	0.01	0.00	-0.04	0.00	0.02	-0.01	0.04	-0.06	0.14	-0.15	-0.05	0.60					
20	10	fci _{C4}	-0.01	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	-0.06	-0.01	0.99				
21	All	$C_{ m W}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00			
22	All	h_0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.00	1.00		
23	All	ĥ	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	1.00	
24	All	$W_{\rm max}$	0.04	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	-0.01	0.00	0.00	-0.01	0.04	0.06	0.07	0.06	0.02	0.00	0.00	0.00	0.06

Table A2. Matrix of error co-variance in parameters in posterior runs assimilating the FAPAR (Experiment 2). Top rows: physiology, middle: phenology, bottom: energy and water budgets. Units of physiology parameters are V_{max} : $\mu \text{mol}(\text{CO}_2)\text{m}^{-2}\text{s}^{-1}$, k: $\mu \text{mol}(\text{air})\text{m}^{-2}\text{s}^{-1}$, k: $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}\text{O}^{-1}$, K_{C} in $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, k: $\mu \text{mol}(\text{co}_2)\text{mol}(\text{air})^{-1}$, k_{C} in $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{mol}(\text{CO}_2)\text{mol}(\text{$

Italic: absolute number of error co-variance, except for diagonal values, > 0.1

Data assimilation for semi-arid woodland

 $\mathbf{2}$

Table A3. Matrix of error co-variance in parameters in posterior runs assimilating the combination of LHF and FAPAR (Experiment 3). Top rows: physiology, middle: phenology, bottom: energy and water budgets. Units of physiology parameters are V_{max} : $\mu \text{mol}(\text{CO}_2)\text{m}^{-2}\text{s}^{-1}$, k: $\mu \text{mol}(\text{air})\text{m}^{-2}\text{s}^{-1}$, $\alpha_{\Gamma, T}$: $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, K_{C} in $\mu \text{mol}(\text{CO}_2)\text{mol}(\text{air})^{-1}$, activation energies E in J mol⁻¹, τ_{W} in days, C_{W0} in mm hour⁻¹, W_{max} in mm, others unitless.

Nun	1.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	1	16	17	18	19	20	21	22	23	24
	PFT		2	10	2	10	All	All	2	2	10	2	10	2	2	2	All	All	2	10	2	10	All	All	All	All
		Parameter	$V_{\rm max}^{~~25}$	$V_{\rm max}^{~~25}$	$a_{\rm J,V}$	k^{25}	$E_{ m Rd}$	$E_{\rm Vmax}$	$E_{\rm KO}$	$E_{\rm KC}$	$E_{\rm k}$	$lpha_{ m q}$	$lpha_{i}$	$K_{\rm C}^{25}$	K_0^{25}	$\alpha_{\Gamma, T}$	$\tilde{\Lambda}$	ξ	$\tau_{ m W}{}^{*}$	$\tau_{ m W}{}^*$	fci _{C3}	fci _{C4}	$C_{ m W0}$	$f_{\rm RD0}$	$f_{\rm RH0}$	W _{max}
1	2	$V_{\rm max}^{25}$	0.33																							
2	10	$V_{ m max}^{25}$	-0.01	0.19																						
3	2	$a_{\rm J,V}$	0.00	0.00	1.00																					
4	10	k^{25}	0.00	0.00	0.00	1.00																				
5	All	$E_{ m Rd}$	0.00	0.00	0.00	0.00	1.00																			
6	All	$E_{ m Vmax}$	-0.01	-0.03	0.00	0.00	0.00	1.00																		
7	2	$E_{\rm KO}$	0.00	0.00	0.00	0.00	0.00	0.00	1.00																	
8	2	$E_{\rm KC}$	0.01	0.00	0.00	0.00	0.00	0.00	0.00	1.00																
9	10	$E_{\rm k}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00															
10	2	$lpha_{ m q}$	0.03	0.00	-0.02	0.00	0.00	0.01	0.00	-0.01	0.00	0.93														
11	10	$lpha_{ m i}$	0.00	-0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.99													
12	2	$K_{\rm C}^{25}$	0.07	0.00	-0.01	0.00	0.00	0.01	0.00	-0.01	0.00	-0.04	0.00	0.98												
13	2	K_{0}^{25}	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	1.00											
14	2	$\alpha_{\Gamma, T}$	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.01	0.00	0.98										
15	All	Λ_{\max}	-0.02	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.01	0.00	-0.01	0.94									
16	All	ξ	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.01	0.73								
17	2	$ au_{ m W}^{*}$	0.01	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.02							
18	10	${ au_{ m W}}^{*}$	-0.03	-0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.20	0.01	0.00	0.19						
19	2	fci _{C3}	-0.43	0.01	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	-0.07	-0.01	0.00	0.00	0.05	0.00	0.00	-0.02	0.01	0.60					
20	10	fci _{C4}	0.00	-0.25	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	-0.01	-0.02	0.00	0.00	0.00	0.01	-0.01	0.00	0.01	0.00	0.92				
21	All	$C_{ m W}$	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	-0.02	0.00	0.00	1.00			
22	All	h_0	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	1.00		
23	All	\hat{h}	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.00	1.00	
24	All	$W_{\rm max}$	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00

Italic: absolute number of error co-variance, except for diagonal values, > 0.1

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