

## **Interactive comment on “Reviews and syntheses: Parameter identification in marine planktonic ecosystem modelling” by Markus Schartau et al.**

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### **Specific comments by Referee #1**

**Comment 1:** In section 2.3, the authors provide two forms of probability densities and cost functions based on the statistical properties of errors. When one consider the lognormal distribution for errors, the error covariances in (11) and (12) should represent the uncertainty in logarithm space and be different from those in (9) and (10) (e.g., Fletcher, 2010; Song et al., 2012). If I am not wrong, the optimal solution for (12) represents the mode while that for (12) without  $2 \sum_{l=1}^{N_{\Theta}} \log(\Theta_l)$  represents the median in lognormal probability density function. If this is right, I think that it would be useful if you mention mode and median in the sentence after (12).

C1

**Author’s response:** We have corrected the notation to also include tildes on residuals and covariances evaluated on the transformed scale. Because we introduced additional equations to the Theoretical Background section, the equation numbers have changed (equation 11 now is 14 and equation 12 is 15). The lognormal equations now read:

“Alternatively, nonnegativity constraints on the variables and parameters may lead us to prefer the lognormal observational error model. Likewise, we can assume lognormal priors for the parameters. In this case the posterior density becomes:

$$p(\Theta | \vec{y}) \propto \frac{1}{\sqrt{(2\pi)^{N_y} \det \tilde{R} \prod_j y_j}} \cdot \exp \left[ -\frac{1}{2} \tilde{d}^T \tilde{R}^{-1} \tilde{d} \right] \cdot \frac{1}{\sqrt{(2\pi)^{N_{\Theta}} \det \tilde{B} \prod_l \Theta_l}} \cdot \exp \left[ -\frac{1}{2} \tilde{\Delta}_{\Theta}^T \tilde{B}^{-1} \tilde{\Delta}_{\Theta} \right] \quad (14)$$

where the data-model residuals and parameter corrections on the transformed scale are defined by  $\tilde{d} = \log(\vec{y}) - \log(H(\vec{x})) + \frac{\tilde{\sigma}^2}{2}$  and  $\tilde{\Delta}_{\Theta} = \log(\Theta) - \log(\Theta^b) + \frac{(\tilde{\sigma}^b)^2}{2}$ . A MAP estimator of  $\Theta$  is then obtained by minimising:

$$J(\Theta) = \tilde{d}^T \tilde{R}^{-1} \tilde{d} + 2 \sum_{l=1}^{N_{\Theta}} \log(\Theta_l) + \tilde{\Delta}_{\Theta}^T \tilde{B}^{-1} \tilde{\Delta}_{\Theta} \quad (15)$$

”

The optimal solution for Eq. (15) does indeed represent a posterior mode for  $\Theta$ , and maximising Eq. (15) without the second term will indeed yield a posterior median estimate for  $\Theta$  (and also  $\log \Theta$ , since quantiles are invariant under scale transformation).

C2

We have modified the text to:

“ The MAP or posterior mode estimator of  $\log(\Theta)$  is equivalent here to the posterior median estimate and is obtained by maximising  $p(\log(\Theta) | \vec{y})$ . This leads to a cost function given by Eq. (15) without the second term,  $2 \sum_{l=1}^{N_e} \log(\Theta_l)$  (cf. , Fletcher, 2010).”

We are particularly thankful to Referee #1 for spotting grammatical- and typing errors. All corrections listed by Referee #1 have been applied.

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