

Supplemental materials for “**Inconveniently large uncertainty in ecosystem carbon dynamics resulting from ambiguous numerical coupling of carbon and nitrogen biogeochemistry: a demonstration with the ACME land model**”

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## **1. Introduction of contents**

The first part of this supplemental material reports the proof of the sequentially weaker nitrogen limitation in the application of the MNL, NUL and PNL nitrogen limitation schemes. The second part contains figures (**S1-S6**) that provide complementary information to support our results and conclusions in the main text.

## **2. Proof of the progressively weaker nitrogen limitation**

We prove under the same soil mineral nitrogen availability and fluxes of  $F_{S,input}$  and  $F_{S,uptake}$  that the application of MNL, NUL and PNL schemes leads to progressively weaker nitrogen limitation.

We first prove  $\bar{F}_{MNL,uptake} < \bar{F}_{NUL,uptake}$ , where, without confusing the readers, the subscript  $S$  was removed.

Because substrate  $S$  is limited,  $\bar{F}_{MNL,uptake} < \bar{F}_{NUL,uptake}$  is equivalent to

$$\frac{S(t)/\Delta t}{F_{S,uptake}} < \frac{S(t)/\Delta t}{F_{S,uptake} - F_{S,input}} \quad (\text{S-1})$$

which is reduced to  $F_{S,uptake} - F_{S,input} < F_{S,uptake}$ , a condition always holds.

We now prove  $\bar{F}_{NUL,uptake} < \bar{F}_{PNL,uptake}$ . This requires

$$\frac{S(t)/\Delta t}{F_{S,uptake} - F_{S,input}} < \frac{F_{S,input} + S(t)/\Delta t}{F_{S,uptake}} \quad (\text{S-2})$$

By rearranging the terms of (S-2), we have to show

$$F_{S,uptake} S(t)/\Delta t < (F_{S,input} + S(t)/\Delta t)(F_{S,uptake} - F_{S,input}) \quad (\text{S-3})$$

which after some rearrangement becomes

$$S(t)/\Delta t < (F_{S,uptake} - F_{S,input}) \quad (\text{S-4})$$

Since (S-4) is the definition of substrate limitation for the NUL scheme, it always holds under substrate limitation.

We now finish our proof.

## List of supplemental figures

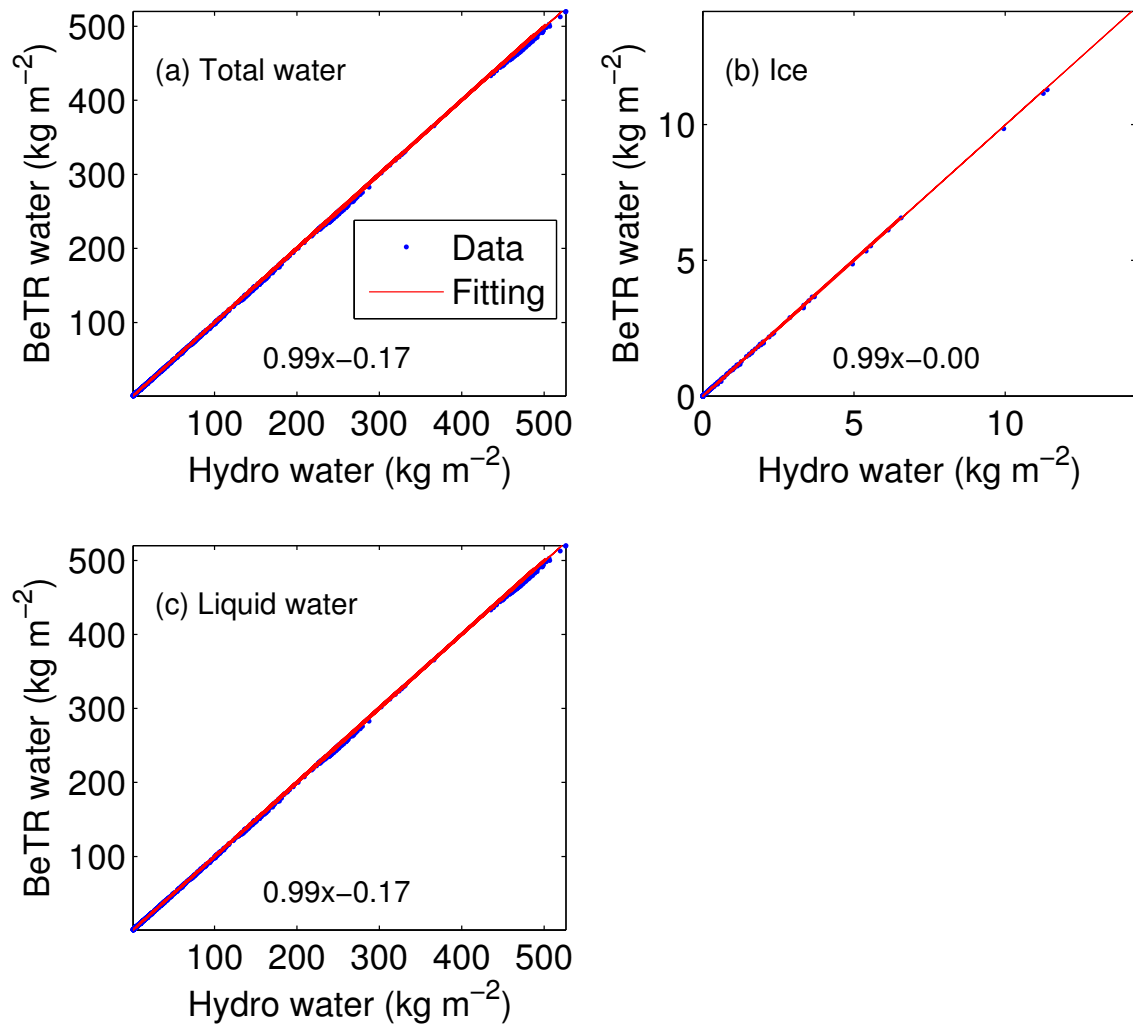


Figure S1. A demonstration of the tracer transport accuracy of BeTR. The Hydro water is water simulated with the biophysics module in the ACME land model. BeTR water is water tracer tracked in BeTR. Ideally, the linear fit should be one to one.

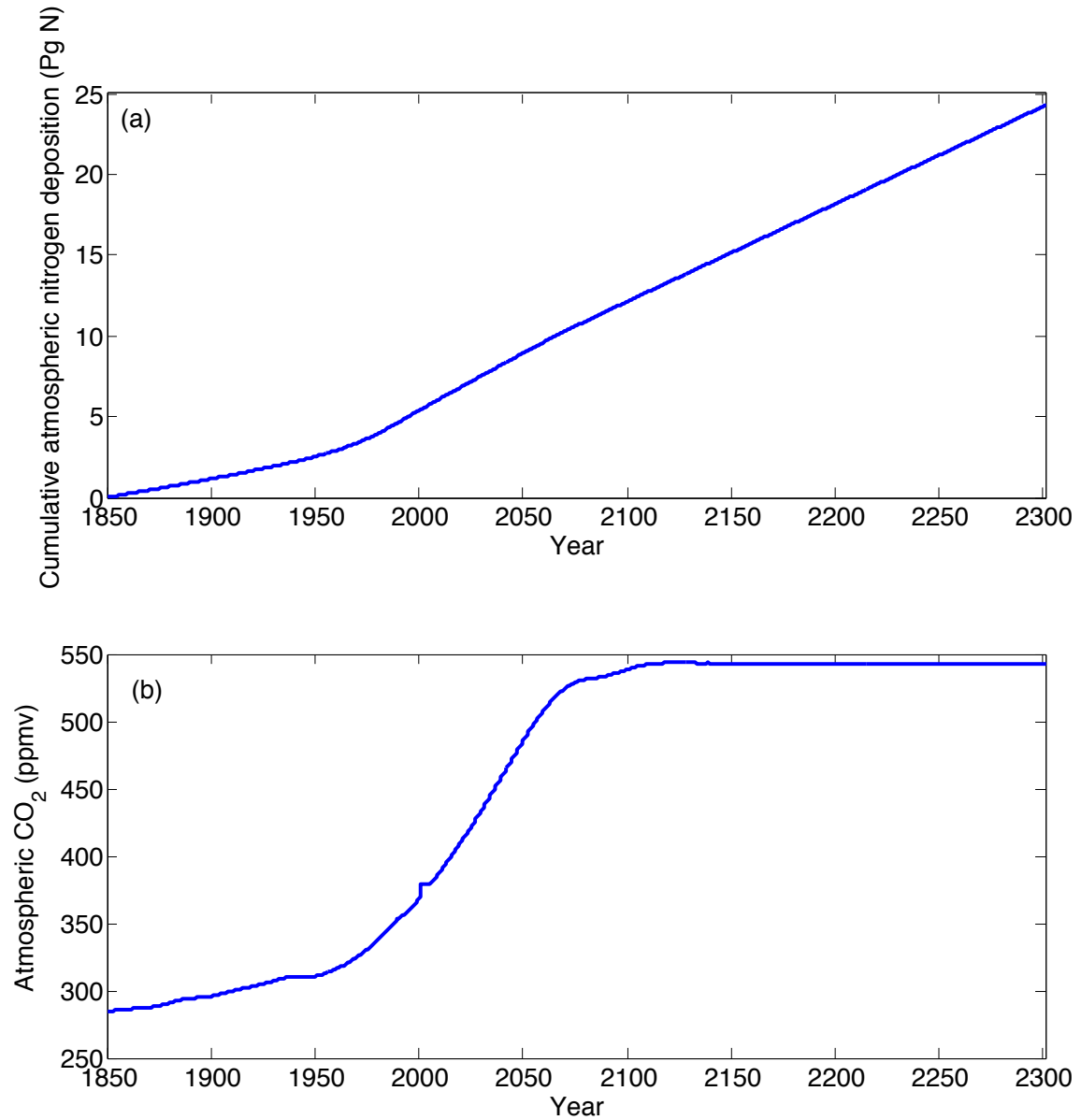


Figure S2: (a) Cumulative atmospheric deposition from 1850 to 2300. (b) Atmospheric CO<sub>2</sub> from 1850 to 2300. The small zigzag in (b) is due to that RCP4.5 CO<sub>2</sub> starts from 2006.

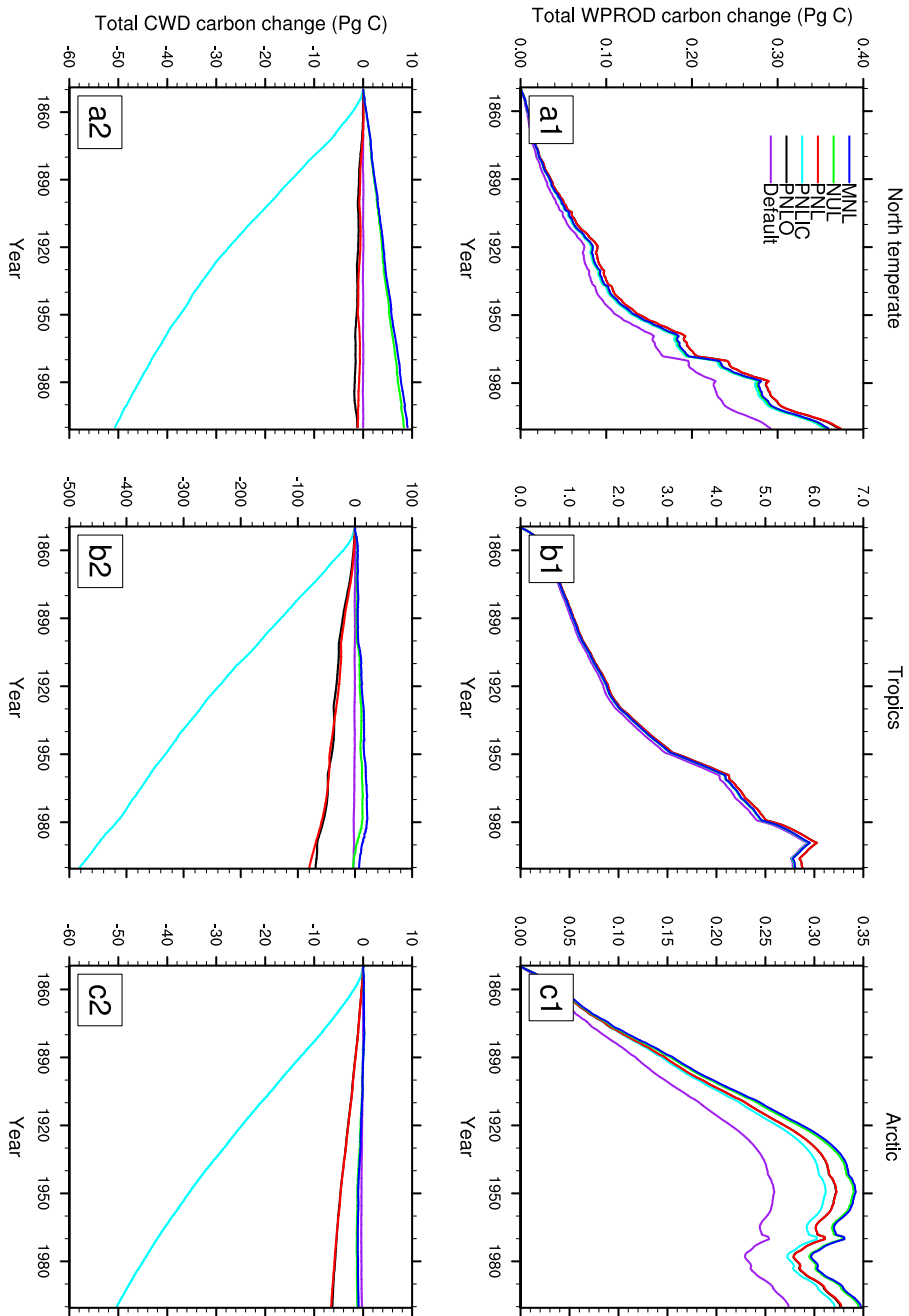


Figure S3: (a1), (b1) and (c1) are carbon changes in total wood product. (a2), (b2) and (c2) are changes in total coarse woody debris.

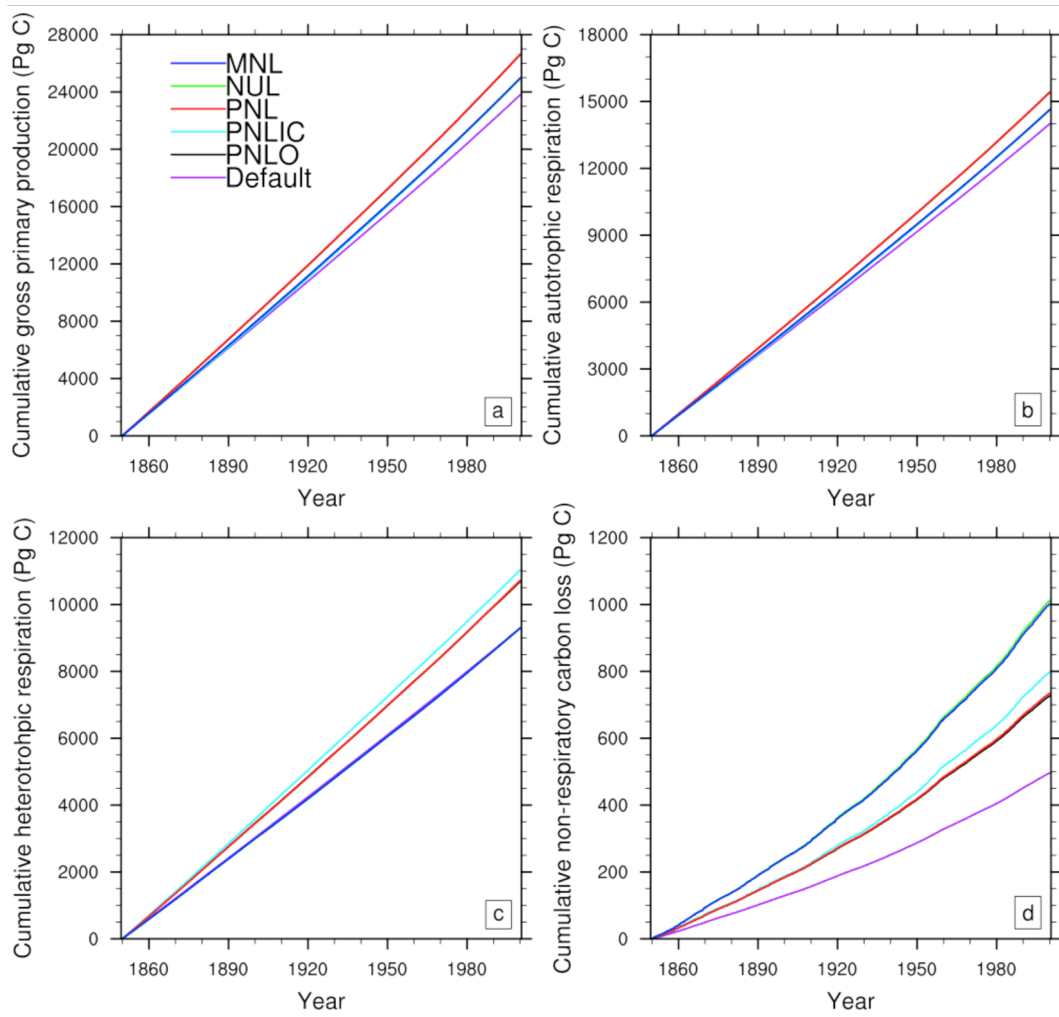


Figure S4. Simulated cumulative carbon fluxes in the contemporary period 1850-2000.

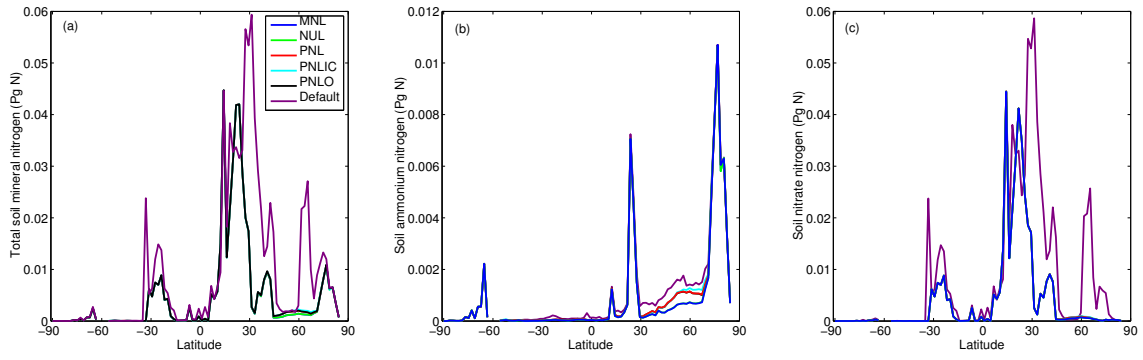


Figure S5: Latitudinal distribution of simulated soil mineral nitrogen for 1991-2000. (a) Total soil mineral nitrogen; (b)  $\text{NH}_4^+$  and (c)  $\text{NO}_3^-$ .

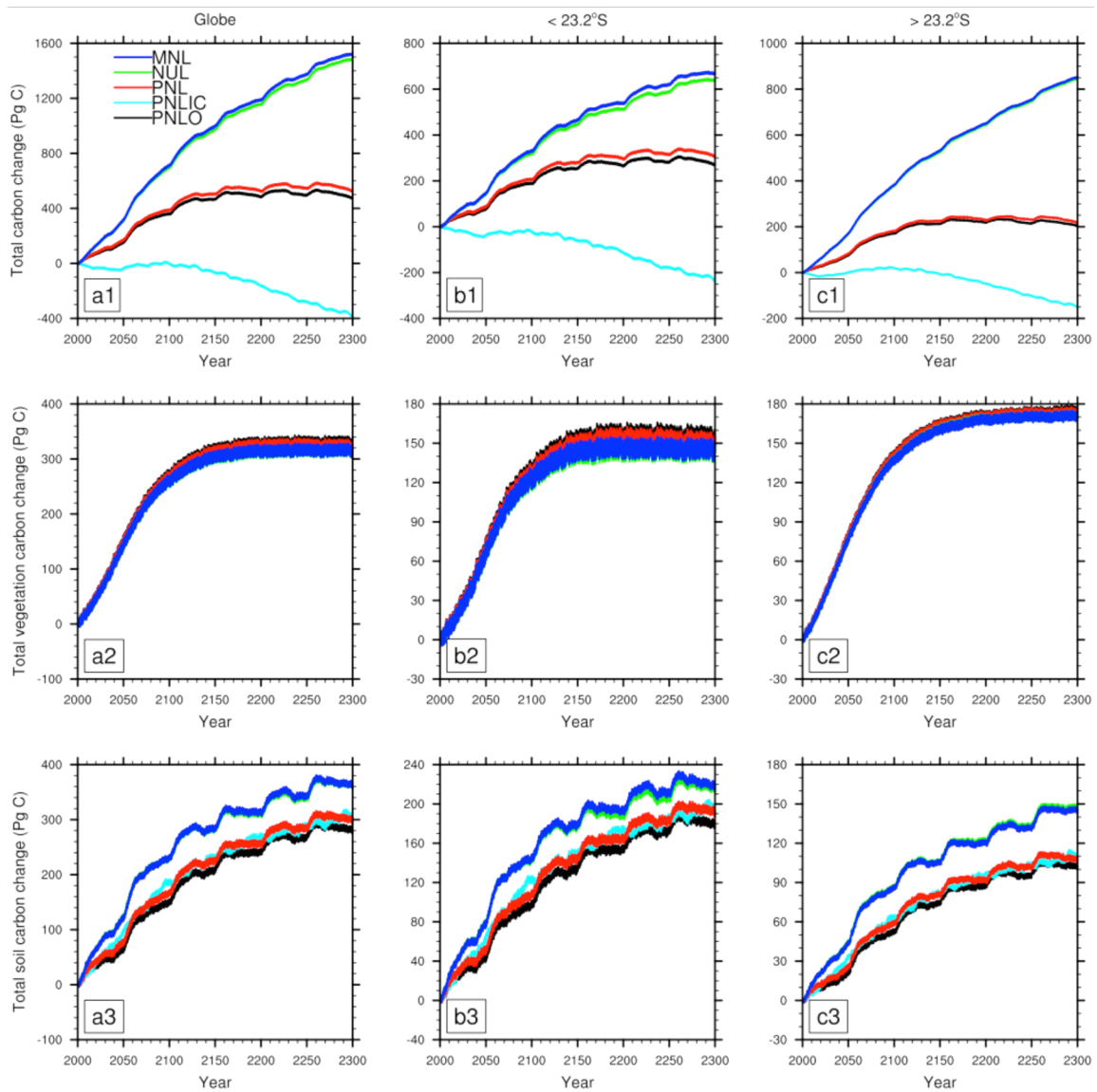


Figure S6: Model simulations for the scenario Representative Concentration Pathway 4.5 (RCP4.5) atmospheric CO<sub>2</sub> for the years 2001-2300. Here total soil carbon includes litter carbon and soil organic matter as defined in CLM4.5; coarse woody debris is excluded. All changes are calculated as relative to each of their initial carbon pool sizes at the start of the simulation (year 2000).



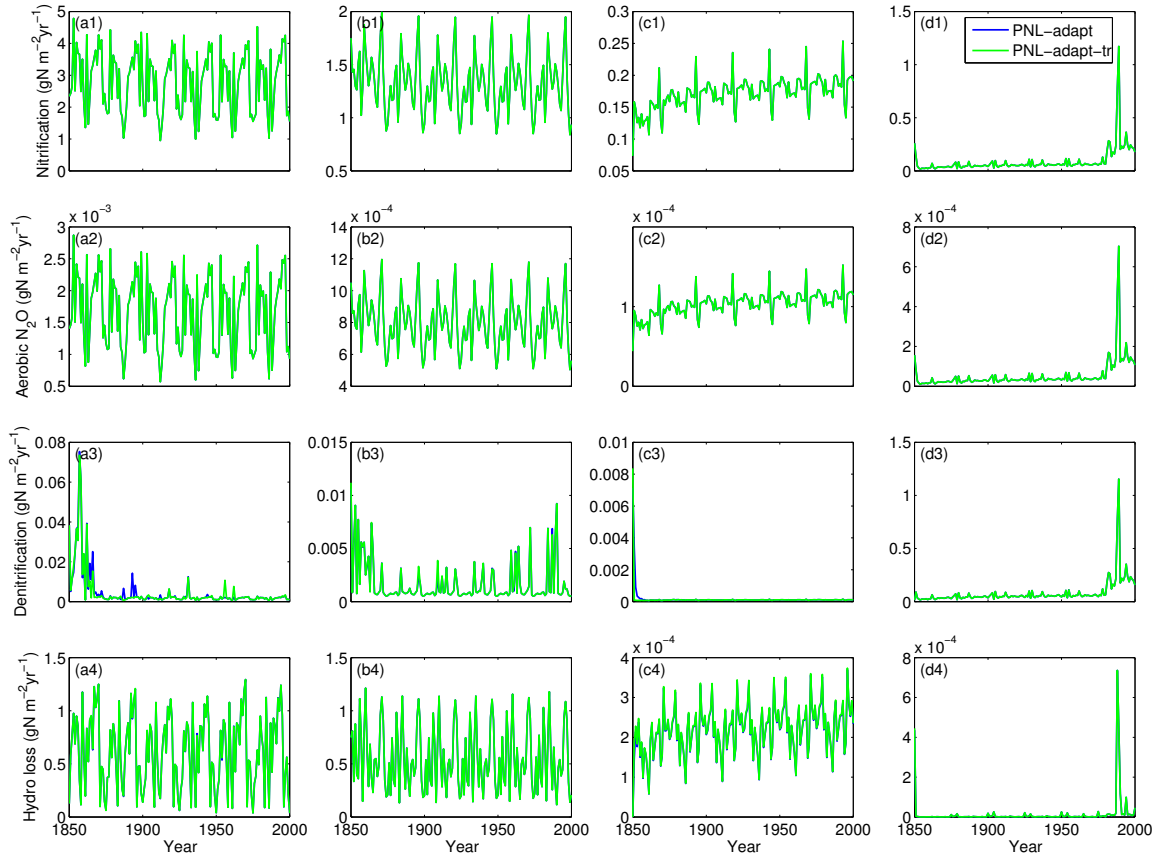


Figure S7: Evaluation of the ordering effect for the point simulations. PNL-adapt-tr simulates transports ahead of biogeochemical calculations, whereas PNL-adapt does the opposite order. From left to right, the four columns are representing sites that are corresponding to the locations specified in Figure 3 of the main text.

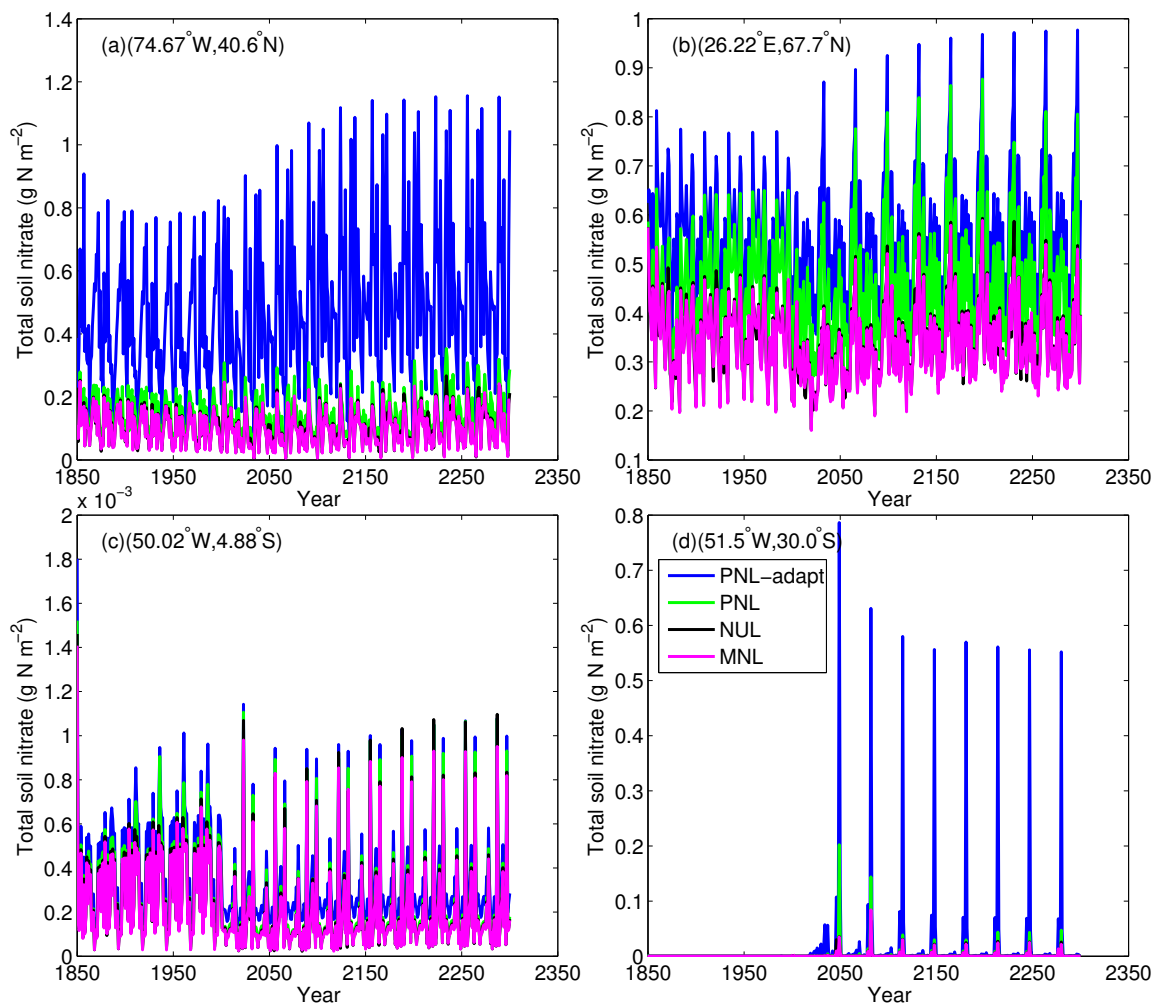


Figure S8: Soil nitrate concentrations for the point simulations as obtained from different model configurations.

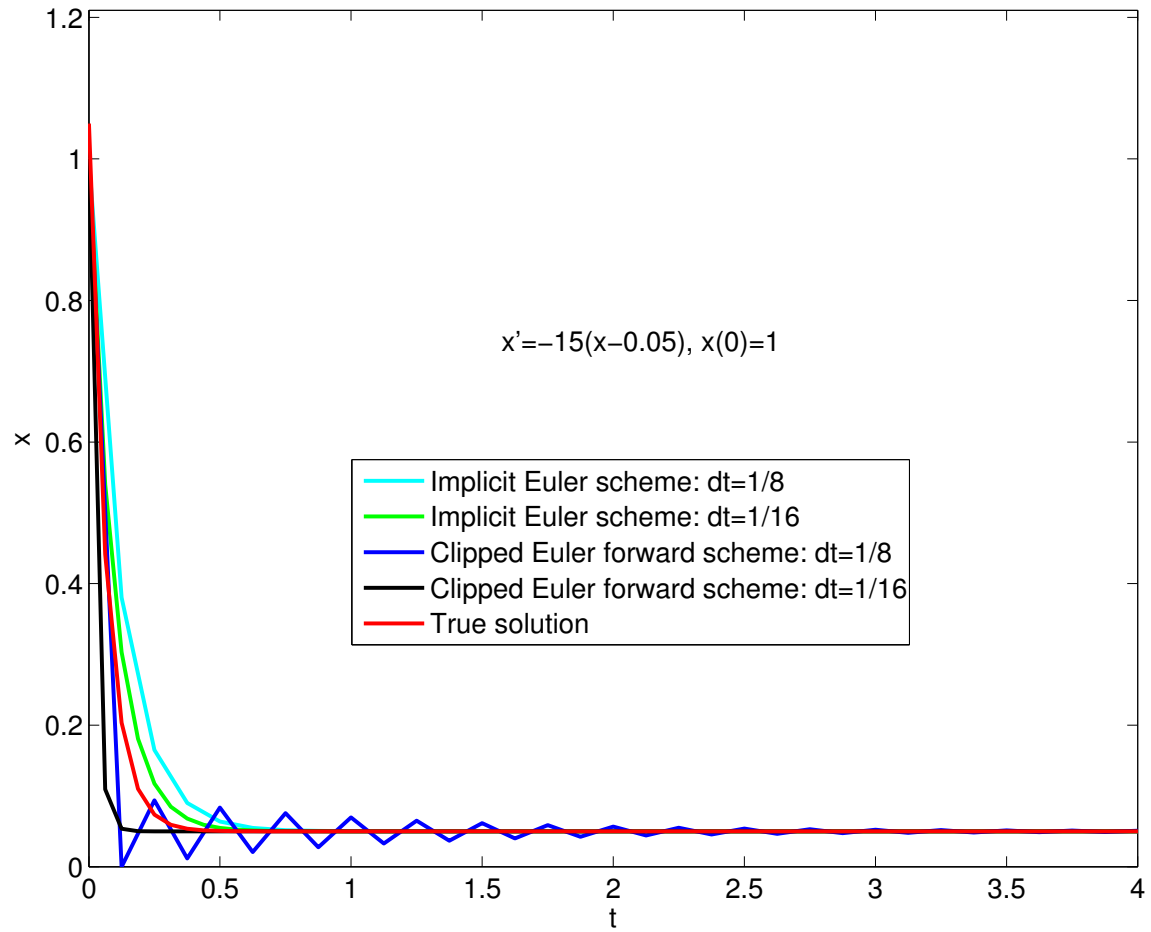


Figure S9: A demonstration of the zigzag phenomena and the strong time-stepping dependence of the numerical solution using Euler methods.