Dear Dr. Middelburg,

We thank you for handling our manuscript. Following your recommendation, we have added "Technical note" to the beginning of the title. Below you will find our point-by-point response to both reviewers, including all changes to this updated version of the manuscript (noted with page and line numbers corresponding to the original submission). Please also find all edits marked as red text in the attached marked-up version of the manuscript.

Sincerely, Jordon Hemingway

Reviewer 1: David Burdige

Prof. Burdige,

We thank you for your helpful concerns and feedback on our manuscript. Below you will find our response to your comments, along with some corresponding changes that we will make to the text once the discussion period ends. We invite further dialogue if anything is unclear or if you believe more explanation is required.

This manuscript describes a mathematical technique for the analysis of serial oxidation results (i.e., thermograms), which effectively allows one to convert the temperature of oxidation on the x-axis of a thermogram to an activation energy (also on the x-axis) for the oxidation of organic matter occurring at that temperature. When combined with isotopic measurements of the CO₂ being produced by the oxidation, the authors suggest that this can be used to infer information about the reactivity of organic matter in sediments.

We would like to clarify an important point (also see our response to Reviewer 2, below): we do not suggest that *thermal* reactivity measured here is equivalent to, or even necessarily scales with, *microbial* reactivity within sediments. Rather, we present thermal E distributions as a proxy for the distribution of chemical bonding environments. Then, by comparing E distributions and corresponding isotope compositions between multiple environmental samples, we propose that this method is able to probe how, if at all, chemical composition controls OC turnover time. For example, it is entirely possible that OC described by high thermal E (likely condensed, aromatic material) is consumed rapidly in certain environmental settings. Using our method, this would result in a high Fm value for this material (if it is constantly replaced by "new" material with the same chemical structure that is high in 14 C) and/or a rapid drop in the fractional contribution of this material in older samples (if no replacement occurs). We *do not* suggest that OC with high thermal E will *inherently* be described by a slow turnover time in the environment (and thus a low Fm value).

To emphasize this point, we add the following paragraphs starting on P2, L33:

"We note that the modeling approach developed here is broadly applicable to any serial oxidation technique, although the resulting *E* distributions will differ depending on oxidation pathway. For example, aromatic compounds such as lignin have been shown to be highly

photoreactive (Spencer et al., 2009) despite their relatively high thermal recalcitrance (Williams et al., 2014) and will likely be described by lower E values when oxidized with uv light relative to PRO analysis. Here, we choose RPO because analysis is rapid (~3 hours per sample), requires little material (150 to 250 μ g C), contains minimal preparation steps, and results in small kinetic isotope fractionation (Hemingway et al., 2017).

We therefore treat E as a proxy for OC chemical structure and emphasize that thermal reactivity is not equivalent to microbial reactivity in the environment (Leifeld and von Lützow, 2014). Rather, by comparing E profiles and corresponding isotope compositions across environmental samples, our method provides a framework to probe how, if at all, OC source and turnover time (as measured by δ^{13} C and Fm) is related to its chemical composition (as predicted by thermal E distributions)."

I think that the results presented here for the one sample that was analyzed are interesting and provide intriguing results. Some of the proposed uses of this approach (e.g., p. 17 line 4 p. 21 line 8) may indeed turn out to be correct, but these conclusions may also be a bit premature based on the information presented here.

Thank you for the positive review of our approach. With regards to the two specific proposed uses, we will change the language in the modified draft to read:

P17 L3-4: "We therefore propose combining p(0,E) with serial oxidation isotope measurements to test the effects of..." By removing "is an ideal method to," this should remove any perceived speculation.

P21 L8: "This result provides initial evidence for the utility of RPO *E* vs. isotope relationships..." Again, this rephrasing should remove any speculation, and more explicitly acknowledges that the results in the manuscript are indeed initial.

Related to this I am concerned about the "validation" of the approach based on the analysis of a single suspended sediment sample, in part because little is presented in this analysis to independently verify the results.

We agree that analyzing a single "test" sample is not entirely satisfactory. However, we believe that inclusion of additional samples is beyond the scope of this technical manuscript. As presented, central focus of this manuscript is the mathematical derivation of the inverse distributed activation energy model, *not* the interpretation of any given sample within the global carbon cycle. To include additional samples, and to properly discuss and interpret their results within a geochemical context, would lengthen the manuscript considerably and, in our opinion, would detract from this central focus. We therefore leave the analysis of large sample sets, and the corresponding geochemical interpretation, to a companion manuscript that is currently in preparation.

It seems to me that one way to verify this approach would involve taking well-defined organic compounds whose activation energy for oxidation is either known or can be estimated, subjecting them to ramped pyrolysis/oxidation and seeing if the activation energies that the

analysis of these thermograms gives back agrees with these values. To be honest, I'm not sure how well-known or easy it is to obtain the activation energy for the oxidation of single organic compounds. However, a quick Google search of "activation energy oxidation of organic compounds" yielded what looked like a number of promising hits.

We agree that, in theory, analyzing single organic compounds and verifying the resulting E values would be an ideal check of our model. In fact, we *have* analyzed several individual compounds (cellulose, n- C_{30} alkane, calcite). However, this is significantly more challenging in practice. The challenge arises from the fact that each carbon atom within any compound experiences a unique bonding environment and will be described by a unique activation energy when exposed to thermal analysis. Any single compound therefore does not result in a thermogram with a single peak (and thus a single E value), but rather a complex distribution (see, for example, results from pure cellulose in Williams et al., 2014).

Additionally, any calculated / estimated E value will be specific to one oxidation pathway. Literature values for decay by, for example, uv light will therefore not be applicable to the thermal decay presented here. While there exist literature thermal E values for specific compounds, all of the studies that we have encountered use similar methods to those presented here (i.e. they are experimental rather than theoretical values) and results are highly variable (e.g. cellulose ranging from $150 - 250 \text{ kJ mol}^{-1}$; see references compiled in Williams et al., 2014).

At the same time, I think there is actually some data in the literature that could be used in some simple, albeit qualitative, verification of the results discussed in section 5.3. For example, Westrich and Berner (1988) suggest, at least in the coastal sediments they studied, that organic matter which is less susceptible to decomposition may have a higher activation energy for decomposition (as one might infer from the results here in section 5.3). Similar observations are also presented in Middelburg et al. (1996). Although I'm not familiar with the papers cited on p. 12 lines 17-18, I also wonder whether information in these papers might be useful here as well.

For the reasons outlined above, our focus is to derive a method to compare the distribution of E between samples rather than to interpret the absolute values of E. Again, we emphasize that thermal E distributions are not necessarily predictive of microbial E, but rather serve as a proxy for the differences in OC chemical structure between samples. Because of this point, we believe that any discussion including comparisons to OC decay in environmental samples would be misleading within the present study.

That said, the literature mentioned by the reviewer does pose an intriguing question: how is thermal *E* related to microbial decay? Based on the results of Westrich and Berner (1988), Middelburg et al. (1996), and this study (albeit with a single sample), one might infer that they are indeed correlated. However, any such relationships are certainly speculative at this time and are outside the scope of our present manuscript.

In general, the presentation of the method is rather dense in places and there are several places where I found things confusing and/or where more information about the mathematical derivation is needed. Note that (x,y) refers to page x, line y.

1. Starting near the bottom of p. 6 (line 23) "thermograms" and "mass-normalized decay rates" seemed to be used somewhat interchangeably (also see the captions for Fig. 2-4). However, there was not a clear explanation (at least that I could find) about why this is the case. This may need to be clarified. Addressing this question should also help explain why the y-axes in Figs. 2, 3 and 4(b) all have different units.

We apologize for the confusion – "thermograms" refers to the measured data (i.e. with units of ppm CO_2), while "mass-normalized decay rates" refers to thermograms that have been normalized by the initial amount of OC loaded into the system, G_0 (i.e. they integrate to unity). For Fig. 3, thermograms have additionally been normalized by the ramp rate, β , in order to properly compare between different ramp rates.

To avoid confusion, we will make the following changes in the updated manuscript:

P5 L 6: We will add the following sentences: "At each time point, the measured thermogram (in units of ppm CO₂) can be converted to an instantaneous OC decay rate (in units of μgC s⁻¹) using the measured gas flow rate and the ideal gas constant. 'Thermogram' and 'decay rate' are therefore used interchangeably throughout this manuscript."

Fig. 2 caption: "Mass-normalized thermograms (gray shaded region, unitless)" will be changed to "Measured thermograms (gray shaded region, ppm CO₂ axis not shown)"

Fig. 3: We will change the y axis label to: " G_0 - and β -normalized decay rate x 10^3 (°C⁻¹)" for clarity.

Fig. 4 (Also see response to reviewer's point 4 below): We will change the y axis labels in panels **(b)** and **(d)** to: " G_0 -normalized decay rate, $m(t) \times 10^4$ (s⁻¹)" for clarity.

2. (9, 20) - It was not clear which model is being referred to here by "our" model.

We will replace "our model" with "the distributed activation energy model."

3. (10,19) – What are "short-range-order" minerals?

"Short-range-order" is a term frequently used in the soil sciences community to refer to the crystalline state of specific minerals. To avoid confusion, we will change "short-range-order minerals" to "clay minerals" in the updated manuscript.

4. (10, 22) – Is there are reason why here and in Fig. 4 the x-axis has changed from temperature to time?

Yes, the x axis has changed in Fig. 4 from temperature to time because the test for first-order kinetics requires the time derivative of the amount of OC remaining, i.e. dG(t)/dt. Fig. 4 panels (a) and (c) are then the results of the test for first-order [Eq. (25)], shown in graphical form. While Fig. 4 panels (b) and (d) *could* be plotted with temperature on the x axis, this would make their connection to panels (a) and (c), as well as to Eq. (25), less clear. In contrast, Fig. 2 is

showing the "raw" measured data while Fig. 3 is shown specifically to illustrate the relationship between elution temperature and ramp rate as discussed on P6 L23-26. Both of these require temperature as the x axis.

However, on P10 L22-24, we realize that the use of time rather than temperature could be confusing and is technically not accurate when referencing Fig. 2b. We therefore will make the following changes for clarity in the revised manuscript:

P10 L22 and L24: We will change " $t \sim 4500 \text{ s}$ " to "T $\sim 500 \text{ °C}$ (corresponding to $t \sim 4500 \text{ s}$)" in both instances.

Fig. 4: We will change the y axis labels in panels (b) and (d) to: " G_0 -normalized decay rate, m(t) x 10^4 (s⁻¹)" to more explicitly connect these panels to panels (a) and (c), as well as Eq. (25).

Fig. 4 caption: We will also add the following sentence at the end of the caption to make the connection clearer: "For each time point in panel (a), the regression slope is equivalent to m(t) for that time point as shown in panel (b)."

5. (13, 6) – Should it say "Eq. (30) can be solved for p by multiplying . . . "?

Yes, specifying that we are solving for **p** would clarify this statement. We will change this in the updated manuscript.

6. (13, 8) - How exactly do you find the solution (i.e., the p vector) that satisfies Eq. (32)?

We will add "...using the non-negative least squares algorithm of Lawson and Hanson (1995) as implemented by the SciPy package for Python" after Eq. (34) to clarify how this is done.

7. (14, 7-9) – How exactly does the method of Miura and Maki (1998) differ from that used here?

As mentioned briefly on P13 L24-26, the method of Miura and Maki (1998) involves analyzing a given sample at multiple (at least 3) ramp rates and generating a plot of β/T^2 vs. 1/T for each value of α , the fraction of initial OC that has been oxidized. Because any given α value will occur at a slightly different T for each ramp rate (e.g. Fig. 3), this will result in a straight line in β/T^2 vs. 1/T space for each α value. The slope and y intercept of this line can then then be used to calculate E and ω values that correspond to that particular α value. To estimate the KCE slope and intercept, one simply generates a plot of β/T^2 vs. 1/T for multiple α values (i.e. 5 % of initial OC oxidized, 10 %, 15 %, etc.) and plots the resulting E and ω estimates. However, as we mention in the text (P13 L26-27), this method requires large extrapolations and is thus subject to large uncertainty.

In contrast, our method to generate Fig. 5 requires choosing a range of ω values *a priori*, solving Eq. (32) – (34) for each value, and calculating the residual norm between the measured and predicted thermograms. These methods are quite different – ours is a "brute force" method that

does not require one to analyze a sample at multiple ramp rates. Here we simply use the method of Miura and Maki (1998) to independently justify our choice of a constant ω value (i.e. KCE slope = 0).

Because we only invoke the Miura and Maki (1998) method as an independent justification, and because they clearly outline their method within their original manuscript, we believe that further description is not necessary here. In our opinion, explaining their method in detail would only add unnecessary equations and could cause confusion.

8. (14, 21-22) – If a higher ω value results in a broader p(0,E) how can it also have "no effect on the shape of the distribution"? What am I missing?

Perhaps this wording is confusing. What we mean is that ω is simply a scaling factor and changing its value will have no effect on the *relative* shape of the distribution, although the reviewer is certainly correct in that broadening the distribution does affect its overall shape.

As an arbitrary example, assume a sample is described by two peaks, one centered at E=150 kJ/mol containing 75% of total OC and a second centered at E=200 kJ/mol containing 25% of total OC. Increasing ω will increase E for both peaks and will increase the width between them accordingly, but will not change the fact that there are 2 peaks and will not affect the relative peak sizes (i.e. the 75 % and 25 % of total OC).

We will change this line to: "...no effect on the *relative* shape of the distribution" in order to clarify this point.

9. (15, 3) – What are the dimensions of R, and do the bold 0's in the description of the first and last rows of this matrix mean that all of the other values in the row are 0? If this is so, other may also not be familiar with this notation, and I think this could be made a little clearer.

R has dimensions $[n_E \times n_E]$ and, yes, $[1 \ 0]$ refers to a row of $[1 \ 0 \ 0 \ 0 \ \dots]$ and $[0 \ 1]$ refers to $[\dots \ 0 \ 0 \ 1]$ as the reviewer assumes.

To avoid confusion, we will update P15 L13-14 in the revised manuscript to read:

- "...where **R** is the bi-diagonal first derivative operator matrix with dimensions $[n_E \times n_E]$. To account for **p** being equal to zero outside the range $E_{\min} < E < E_{\max}$, we set the first and last rows of **R** to be equal to [1 0] and [0 1], respectively, where 0 refers to a zero vector of length $n_E 1$."
- 10. (15, 5) What is meant here by "solving the constrained least squares"? I kept thinking this was similar in someway to how Eq. (32) was used to solve for the p vector, but Eq. (39) just didn't make sense to me in that way. Again, am I missing something here?

Here, "solving the constrained least squares" refers to the constraints that each value in \mathbf{p} is nonnegative and that \mathbf{p} sums to unity [i.e. Eqs. (33) – (34)]. The reviewer is correct in thinking that Eq. (39) is analogous to Eq. (32), with the only difference being that Eq. (39) now contains a roughness term.

In an attempt to clarify this point, we will change P15 L5-7 to read:

"Similar to Eq. (32), the regularized inverse problem can then be solved for **p** by including this roughness term in the constrained least squares. That is, we solve

[Eq. (39) goes here]

for **p** subject to the constraints presented in Eqs. (33) – (34), where λ is a scalar that determines how much to weight the roughness $\|\mathbf{Rp}\|$ relative to the residual error $\|\mathbf{g} - \mathbf{Ap}\|$."

11. It's also not clear to me how Fig. 6 was generated, and the interpretation of this figure starting on line 7, p. 15.

Fig. 6 was generated by solving Eq. (39) for $\bf p$ using a range of possible λ values (in this case, ranging from $\lambda = 0.001$ to $\lambda = 100$). Each λ value will result in a unique solution for $\bf p$ that is described by a particular roughness norm and residual error norm. The black line in Fig. 6 is simply the line passing through each of these solutions. As described in Hansen (1994), solutions to the bottom left of this line are outside of the possible domain (i.e. it is impossible to have a $\bf p$ vector that is both smoother and fits the data with a lower residual error) while solutions above the line represent a poor fit of the data.

At the heart of this regularization technique is determining which λ value is deemed "best." Tikhonov regularization states that the λ value that "best fits the data but not the noise" is the one corresponding to the point of maximum curvature in a plot of roughness vs. residual error norm (i.e. the white circle in Fig. 6). We refer the reader to Hansen (1994) and Forney and Rothman (2012b) for a detailed description and background of this technique.

To alleviate any confusion, and to refer the reader to the proper references, we will add detail to the caption of Fig. 6 in the revised manuscript to read:

"Figure 6. Tikhonov regularization L-curve for Narayani POC ($\beta = 5$ °C min⁻¹). The black line represents the range of roughness and residual error norms that are the result of solving Eq. (39) for **p** using multiple λ values ranging from 0.001 to 100. The white circle corresponds to the point of maximum curvature along this line, and is thus deemed the "best fit" λ value [see Hansen (1994), Forney and Rothman (2012b) for further details on generating the L-curve and the theory behind Tikhonov regularization]."

12. (16,15) – What exactly is meant by "diversify the distribution of chemical bonds"?

This is meant to convey the phenomenon of increasing chemical complexity of OC with increasing turnover time, as shown in the references cited in these lines. That is, interactions with particles, production of new compounds by heterotrophs, partial oxidation by *uv* light, etc. should lead to a more complex OC mixture with time than was initially present.

However, we now realize that "diversify the distribution" is probably not the best wording. We will change this to "…has been shown to enhance the diversity of chemical bonds…" in the updated manuscript.

13. The general shape of the thermogram in Fig. 2 looks awfully similar to the p(0,E) distribution in Fig. 7. Does that means that activation energy scales linearly (more or less) with temperature of pyrolysis? In general that makes intuitive sense, and this is also discussed here briefly on p. 9, line 15. It will be interesting to see if this is a general trend observed across a broader range of samples.

Yes, the general similarity is striking and does make intuitive sense. The main difference is that the thermogram shape is "smoother," while the p(0,E) distribution contains more features and "sharper" peaks. This can be explained by the fact that material at a single E value (for example, a delta function) will take some amount of time to fully decay and will thus decay over a wide temperature window when analyzed in the RPO instrument. (If interested, this thought experiment is shown quite nicely in Cramer, 2004). We therefore always expect that p(0,E) to contain more features and sharper peaks than the corresponding thermograms, and this is indeed the case for all samples that we have analyzed thus far.

To enforce this idea, we will add the following sentences beginning on P16 L10:

"Note that the p(0,E) distribution broadly resembles the initial thermogram shape (Fig. 2a and Fig. 7), albeit with more defined features and a higher roughness. This is a result of the fact that OC associated with each E value will decay over a wide temperature range in the RPO instrument, thus resulting in a "smoothed" thermogram relative to p(0,E) (Cramer, 2004)."

References cited

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Westrich, J. T., and R. A. Berner. 1988. The effect of temperature on rates of sulfate reduction in marine sediments. Geomicrobiol. J. 6, 99-117.

Reviewer 2: Bernard Boudreau

Prof. Boudreau,

We thank you for your helpful concerns and feedback on our manuscript. Below you will find our response to your two reservations, along with some corresponding changes that we will make to the text once the discussion period ends. Because your two reservations were somewhat broad and open-ended, we have attempted to address them and explain our position and reasoning as best as possible. However, we invite further dialogue if anything is unclear or if you believe more explanation is required. We thank you again for your time and feedback.

Hemingway et al. have offered an inverse model to extract the reactivity of organic matter (OM) and relate it to the isotopic composition via data obtained from a record of thermal decomposition of that organic matter. This is a technique that has been used with respect to petroleum formation and there exists published literature for that application. The present paper hopes to extend the method to the degradation of OM sampled in low-temperature aquatic systems.

The mathematics of the model appear to be solid (better than my own efforts in this area), including the use of a Lagrange multiplier to add constraints to the model solution. I do not think that the paper can be faulted on this account; nevertheless, I have two strong reservations.

Firstly, microbial enzymatic degradation of OM is not the same process as thermal decomposition. Microbes use enzymes to breakdown OM in order to increase the rate of this reaction. According to a very broad interpretation of transition-state theory, that result is obtained by altering the decay (reaction) mechanism so as to lower the activation energy of the reaction. The authors' thermal method is also based on activation energy, but on the activation energy for a thermal decomposition reaction. Thus, the microbial and the thermal activation energies are not guaranteed to be the same or even comparable. Assigning the thermally derived results to the microbial situation is not, at this time, experimentally justified.

We thank the reviewer for allowing us to clarify this issue. While other studies have begun to compare OM thermal *E* values to those for microbial decay, both using laboratory incubations (e.g. Leifeld and von Lützow, 2014) as well as long-term bare fallow soil experiments (e.g. Barré et al., 2016), this is never our intention in the present study. At no point do we imply that *E* distributions determined using the thermal analysis described here are identical, or even comparable, to those that would be obtained by serial oxidation by microbial respiration (e.g. Mahmoudi et al., 2017). In fact, to extend the reviewer's point, we expect that *every* oxidation pathway (microbial respiration, thermal analysis, *uv* light, chemical hydrolysis, etc.) will involve a unique transition state intermediate and will therefore likely result in a different *E* distribution. For example, lignin is highly thermally recalcitrant (Williams et al., 2014) yet degrades rapidly under *uv* light (Spencer et al., 2009).

Rather, here we present thermally derived E as a proxy for the range of the strength of chemical bonds experienced by carbon atoms within a sample. We emphasize that this is simply a method to separate a complex OM mixture along a particular lability "axis" (i.e. thermal lability) and

measure the isotope composition at multiple points along that "axis." Thermally derived E, along with the corresponding isotope distributions, can then be directly compared across a sample set in order to infer differences in the molecular and isotope compositions between samples. For example, the observation that our test sample approaches an Fm value of 0 at $E > 200 \text{ kJ mol}^{-1}$ implies that this material is derived from OM-rich bedrock (see P21, L3-5), but says nothing about microbial recalcitrance (in fact, it is possible that rock-derived OM is highly bioavailable; see Petsch et al., 2001).

To emphasize this point, we will add and/or modify the following lines within the text:

- 1. (P1, L2-13). Throughout the abstract, we will add "thermal" before each use of the word "reactivity" in order to clarify that *E* values calculated here apply only to thermal analysis.
- 2. (P2, L33). We will add the following paragraphs:

"We note that the modeling approach developed here is broadly applicable to any serial oxidation technique, although the resulting E distributions will differ depending on oxidation pathway. For example, aromatic compounds such as lignin have been shown to be highly photoreactive (Spencer et al., 2009) despite their relatively high thermal recalcitrance (Williams et al., 2014) and will likely be described by lower E values when oxidized with uv light relative to PRO analysis. Here, we choose RPO because analysis is rapid (\sim 3 hours per sample), requires little material (150 to 250 μ g C), contains minimal preparation steps, and results in small kinetic isotope fractionation (Hemingway et al., 2017).

We therefore treat E as a proxy for OC chemical structure and emphasize that thermal reactivity is not equivalent to microbial reactivity in the environment (Leifeld and von Lützow, 2014). Rather, by comparing E profiles and corresponding isotope compositions across environmental samples, our method provides a framework to probe how, if at all, OC source and turnover time (as measured by δ^{13} C and Fm) is related to its chemical composition (as predicted by thermal E distributions)."

Secondly, only two samples were tested with the method, and one, OM from a marine sediment, failed the test of the model assumptions. I am unaware of any other papers that have applied this technique to aquatic "low-temperature" sediments. That represents meager testing of the applicability of the model. The low-temperature geochemical community does not at this stage know if the method is useful, and the theory has significantly overstepped the acceptance of the methodology itself.

Again, we thank the reviewer for raising this concern. However, we disagree with the reviewer's interpretation that the marine sediment OM sample "failed the test of the model assumptions." Rather, it is the analysis of OM *combined with* inorganic carbon (IC) that failed the model assumptions. This distinction is critical. The results from this sample (e.g. Fig. 4d) emphasize the need to decarbonate sediment samples prior to RPO analysis. Because decarbonation likely alters OM composition to some degree, there exists longstanding discussion on this topic in the organic geochemistry literature at large and the RPO literature specifically (Plante et al., 2013). By including this sample within the present study, we make a kinetic argument in favor of decarbonation – that is, we show that IC decay is mass-dependent in the presence of OM and

therefore does not follow first-order kinetics. This result *does not* imply that OM decay from this sample fails the model assumptions.

To the reviewer's larger point, we agree that additional samples would aid in solidifying the utility of the method in describing "low-temperature" OM. However, we believe that this is beyond the scope of this technical manuscript. As presented, central focus of this manuscript is the mathematical derivation of the inverse distributed activation energy model. To include additional samples, and to properly discuss and interpret their results within a geochemical context, would lengthen the manuscript considerably and, in our opinion, would detract from this central focus.

We note that this exact concern is the focus of a companion manuscript that is currently in preparation. In that publication, we subject dozens of samples to the model treatment presented here and interpret the environmental factors controlling differences in *E* distributions. Combining these two manuscripts would result in the mathematical treatment presented here being relegated to a supplemental discussion (as was the case for the original submission of our companion manuscript, to which those reviewers and editor suggested we separate the mathematical treatment). We believe that the mathematics and theory contain adequate nuance and require sufficient discussion to warrant this technical manuscript, rather than being relegated to a supplemental discussion. Thus, we believe that inclusion of multiple samples (and the corresponding geochemical discussion) is beyond the scope of this manuscript.

Technical note: An inverse method to relate organic carbon reactivity to isotope composition from serial oxidation

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Abstract.

Serial oxidation coupled with stable carbon and radiocarbon analysis of sequentially evolved CO₂ is a promising method to characterize the relationship between organic car-5 bon (OC) chemical composition, source, and residence time in the environment. However, observed decay profiles depend on experimental conditions and oxidation pathway. It is therefore necessary to properly assess serial oxidation kinetics before utilizing decay profiles as a measure of OC reactiv-10 ity. We present a regularized inverse method to estimate the distribution of OC activation energy (E), a proxy for bond strength, using serial oxidation. Here, we apply this method to ramped temperature pyrolysis/oxidation (RPO) analysis but note that this approach is broadly applicable to any serial 15 oxidation technique. RPO analysis directly compares thermal reactivity to isotope composition by determining the E range for OC decaying within each temperature interval over which CO₂ is collected. By analyzing a decarbonated test sample at multiple masses and oven ramp rates, we show that OC de-20 cay during RPO analysis follows a superposition of parallel first-order kinetics and that resulting E distributions are independent of experimental conditions. We therefore propose the E distribution as a novel proxy to describe OC thermal reactivity and suggest that E vs. isotope relationships can 25 provide new insight into the compositional controls on OC source and residence time. This manuscript is accompanied by an open-source Python package for performing all analyses.

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1 Introduction

Natural organic matter present in aquatic environments, sediments, soils, and vegetation contains roughly three-fold more carbon than the pre-industrial atmosphere (Bianchi, 2011). As such, the balance between organic carbon (OC) synthesis and remineralization exerts a major control on the global 35 carbon cycle and on atmospheric CO₂ levels (Lasaga et al., 1985). However, OC remineralization rates are spatiotemporally heterogeneous, leading to decay timescales that range from minutes to millions of years (Boudreau and Ruddick, 1991; Forney and Rothman, 2012a; Middelburg, 1989). To 40 explain this variability, it has been hypothesized that remineralization depends on multiple chemical and environmental factors such as OC molecular structure (Burdige, 2007; Tegelaar et al., 1989), microbial community composition (Pedler et al., 2014; Schmidt et al., 2011), secondary chemical interactions (Schmidt et al., 2011), and physical protection by particles (Mikutta et al., 2006; Keil and Mayer, 2014). The

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relative importance of these governing mechanisms remains actively debated and is thought to vary depending on environmental setting (Hedges et al., 2001; Rothman and Forney, 2007; Schmidt et al., 2011), thus limiting our mechanistic understanding of OC decay.

This limitation is partially methodological in nature; traditional geochemical analyses often target either "bulk" OC or trace "biomarker" molecules such as plant-wax fatty acids (Galy et al., 2011; Galy and Eglinton, 2011; Hemingway et al., 2016). While bulk measurements include all OC contained within a sample, they offer no information on the distribution of chemical structure or reactivity within a complex mixture. In contrast, biomarker analysis is highly specific but individual compounds nonetheless still represent the average of multiple sources. Furthermore, biomarkers typically constitute ≤1% of total OC and can be subject to production, transport, and preservation biases (Hemingway et al., 2016).

To bridge the information gained by these methods, a novel class of analytical techniques, termed "serial oxida-20 tion," has emerged. Such analyses separate carbon within a bulk sample based on its susceptibility to decomposition by chemical hydrolysis (Helfrich et al., 2007), uv light (Beaupré et al., 2007; Follett et al., 2014), heat (Rosenheim et al., 2008), or microbial respiration (Beaupré et al., 2016) and ₂₅ measure the stable carbon (13 C/ 12 C, expressed as δ^{13} C) and radiocarbon (14C/12C, here expressed as fraction modern or Fm) content of evolved CO2. By separating CO2 into multiple lability intervals, isotope ratios are obtained for carbon atoms exhibiting similar physical and/or chemical properties. ₃₀ Because δ^{13} C provides information on the source of organic matter while Fm reflects the amount of time that has passed since organic compounds were initially synthesized, serial oxidation is a promising method to directly probe the compositional controls on OC source and residence time.

Still, a theoretical treatment of serial oxidation kinetics is lacking, hindering our ability to correlate measured isotope distributions with intrinsic chemical properties and reactivity. In this study, we relate OC thermal recalcitrance to its corresponding δ^{13} C and Fm values using ramped-temperature py-40 rolysis/oxidation (RPO). This method involves heating OC at a controlled rate while continuously quantifying and collecting evolved CO₂, which is binned over user-defined time intervals (termed "fractions") and analyzed for δ^{13} C and Fm (Rosenheim et al., 2008; Hemingway et al., 2017). We de- $_{45}$ scribe non-isothermal OC decay rates as a function of E, the Arrhenius activation energy, using a novel inverse solution to the distributed activation energy model (Braun and Burnham, 1987; Burnham and Braun, 1999; Cramer, 2004; White et al., 2011). By conducting a set of kinetic experiments, we $_{50}$ show that the E distribution within a given OC mixture does not depend on experimental conditions and is thus a reliable proxy for bond strength and OC chemical composition.

We note that the modeling approach developed here is broadly applicable to any serial oxidation technique, also though the resulting E distributions will differ depending

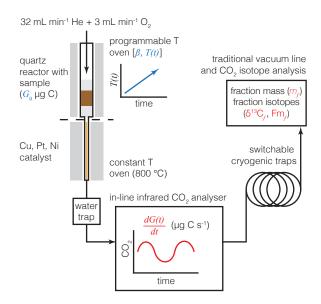


Figure 1. RPO instrument schematic. User-defined inputs are printed in blue, while observed measurements are printed in red (See Table 1 for symbol definitions).

on oxidation pathway. For example, aromatic compounds such as lignin have been shown to be highly photoreactive (Spencer et al., 2009) despite their relatively high thermal recalcitrance (Williams et al., 2014) and will likely be described by lower E values when oxidized with uv light relative to PRO analysis. Here, we choose RPO because analysis is rapid ($\approx 3 \, \mathrm{h}$ per sample), requires little material ($150 \, \mathrm{\mu g} \, \mathrm{C}$ to $250 \, \mathrm{\mu g} \, \mathrm{C}$), contains minimal preparation steps, and results in small kinetic isotope fractionation (Hemingway et al., 2017).

We therefore treat E as a proxy for OC chemical structure and emphasize that thermal reactivity is not equivalent to microbial reactivity in the environment (Leifeld and von Lützow, 2014). Rather, by comparing E profiles and corresponding isotope compositions across environmental sam- 70 ples or experimental conditions (e.g. before and after microbial degradation), our method provides a framework to probe how, if at all, OC source and turnover time (as measured by δ^{13} C and Fm) are related to its chemical composition (as predicted by thermal E distributions). We begin in Section 3 by deriving the governing equations to describe a parallel superposition of first-order, non-isothermal decay. Then, in Section 4, we describe a method to solve for the distribution of E using a regularized inverse approach. Finally, in Section 5, we determine the subset of E that is contained within each RPO fraction and directly relate OC reaction energetics to corresponding isotope values. All calculations were performed using the accompanying 'rampedpyrox' Python package (Hemingway, 2017).

Table 1. List of mathematical symbols used throughout this study.

Symbol	Parameter	Units
A	Dynamic disordered kinetic design matrix	$kJ \text{mol}^{-1}$
$\alpha(t)$	Integral of G_0 -normalized thermogram at time t	_
β	Temperature ramp rate	$\mathrm{K}\mathrm{s}^{-1}$
$\delta^{13} C_f$	$^{13}\text{C}/^{12}\text{C}$ ratio of RPO fraction f, expressed in per mille VPDB	$% c = \frac{1}{2} $
ΔE	Activation energy step	$k J \text{mol}^{-1}$
Δt_j	Time step for point j in \mathbf{t}	S
$^{13-12}\Delta E$	E difference between ¹³ C- and ¹² C-containing compounds	$k J \text{mol}^{-1}$
E_{i}	Activation energy for component i	$kJ \text{mol}^{-1}$
E	Continuous form of E_i	$kJ \text{mol}^{-1}$
${f E}$	Vector of discretized activation energy	$kJ \text{mol}^{-1}$
Fm_f	$^{14}\text{C}/^{12}\text{C}$ ratio for RPO fraction f , expressed as fraction modern	_
G_0	Total initial mass of carbon	$\mu \mathrm{g}\mathrm{C}$
$g_i(0)$	Initial mass of carbon in component i	$\mu \mathrm{g}\mathrm{C}$
$g_i(t)$	Mass of carbon in component i remaining at time t	$\mu \mathrm{g}\mathrm{C}$
G(t)	Mass of total carbon remaining at time t	$\mu \mathrm{g}\mathrm{C}$
g(0,E)	Continuous form of $g_i(0)$	$\mu \mathrm{g}\mathrm{C}$
g(t,E)	Continuous form of $g_i(t)$	$\mu \mathrm{g}\mathrm{C}$
\mathbf{g}	Vector of $G(t)/G_0$ at each time point	
$k_i(t)$	First-order rate coefficient for component i at time t	s^{-1}
k(t,E)	Continuous first-order rate coefficient for energy value E at time t	s^{-1}
$\kappa_i(t)$	Discrete, time-integrated first-order decay coefficient for component \boldsymbol{i} at time \boldsymbol{t}	_
$\kappa(t,E)$	Continuous, time-integrated first-order decay coefficient for energy value ${\cal E}$ at time t	_
λ	Regularization weighting factor	_
\mathbf{m}_f	Mass of carbon (as CO_2) contained in RPO fraction f	μgC
m(t)	G_0 -normalized decay rate at time t	s^{-1}
n_E	Number of nodes in E	_
n_t	Number of nodes in t	_
$p_i(0)$	Fraction of G_0 initially in component i	_
$p_i(t)$	Fraction of G_0 remaining in component i at time t	_
p(0,E)	Continuous form of $p_i(0)$	_
p(t,E)	Continuous form of $p_i(t)$	- (1. T. 1-1)-1
p	Vector of $p(0,E)/\Delta E$ at each energy point	$(kJ \text{mol}^{-1})^{-1}$
$\Pi_f(E)$	Subset of $p(0, E)$ contained in RPO fraction f	_
$^{13/12}$ r (t)	Ratio of ${}^{13}\text{C}/{}^{12}\text{C}$ decay at time t	- 1 7 1-1 77-1
R	Ideal gas constant	$kJ \operatorname{mol}^{-1} K^{-1}$
\mathbf{R}	First derivative operator matrix	_ TZ
T(t)	Temperature at time t	K
t	Vector of discretized time	$\frac{\mathrm{S}}{\mathrm{S}}^{-1}$
ω	Arrhenius pre-exponential ("frequency") factor	s -

2 Materials and Methods

2.1 Sample selection and preparation

As a representative sample, we analyzed particulate organic carbon (POC) contained in suspended sediments from the surface of the Narayani River. This sample (PB-60) was collected at the base of the Himalayas (27.70° N, 84.43° E) and has been analyzed for bulk OC and plant-wax carbon isotopes (Galy et al., 2008; Galy and Eglinton, 2011; Galy et al., 2011). Aliquots were taken for RPO analysis from freezedried, archived material and acidified under HCl fumes at 60 °C for 72 h to remove carbonates (Whiteside et al., 2011).

Because residual chloride has been shown to interact with the RPO catalyst wire (Hemingway et al., 2017), acidified aliquots were rinsed $3\times$ in $18.2\,\mathrm{M}\Omega$ MilliQ water and freezedried overnight at $-40\,^{\circ}\mathrm{C}$ prior to analysis. For consistency and to properly calculate RPO isotope mass balance, bulk %OC, $\delta^{13}\mathrm{C}$, and Fm values were re-measured using fumigated and rinsed material (McNichol et al., 1994a, b). Resulting Fm for rinsed material is 0.04 lower than that for unrinsed aliquots (Galy et al., 2008), reflecting a minor loss of acid soluble OC for this sample during the rinsing step.

To test if the presence of inorganic carbon (IC; *e.g.* CaCO₃) affects decay kinetics, we additionally analyzed a pure CaCO₃ laboratory working standard (Icelandic spar;

Hemingway et al., 2017) as well as carbonate-rich sediment from the Southern Ocean (60.24° S, 170.19° W) collected for the Joint Global Ocean Flux Study (JGOFS; Sayles et al., 2001). JGOFS aliquots were taken from archived core-top material ($0 \,\mathrm{cm}$ to $0.5 \,\mathrm{cm}$, stored at $-80 \,^{\circ}\mathrm{C}$), freeze-dried overnight at $-40 \,^{\circ}\mathrm{C}$, and homogenized prior to RPO analysis. IC content, OC content, and bulk $^{13}\mathrm{C}$ composition were re-quantified at NOSAMS (McNichol et al., 1994a).

2.2 Instrumental setup

10 RPO analysis has been described in detail previously (Rosenheim et al., 2008; Hemingway et al., 2017). In summary, a solid sample containing $\approx 150 \,\mu \mathrm{g\,C}$ to $250 \,\mu \mathrm{g\,C}$ is loaded into a pre-combusted (850 °C, 5 h) quartz reactor and placed into a two-stage oven, as shown in Fig. 1. The reactor is 15 then sealed and the sample is exposed to an atmosphere of 92:8 He:O₂ with a total flow rate of $35 \,\mathrm{mL\,min^{-1}}$ (oxidation mode). During analysis, the oven surrounding the sample is programmed to heat at a user-defined ramp rate, termed β (see Table 1 for symbol descriptions). Instantaneous tem-20 perature within the oven is measured using two thermocouples separated by $\approx 1 \, \mathrm{cm}$ to monitor temperature heterogeneity, which is typically <5 °C. Following standard practice (Rosenheim et al., 2008), a ramp rate of $5 \,^{\circ}\text{C}\,\text{min}^{-1}$ was used for all experiments in which CO2 gas was collected for iso-25 tope analysis. In the second (downstream) oven, eluent gas is passed over a Cu, Pt, and Ni catalyst wire held at 800 °C to facilitate oxidation of reduced carbon-containing gases to CO_2 .

After exiting the second oven, eluent gas is distilled 30 through a water trap and passed into a flow-through infrared gas analyzer (IRGA) to measure CO₂ concentration (in parts per million by volume; $ppm CO_2$) with 1-s temporal resolution. Resulting ppm CO₂ vs. temperature plots are referred to as "thermograms" (Fig. 2). At each time point, the mea-35 sured thermogram (in units of ppm CO₂) can be converted to an instantaneous OC decay rate (in units of $\mu g \, C \, s^{-1}$) using the measured gas flow rate and the ideal gas constant. "Thermogram" and "decay rate" are therefore used interchangeably throughout this manuscript. IRGA measurements were 40 calibrated using a two-point calibration curve before each analysis to account for instrument drift and are precise to ± 5 ppm CO₂ (Hemingway et al., 2017). Downstream of the IRGA, eluent gas is passed into one of two switchable traps and CO_2 is cryogenically frozen while He and O_2 are vented 45 to the atmosphere. Traps are switched at user-defined time points and CO2 is further distilled, quantified, transferred into glass tubes packed with $\approx 100 \,\mathrm{mg} \,\mathrm{CuO}$ and $\approx 10 \,\mathrm{mg} \,\mathrm{Ag}$, and flame sealed. Finally, CO₂ was recombusted at 525 °C for 1 h to remove trace contaminant gases.

2.3 Isotope measurement, blank correction, and data analysis

Radiocarbon compositions of all bulk samples and RPO fractions were measured at NOSAMS following standard graphitization methods (McNichol et al., 1994b). All radiocarbon results are expressed in fraction modern notation (Fm). We 55 note that Fm used here is corrected for ¹³C fractionation and is thus identical to the F¹⁴C notation of Reimer et al. (2004). Bulk and RPO fraction stable carbon isotope compositions were measured on CO₂ gas using a dual-inlet IRMS located at NOSAMS (McNichol et al., 1994a), with resulting ¹³C content expressed in δ^{13} C per mille (%) notation relative to Vienna Pee Dee Belemnite (VPDB). RPO fraction masses, δ^{13} C values, and Fm values were corrected for blank carbon contribution, and δ^{13} C was additionally corrected to ensure ¹³C mass balance as incomplete oxidation to CO₂ has been 65 shown to impart a small fractionation effect (Hemingway et al., 2017). Analytical uncertainty was propagated throughout all corrections. Thermograms and isotope results for both Narayani POC and JGOFS sediment are plotted in Fig. 2, while temperature ranges, carbon masses, and isotope values 70 are additionally reported in Tables 2–3.

3 Deriving a model of decay kinetics

We derive the distributed activation energy model by first considering the case where OC is separated into a set of discrete components with unique E values. We then generalize this description to allow for a continuous E distribution (Braun and Burnham, 1987; Burnham and Braun, 1999; Cramer, 2004).

3.1 Discrete model

During OC remineralization, the decay rate of carbon contained in a particular component i is often described as as a first-order process with respect to $g_i(t)$, the mass of carbon remaining in component i at any time t (Westrich and Berner, 1984; Braun and Burnham, 1987), according to

$$\frac{dg_i(t)}{dt} = -k_i g_i(t),\tag{1}$$

where k_i is the first-order rate coefficient associated with component i. Total OC decay is then treated as the sum over all components. Although it is possible that OC decay in the natural environment additionally depends on oxidant concentration, we omit this dependency here since O_2 is provided in excess in our experimental setup (Fig. 1). In contrast to the "multi-G" and "reactive continuum" models that are often used to describe environmental OC degradation rates (Westrich and Berner, 1984; Boudreau and Ruddick, 1991; Forney and Rothman, 2012a, b), here we allow k_i to vary with time. Because rate coefficients are related to temperature and activation energy, k_i can be written as a time-

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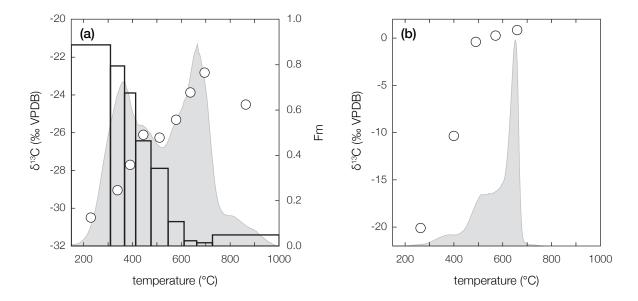


Figure 2. RPO results. Measured thermograms (gray shaded region, ppm CO_2 axes not shown), $\delta^{13}C$ values (white circles, left axes), and Fm values (transparent bars, right axes) for (a) Narayani POC and (b) JGOFS sediment (Fm not measured). Fm bar widths correspond to the temperature range of collection for each RPO fraction. Fm and $\delta^{13}C$ analytical uncertainty is always smaller than point marker and is therefore not shown (see Tables 2–3 for values).

Table 2. Narayani POC RPO temperature ranges, carbon masses, δ^{13} C, Fm, and E for each fraction, f. All masses and isotope values are blank corrected following Hemingway et al. (2017). See Eqs. (41)–(42) for E calcuations.

f	T (°C)		m_f	(μgC)	$\delta^{13} C_f (\% \text{VPDB})^*$		Fm_f		$E (k \text{J} \text{mol}^{-1}) **$	
	min.	max.	mean	std. dev.	mean	std. dev.	mean	std. dev.	\overline{E}_f	σ_f
1	150	310	68.4	0.7	-29.5	0.2	0.891	0.004	134.4	8.1
2	310	367	105.6	1.1	-28.1	0.2	0.795	0.002	147.9	7.1
3	367	412	82.4	0.8	-26.7	0.2	0.676	0.003	159.0	7.5
4	412	475	92.6	0.9	-25.1	0.2	0.464	0.003	173.1	8.5
5	475	545	85.6	0.9	-25.3	0.2	0.342	0.003	190.6	10.9
6	545	610	98.4	1.0	-24.3	0.2	0.107	0.002	209.7	10.7
7	610	661	101.5	1.0	-22.9	0.2	0.022	0.002	223.4	8.0
8	664	725	125.6	1.3	-21.8	0.2	0.014	0.002	231.5	7.1
9	725	997	86.6	0.9	-23.5	0.2	0.042	0.002	260.5	17.7

 $^{*\}delta^{13}$ C $_f$ is additionally corrected following Hemingway et al. (2017) to ensure that the mass-weighted mean matches the measured bulk value.

dependent function of E following the Arrhenius equation:

$$k_i(t) = \omega \exp\left[-\frac{E_i}{RT(t)}\right],$$
 (2)

where ω is the empirically derived Arrhenius pre-exponential $_5$ ("frequency") factor, R is the ideal gas constant, E_i is the activation energy of carbon contained in component i, and T(t) is the measured temperature of the system at time t. For non-isothermal systems, time-dependent decay coefficients can therefore be described by the static property E_i and the observed variable T(t). Although T(t) is related to t by a

constant ramp rate β during RPO analysis, we leave this written as is to emphasize that our model is valid for any measured time-temperature history. Substituting Eq. (2) into Eq. (1), we write the first-order decay at time t during a non-isothermal process as

$$\frac{dg_i(t)}{dt} = -\omega \exp\left[-\frac{E_i}{RT(t)}\right]g_i(t). \tag{3}$$

The mass of carbon remaining in component i at time t can be determined by integrating Eq. (3) from an initial time t =

^{**}Assuming L-curve best-fit λ value and $\omega = 10 \times 10^{10} \text{ s}^{-1}$.

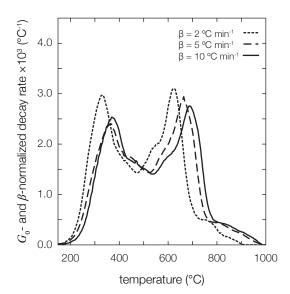


Figure 3. Testing the ramp-rate effect. Measured thermograms are shown for Narayani POC analyzed using multiple ramp rates (β) . Decay rates have been normalized by G_0 and β in order to properly compare y axis values between each analysis.

0:

$$g_i(t) = g_i(0)e^{-\kappa_i(t)},\tag{4}$$

where

$$\kappa_i(t) = \omega \int_0^t \exp\left[-\frac{E_i}{RT(t')}\right] dt'$$
 (5)

 $_{5}$ is the time integrated decay coefficient at time t and $g_{i}(0)$ is the initial mass of carbon contained in component i. Equation (5) states that $g_{i}(t)$ depends on the entire time-temperature history of the experiment. That is, the evolution of $dg_{i}(t)/dt$ reflects both a decrease in $g_{i}(t)$ as OC is remineralized and an increase in $k_{i}(t)$ with increasing T(t) as the experiment progresses. This results in a predictable shift in RPO thermograms toward higher elution temperatures with increasing β (Miura and Maki, 1998), as shown in Fig. 3.

Following Boudreau (1997) and Westrich and Berner (1984), an environmental sample containing a complex OC mixture can be described as a superposition of a finite set of n components, each decaying according to a unique $k_i(t)$ and thus corresponding to a unique E_i value. G(t), the total carbon mass remaining at t, is then the sum of the mass 20 remaining in each component:

$$G(t) = \sum_{i=1}^{n} g_i(t). \tag{6}$$

Substituting Eq. (4) into Eq. (6), this can be written as

$$G(t) = \sum_{i=1}^{n} g_i(0)e^{-\kappa_i(t)}.$$
 (7)

We then define G_0 , the initial OC mass present in the entire sample, as the sum of initial mass contained in each component:

$$G_0 = \sum_{i=1}^n g_i(0). (8)$$

Finally, we define $p_i(0)$, the fraction of total carbon initially contained in component i, as

$$p_i(0) = \frac{g_i(0)}{G_0} \tag{9}$$

and note that

$$\sum_{i=1}^{n} p_i(0) = 1. (10)$$

Substituting Eq. (9) into Eq. (7) yields

$$\frac{G(t)}{G_0} = \sum_{i=1}^{n} p_i(0)e^{-\kappa_i(t)},\tag{11}$$

which describes the evolution of the fraction of initial carbon 35 remaining at any time. The fraction of OC initially present within each component, $p_i(0)$, can be determined by fitting Eq. (11) to the observed G(t) profile measured by the RPO instrument. While informative, this discrete description of the model suffers from two major limitations: (i) n must be 40 set a priori or determined empirically (Boudreau and Ruddick, 1991) and (ii) any noise recorded in the data will result in large uncertainty in best-fit $p_i(0)$ and E_i values (Forney and Rothman, 2012b). To circumvent the first of these issues, we derive a more general description of non-isothermal first-order decay that does not assume a finite set of components with unique E_i , but rather allows E to vary continuously (Boudreau, 1997; Braun and Burnham, 1987; Burnham and Braun, 1999; Cramer, 2004). Limitation (ii) is then solved using Tikhonov regularization (Section 4.2; Forney 50 and Rothman, 2012b; Hansen, 1994).

3.2 Continuous model

In the continuous model, discrete components $g_i(t)$, $\kappa_i(t)$ and E_i are replaced by continuous variables g(t,E), $\kappa(t,E)$ and E, respectively (Table 1). Analogous to Eq. (3), we calculate the decay of OC associated with an infinitesimal range dE about any non-negative value of E following first-order Arrhenius kinetics as

(6)
$$\frac{dg(t,E)}{dt} = -\omega \exp\left[-\frac{E}{RT(t)}\right]g(t,E). \tag{12}$$

Table 3. JGOFS RPO temperature ranges, carbon masses, and δ^{13} C for each fraction, f. All masses and isotope values are blank corrected following Hemingway et al. (2017).

\overline{f}	T (°C)		$m_f (\mu g C)$		$\delta^{13} C_f (\% \text{oVPDB})^*$		
	min.	max.	mean	std. dev.	mean	std. dev.	
1	163	363	38.5	0.4	-20.1	0.2	
2	363	435	45.9	0.5	-10.3	0.2	
3	435	543	217.6	2.2	-0.4	0.2	
4	543	597	154.4	1.5	0.3	0.2	
5	597	720	497.7	5.0	0.9	0.2	

 $^{*\}delta^{13}{
m C}_f$ is additionally corrected following Hemingway et al. (2017) to ensure that the mass-weighted mean matches the measured bulk value.

The mass of carbon associated with any value of E that remains unreacted at time t is then calculated by integrating Eq. (12) to obtain

$$g(t,E) = g(0,E)e^{-\kappa(t,E)},\tag{13}$$

 $_{5}$ where g(0,E) is the initial mass of carbon associated with activation energy value E and

$$\kappa(t, E) = \omega \int_{0}^{t} \exp\left[-\frac{E}{RT(t')}\right] dt'. \tag{14}$$

The total carbon remaining at time t can now be written as the integral of g(t, E) over all possible values of E as

10
$$G(t) = \int_{0}^{\infty} g(t, E) dE$$
. (15)

Substituting Eq. (13) into Eq. (15), we obtain

$$G(t) = \int_{0}^{\infty} g(0, E)e^{-\kappa(t, E)}dE.$$
(16)

Analogous to Eq. (9), we then define the fraction of total carbon initially associated with any value of E as

15
$$p(0,E) = \frac{g(0,E)}{G_0}$$
 (17)

where

$$\int_{0}^{\infty} p(0, E)dE = 1. \tag{18}$$

Substituting Eq. (17) into Eq. (16) yields:

$$\frac{G(t)}{G_0} = \int_0^\infty p(0, E)e^{-\kappa(t, E)}dE.$$
 (19)

The distribution of p(0,E) over all values of E describes the 20 initial probability density function (pdf) of E that will lead to the observed OC decay profile when a sample is analyzed in the RPO instrument. As RPO analysis proceeds, this pdf must evolve with time to reflect the fact that some carbon has been remineralized to CO_2 . Like g(t,E), p(t,E) follows 25 first-order Arrhenius kinetics according to

$$\frac{dp(t,E)}{dt} = -\omega \exp\left[-\frac{E}{RT(t)}\right]p(t,E),\tag{20}$$

where p(t, E) is the fraction of initial carbon mass that remains associated with E at time t. This can be obtained by integrating Eq. (20) from an initial time t = 0:

$$p(t,E) = p(0,E)e^{-\kappa(t,E)}$$
. (21)

This implies that the carbon initially remineralized to CO_2 must be associated with the lowest E values, as low E will lead to a $\kappa(t,E)$ term in Eq. (21) that approaches zero most rapidly. Put differently, OC that is described by higher E values will resist remineralization until more time has passed and, therefore, higher temperatures have been reached – *i.e.* it is more thermally recalcitrant.

3.3 First-order verification

Because the distributed activation energy model is a specific case of $n^{\rm th}$ -order non-isothermal kinetic models (Braun and Burnham, 1987; White et al., 2011), we must verify that carbon degradation within the RPO instrument behaves according to a superposition of parallel first-order reactions rather than higher-order processes. By replacing g(t,E) with $G_0p(t,E)$ on the right-hand side of Eq. (12), it can be seen that

$$\frac{dg(t,E)}{dt} = -G_0\omega \exp\left[-\frac{E}{RT(t)}\right]p(t,E). \tag{22}$$

By integrating over all non-negative values of E and utilizing the definition of G(t) from Eq. (15), this can be written as

$$\frac{dG(t)}{dt} = -G_0 \int_0^\infty \omega \exp\left[-\frac{E}{RT(t)}\right] p(t, E) dE.$$
 (23)

The first-order model describes dG(t)/dt as a linear function of G_0 multiplied by an integral term that depends on p(t,E) but is independent of G_0 . In contrast, if carbon decomposition within the RPO instrument were to follow a higher-order process, then the relationship between dG(t)/dt and G_0 would be nonlinear and would evolve as a function of time (Follett et al., 2014). If we define

(19)
$$m(t) = \int_{0}^{\infty} \omega \exp\left[-\frac{E}{RT(t)}\right] p(t, E) dE,$$
 (24)

then the carbon decay at time t as predicted by parallel first-order kinetics simplifies to

$$\frac{dG(t)}{dt} = -G_0 m(t). (25)$$

Therefore, similar to the isothermal case (Follett et al., 2014), a superposition of parallel first-order decay reactions will result in a linear relationship between dG(t)/dt and G_0 with a zero intercept and a time-dependent slope. Thus, m(t) can be interpreted as the G_0 -normalized OC decay rate at time t.

We verify that OC remineralization within the RPO instrument follows parallel first-order kinetics by assessing the linearity between Narayani POC dG(t)/dt and G_0 at any time t across a range of G_0 values. For 4 arbitrarily chosen time points, this relationship is linear with an ordinary least squares $R^2 \geq 0.999$, resulting in identical G_0 -normalized thermograms within analytical uncertainty (Fig. 4a–b). Thus, the decay of complex OC mixtures contained in carbonate-free samples during RPO analysis can indeed be accurately described as a superposition of parallel first-order reactions.

3.4 A note of caution on carbonates

20 While most RPO studies to date have isolated OC by acidifying to remove carbonates (e.g. Rosenheim et al., 2008; Rosenheim and Galy, 2012; Rosenheim et al., 2013; Schreiner et al., 2014; Bianchi et al., 2015), it has been argued that acid hydrolysis and/or dissolution of clay minerals 25 during acid treatment can alter the OC chemical bonding environment and therefore affect thermal stability (Plante et al., 2013). While analyzing samples without acid treatment can circumvent these issues, the effect of carbonates on decay kinetics has not yet been considered. To test if carbonate-30 rich samples follow parallel first-order kinetics, we analyzed JGOFS sediment for a range of G_0 values (Fig. 4c-d). Prior to $T \approx 500$ °C (corresponding to $t \approx 4500$ s), when δ^{13} C values of eluted CO₂ indicate a predominantly OC source (Table 3; Fig. 2b), dG(t)/dt can be accurately described as a linear 35 function of G_0 ($R^2 \ge 0.999$). However, as carbonate begins to decompose above $T \approx 500$ °C, the relationship between dG(t)/dt and G_0 becomes nonlinear and the carbonate peak shifts toward higher t with increasing G_0 (Fig. 4d).

To investigate if non-first-order decomposition is an in- trinsic property of CaCO₃ or if this is due to interactions with other materials within the sample (so-called "matrix effects"), we analyzed a pure Icelandic spar CaCO₃ labroatory standard at multiple masses ($G_0 = 258 \, \mu \mathrm{g} \, \mathrm{C}$, $492 \, \mu \mathrm{g} \, \mathrm{C}$ and $1014 \, \mu \mathrm{g} \, \mathrm{C}$; $\beta = 5 \, ^{\circ} \mathrm{C} \, \mathrm{min}^{-1}$; not shown). Results indicate that 45 pure carbonate, unlike JGOFS sediment, does follow first-order kinetics with a maximum decomposition rate occurring at $(700 \pm 6) \, ^{\circ} \mathrm{C}$ independent of G_0 . Interaction with reduced organic carbon, corresponding hetero-atoms ($e.g. \, \mathrm{N}, \, \mathrm{P}, \, \mathrm{S}$), or trace metals contained within the sample matrix are thereso fore the likeliest cause of non-first-order CaCO₃ decomposition when analyzing environmental samples. Thus, while

avoiding the issues of acid treatment, the presence of carbonate will result in thermograms that cannot be accurately described by the model presented here, and we therefore argue in favor of acid treatment when using the RPO instrument to determine reaction energetics of carbonate-containing samples.

4 Finding the regularized inverse solution

Following Forney and Rothman (2012a, b), we present a method to estimate p(0,E) by finding an inverse solution to Eq. (19). In contrast to previous solutions (Braun and Burnham, 1987; Burnham and Braun, 1999; Cramer, 2004), this approach does not require an *a priori* assumption about the form of p(0,E) (e.g. Gaussian). Because this problem is sensitive to noise at the level of our analytical uncertainty (Forney and Rothman, 2012b), we seek a smooth solution using Tikhonov regularization (Section 4.2; Forney and Rothman, 2012b; Hansen, 1994).

To numerically calculate p(0,E), we discretize the continuous variable t over the time course of the experiment into a vector \mathbf{t} containing n_t nodes such that

$$\Delta t_j = \frac{1}{2} (t_{j+1} - t_{j-1}), \quad j = 2, \dots, n_t - 1.$$
 (26)

For j=1 and $j=n_t,\,t_{j-1}$ and t_{j+1} in Eq. (26) are, respectively, replaced by t_j since ${\bf t}$ is undefined outside of this range. For generality, and because the distributed activation renergy model is frequently applied over geologic timescales with non-uniformly distributed time measurements (Braun and Burnham, 1987; Burnham and Braun, 1999; Cramer, 2004), Eq. (26) does not require a uniform time step (*i.e.* it is possible that $\Delta t_j \neq \Delta t_{i \neq j}$). Similarly, we generate a vector E containing n_E nodes over the range values considered for the model solution such that

$$\Delta E = \frac{E_{\text{max}} - E_{\text{min}}}{n_E},\tag{27}$$

noting that **E** is uniformly spaced since this vector is not constrained by observations. We constrain E to be within $_{50}\,\mathrm{kJ\,mol^{-1}}$ to $350\,\mathrm{kJ\,mol^{-1}}$ based on published biomass and petroleum E ranges (Braun and Burnham, 1987; Burnham and Braun, 1999; Cramer, 2004; White et al., 2011).

It can be seen from Eq. (19) that the model can be separated into two components: (i) p(0,E) and (ii) the exponentiated, time integrated decay coefficient, $\exp\left[\kappa(t,E)\right]$. Analogous to the Laplace transform for the isothermal reactive continuum model (Forney and Rothman, 2012b), $\exp\left[\kappa(t,E)\right]$ determines the fraction of carbon initially associated with an activation energy value E that has decayed by time t. While this integral can be calculated analytically for a constant ramp rate β , here we approximate the solution numerically so that our model can be applied to any time-temperature history. Thus, we populate a matrix $\bf A$ by calculating $\exp\left[\kappa(t,E)\right]$ for each t_j and E_l contained in $\bf t$ and

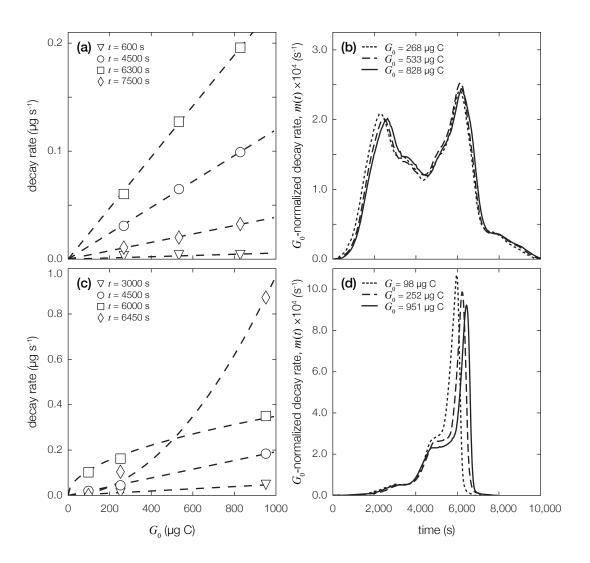


Figure 4. First-order kinetic assessment. Left column shows decay rate, dG(t)/dt, vs. G_0 relationships at four arbitrarily chosen time points (including best-fit regression lines, dashed lines) and right column shows the mass-normalized decay rates [termed m(t) in Eq. (24)–(25)] at all time points for (a)–(b) Narayani POC and (c)–(d) JGOFS sediment. Linear relationships and nearly identical normalized decay rates in panels (a)–(b) confirm the first-order nature of OC decay, while non-linear relationships and a shifting carbonate peak in panels (c)–(d) indicate non-first-order CaCO₃ decay kinetics. For each time point in panel (a), the regression slope is equivalent to m(t) for that time point as shown in panel (b).

E as

$$A_{j,l} = \exp\left\{-\sum_{u=1}^{j} \omega \exp\left[-\frac{E_l}{RT(t_u)}\right] \Delta t_u\right\} \Delta E,$$

$$j = 1, \dots, n_t,$$

$$l = 1, \dots, n_E.$$
(28)

The **A** matrix is often termed the model "design matrix." We then calculate the fraction of initial carbon remaining at each 5 time point as

$$\frac{G(t)}{G_0} = 1 - \alpha(t),\tag{29}$$

where $\alpha(t)$ is the G_0 -normalized, integrated RPO thermogram at time t. We generate a discretized vector \mathbf{g} by interpolating $G(t)/G_0$ onto each t_j in \mathbf{t} $(j=1,\ldots,n_t)$. Our model can then be written in matrix form as

$$\mathbf{g} = \mathbf{A} \cdot \mathbf{p},\tag{30}$$

where **p** is an unknown, discretized vector of p(0, E) with length n_E such that

$$p_{l} = \frac{1}{\Delta E} \int_{E_{l} - \frac{1}{2}\Delta E}^{E_{l} + \frac{1}{2}\Delta E} p(0, E) dE, \quad l = 1, \dots, n_{E}.$$
 (31)

While Eq. (30) can be solved for p by multiplying g by the computed inverse of A, if g contains noisy data this may result in negative values of p_l that are mathematically possible but physically unreasonable (Forney and Rothman, 2012b). $_5$ Here, we find the solution that satisfies

$$\min_{\mathbf{p}} \|\mathbf{g} - \mathbf{A} \cdot \mathbf{p}\| \equiv \left[\sum_{j=1}^{n_t} \left(g_j - \sum_{l=1}^{n_E} A_{j,l} p_j \right)^2 \right]^{\frac{1}{2}}, \quad (32)$$

subject to the constraints

$$p_l \ge 0, \quad l = 1, \dots, n_E,$$
 (33)

and

using the non-negative least squares algorithm of Lawson and Hanson (1995) as implemented by the SciPy package for Python. Eqs. (32)–(34) describe the model solution that minimizes the norm of the residual error (*i.e.* the root mean square error, or RMSE) while fulfilling the constraints that p is non-negative and sums to unity.

4.1 Choice of frequency factor

In order to construct the A matrix and solve for p, our method requires that the Arrhenius frequency factor ω is pre-20 scribed a priori. There exists significant discussion in the literature on the best choice of ω , as multiple values can describe laboratory results equally well but will result in drastically different predictions of OC degradation rates over geologic timescales (Braun and Burnham, 1987; Dieckmann, 25 2005). Furthermore, it has been argued that ω represents a variable change in entropy associated with the decay of specific organic compounds and should therefore be parameterized as a function of E (the so-called "kinetic compensation effect" or KCE; Dieckmann, 2005; Lakshmanan et al., 1991; ₃₀ Tang et al., 2000). For example, a linear ω increase with E from $\approx 10^8 \,\mathrm{s}^{-1}$ (E = 175 kJ mol⁻¹) to $\approx 10^{26} \,\mathrm{s}^{-1}$ (E = 400 kJ mol⁻¹) has been utilized to better predict petroleum formation rates (Dieckmann, 2005). To circumvent the issue of multiplicity, and to account for the KCE, Miura and Maki 35 (1998) developed a method to estimate the best-fit ω for each E value by comparing the shift in elution temperatures when a sample is analyzed at multiple ramp rates. However, because this approach is based on large extrapolations in 1/Tvs. β/T^2 space, it is highly sensitive to noise in temperature ₄₀ and β measurements (Burnham and Braun, 1989).

To select a best-fit ω , we construct the **A** matrix for a range of ω values and determine the residual error norm between measured $G(t)/G_0$ and that predicted by the resulting **p** vector determined by Eqs. (32)–(34). We consider the KCE by as calculating ω as a function of E according to

$$\log_{10} \omega = (\text{KCE slope})E + (\text{KCE intercept}).$$
 (35)

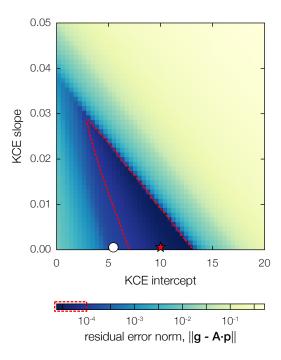


Figure 5. Frequency factor assessment. Model residual error norm using a range of KCE slopes and intercepts for Narayani POC (β = 5 °C min⁻¹). Each pixel represents the best-fit solution to Eqs. (32)–(34) for a given ω as determined by Eq. (35). "Acceptable" fits with residual error norm $\leq 10^{-4}$ are contained within the red dotted line. Estimated result using the method of Miura and Maki (1998) for 3 ramp rates (β = 2 °C min⁻¹, 5 °C min⁻¹ and 10 °C min⁻¹) is plotted as a white circle, while the point corresponding to ω = 10^{10} s⁻¹ (the value chosen for samples in this study) is plotted as a red star.

Resulting residual errors for Narayani POC using a range of KCE slopes and intercepts are shown in Fig. 5 (β = $5 \,^{\circ}\text{C}\,\text{min}^{-1}$, E ranging from $50 \,\text{kJ}\,\text{mol}^{-1}$ to $350 \,\text{kJ}\,\text{mol}^{-1}$). By setting an "acceptable" residual error norm cutoff of 50 $\leq 10^{-4}$, it can be seen that there exist multiple KCE slope and intercept combinations that can equally fit the observed data. Additionally, we estimate the best-fit ω using a range of ramp rates ($\beta = 2$ °C min⁻¹, 5 °C min⁻¹ and 10 °C min⁻¹) following the method of Miura and Maki (1998) (Fig. 5, white circle). While this estimate falls outside of the cutoff range, likely due to noise in temperature and β measurements, it results in a KCE slope near zero and suggests that ω is constant during RPO oxidation of this sample. To accurately compare RPO results between samples, we therefore select a constant 60 ω value of $10^{10}\,\mathrm{s}^{-1}$, well within the cutoff range, for samples analyzed herein (Fig. 5, red star). While a different choice of ω will shift p(0,E) to higher or lower absolute values of E, we emphasize that it will not affect the relative distribution of E and that only relative changes in E between RPO fractions 65 should be interpreted.

For example, a shift in ω from a constant value of $10^7\,\mathrm{s}^{-1}$ to $10^{12}\,\mathrm{s}^{-1}$ results in an increase in the mean of the pdf of E, termed \overline{E} and calculated as

$$\overline{E} = \sum_{l=1}^{n_E} E_l p(0, E_l) \Delta E, \tag{36}$$

 $_{5}$ from $150\,\mathrm{kJ\,mol^{-1}}$ to $224\,\mathrm{kJ\,mol^{-1}}$ for Narayani POC. However, the relative standard deviation of the pdf of E, calculated as σ/\overline{E} , where

$$\sigma^2 = \overline{E^2} - \left(\overline{E}\right)^2,\tag{37}$$

remains constant at 20%. A higher ω value therefore results in a broader p(0,E) distribution that is centered at a higher mean E value but has no effect on the relative shape of the distribution.

4.2 Tikhonov regularization

In principle, after choosing ω and constructing the **A** matrix, the pdf of E that best describes an RPO thermogram can be determined by solving Eqs. (32)–(34). However, the inverse solution is sensitive to noise at the level of RPO instrument precision (± 5 ppm CO₂, ± 5 °C; Hemingway et al., 2017), and is therefore ill-posed (Hansen, 1994; Lakshmanan et al., 20191). We address this sensitivity to data uncertainty using Tikhonov regularization (Hansen, 1994; Forney and Rothman, 2012a, b).

This approach finds an optimal solution that minimizes p(0,E) complexity (as determined by the intensity of fluctuations, or "roughness") while maximizing solution accuracy. Following Forney and Rothman (2012b), we calculate roughness as the first derivative of the solution vector:

$$\left\| \frac{dp(0,E)}{dE} \right\| = \left[\sum_{l=2}^{n_E-1} \left(\frac{p_{l+1} - p_l}{\Delta E} \right)^2 \right]^{\frac{1}{2}} \equiv \| \mathbf{R} \cdot \mathbf{p} \|, \quad (38)$$

where ${\bf R}$ is the bi-diagonal first derivative operator matrix with dimensions $[n_E \times n_E]$. To account for ${\bf p}$ being equal to zero outside the range $E_{\rm min} < E < E_{\rm max}$, we set the first and last rows of ${\bf R}$ to be equal to $[{\bf 1} \quad {\bf 0}]$ and $[{\bf 0} \quad -1]$, respectively, where ${\bf 0}$ refers to a zero vector of length n_E-1 . Similar to Eq. (32), the regularized inverse problem can then be solved for ${\bf p}$ by including this roughness term in the constrained least squares. That is, we solve

$$\min_{\mathbf{p}} \|\mathbf{g} - \mathbf{A} \cdot \mathbf{p}\| + \lambda \|\mathbf{R} \cdot \mathbf{p}\|, \tag{39}$$

for \mathbf{p} subject to the constraints presented in Eqs. (33) – (34), where λ is a scalar that determines how much to weight the roughness $\|\mathbf{R} \cdot \mathbf{p}\|$ relative to the residual error $\|\mathbf{g} - \mathbf{A} \cdot \mathbf{p}\|$. The best choice of λ is considered to be the value that optimizes this balance. As described in Hansen (1994), a common approach is to choose the value corresponding to the

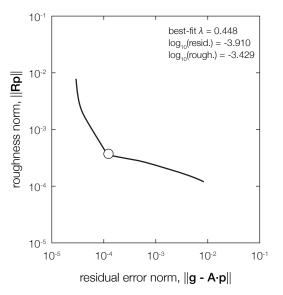


Figure 6. Tikhonov Regularization L-curve for Narayani POC ($\beta = 5 \,^{\circ}\text{C min}^{-1}$). The black line represents the range of roughness and residual error norms that are the result of solving Eq. (39) for **p** using multiple λ values ranging from 0.001 to 100. The white circle corresponds to the point of maximum curvature along this line, and is thus deemed the "best-fit" λ value [see Hansen (1994) and Forney and Rothman (2012b) for further details on generating the L-curve and theory behind Tikhonov regularization.]

point of maximum curvature in a $\log - \log$ plot of residual error and roughness while allowing λ to range over many orders of magnitude (*i.e.* the so-called "L-curve"). From this point, increasing λ strongly increases residual error but has little effect on solution roughness, while decreasing λ greatly increases roughness but has little effect on residual error. Thus, here we choose the λ value corresponding to the corner of the L-curve for each sample, as exemplified in Fig. 6.

4.3 p(0,E): A novel proxy for chemical bond strength

In order to interpret p(0,E) as an intrinsic property of OC contained within a sample, we must show that results do not depend on experimental conditions such as ramp rate β and initial carbon mass G_0 . To test this, we analyzed Narayani POC using a range of masses ($G_0 = 268 \, \mu \mathrm{g\,C}$, $533 \, \mu \mathrm{g\,C}$ and $828 \, \mu \mathrm{g\,C}$) and ramp rates ($\beta = 2 \, ^{\circ}\mathrm{C\,min^{-1}}$, $5 \, ^{\circ}\mathrm{C\,min^{-1}}$ and $10 \, ^{\circ}\mathrm{C\,min^{-1}}$). Fig. 7 shows that the regularized pdfs of E are nearly identical across all experimental conditions. This supports the hypothesis that the p(0,E) distribution is an intrinsic property of a given sample when exposed to a particular oxidation pathway (e.g. thermal decay). Although there exist small differences between individual analyses due to measurement uncertainty and variability in best-fit λ values (ranging from 0.044 to 0.448, n=5), the main features of p(0,E) are robust across all conditions. Note that the p(0,E)

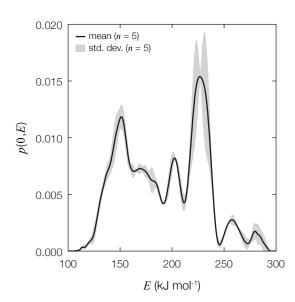


Figure 7. Regularized p(0,E) distribution for Narayani POC. Mean (black line) and standard deviation (gray shaded region) of p(0,E) analyzed using a range of G_0 and β values (n=5). Narrow standard deviation indicates that model results are independent of experimental conditions.

distribution (Fig. 7) broadly resembles the initial thermogram shape (Fig. 2a), albeit with more defined features and a higher roughness. This is a result of the fact that OC associated with each E value will decay over a wide temperature range within the RPO instrument, thus resulting in a "smoothed" thermogram relative to p(0, E) (Cramer, 2004).

While further study is required to assess the general applicability of this technique, we propose p(0,E) as a novel proxy to describe the distribution of carbon bond strength 10 (Braun and Burnham, 1987; Burnham and Braun, 1999; Cramer, 2004). For example, Narayani POC is known to integrate recently fixed biomass, pre-aged soils, and eroded rockderived material (Galy et al., 2008, 2011; Galy and Eglinton, 2011; Rosenheim and Galy, 2012). Such integration should $_{15}$ lead to large chemical diversity and a broad, complex E distribution, as is observed (Fig. 7). Furthermore, slow environmental turnover has been shown to enhance the diversity of chemical bonds due to a combination of microbial alterations (Schmidt et al., 2011), OC aggregation (Keil and 20 Mayer, 2014), and stabilization by mineral surfaces (Keil and Mayer, 2014; Mikutta et al., 2006). Thus, OC reactivity within the RPO instrument and the resulting E distribution likely reflects both the strength of covalent bonds between carbon atoms as well as interactions with mineral sur-25 faces (Keil and Mayer, 2014; Mikutta et al., 2006). We therefore propose combining p(0,E) with serial oxidation isotope measurements to test the effects of mineral interactions and

selective preservation on OC turnover time.

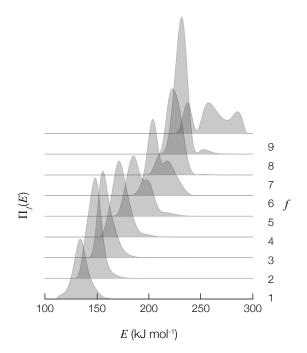


Figure 8. $\Pi_f(E)$ distributions for Narayani POC $(f=1,\ldots,9)$. Each $\Pi_f(E)$ represents the range of E values contained within RPO fraction f. The sum of all $\Pi_f(E)$ distributions shown here thus yields the p(0,E) distribution shown in Fig. 7. Distributions have been staggered along the y axis for visual clarity. $\Pi_f(E)$ distributions do not follow any predictable functional form and are highly overlapping due to the fact that OC associated with a given E value decays over a wide time interval (Cramer, 2004).

5 Relating E and isotope composition

5.1 Determining the distribution of E within each RPO $_{50}$ fraction

To relate p(0,E) distributions to RPO isotope measurements, we calculate the subset of the pdf of E that is contained within each RPO fraction. Because we can predict the evolution of p(t,E) at any time t following Eq. (21), this can be $_{35}$ calculated as

$$\Pi_f(E) = p(t_1, E) - p(t_2, E), \quad f = 1, \dots, n_f,$$
 (40)

where n_f is the number of RPO fractions collected for a given sample, $\Pi_f(E)$ is the subset of p(0,E) contained in RPO fraction f, and t_1 and t_2 are the initial and final time points, respectively, of CO_2 collection for RPO fraction f. Resulting $\Pi_f(E)$ distributions for Narayani POC are shown in Fig. 8. Finally, in order to generate E vs. $\delta^{13}\mathrm{C}$ and E vs. Fm scatter plots, we calculate the mean E value contained in each RPO fraction as

$$\overline{E}_f = \sum_{l=1}^{n_E} E_l \Pi_f(E_l) \Delta E, \quad f = 1, \dots, n_f$$
(41)

and the standard deviation of E contained in each RPO fraction as σ_f , where

$$\sigma_f^2 = \overline{E^2}_f - \left(\overline{E}_f\right)^2, \quad f = 1, \dots, n_f. \tag{42}$$

Resulting \overline{E}_f and σ_f values are reported in Table 2. It can 5 be seen in Fig. 8 that $\Pi_f(E)$ distributions do not follow any particular form (e.g. Gaussian) and are highly overlapping, reflecting the fact that the CO₂ isotope composition for each RPO fraction is itself a weighted average of multiple sources.

5.2 Kinetic isotope fractionation

¹⁰ While not necessary for Fm because it is fractionation-corrected by definition (Reimer et al., 2004), it is important to correct for any kinetic isotope effects occurring within the RPO instrument before interpreting δ^{13} C as a carbon source tracer (Hemingway et al., 2017). If kinetic fractionation is large, as has been observed both during thermogenic methane formation (Tang et al., 2000; Cramer, 2004) and dissolved OC oxidation by uv light (Oba and Naraoka, 2008), then this effect could overprint carbon source δ^{13} C signals. However, when directly measured using single-compound standards, Hemingway et al. (2017) concluded that ¹³C fractionation within the RPO instrument must be $\leq 2\%c$. Still, we correct the measured δ^{13} C values of each RPO fraction using the ratio of carbon-normalized ¹³C and ¹²C decomposition rates at each time point according to

$${}_{25} {}^{13/12} \mathbf{r}(t) = \frac{\left(\frac{d^{13}G(t)}{dt}\right)}{\left(\frac{d^{12}G(t)}{dt}\right)} \left(\frac{{}^{12}G_0}{{}^{13}G_0}\right),\tag{43}$$

where we have added a preceding 12 or 13 superscript to specify isotope-specific variables. Following the Arrhenius equation, $^{13/12}$ r(t) can be written as a function of the difference in E between 13 C- and 12 C-containing molecules:

$$_{30}$$
 $^{13-12}\Delta E = ^{13}E - ^{12}E.$ (44)

Although $^{13-12}\Delta E$ is likely not identical for all compounds due to differences in the entropy and enthalpy of isotope substitution (Tang et al., 2000), the estimated range of values for RPO analysis is small $(0.3\times10^{-3}~{\rm kJ\,mol^{-1}}$ to $1.8\times10^{-3}~{\rm kJ\,mol^{-1}}$; Hemingway et al., 2017). We therefore assume a $^{13-12}\Delta E$ value of $1.8\times10^{-3}~{\rm kJ\,mol^{-1}}$ for all RPO fractions, noting that a choice of $0.3\times10^{-3}~{\rm kJ\,mol^{-1}}$ would result in $\delta^{13}C$ values that are identical to those calculated here within analytical uncertainty.

Values of $^{13/12}$ r(t) can be determined using the ratio of carbon-normalized, isotope-specific decay rates by substituting $p(0, ^{12}$ E) and $p(0, ^{13}$ E) for p(0, E) in Eq. (19). Because carbon is present as $\approx 99 \,\%$ 12 C, we set $p(0, ^{12}$ E) equal to p(0, E) such that

$$_{45}\frac{d^{12}G(t)}{dt} = \frac{dG(t)}{dt}.$$
 (45)

Corresponding $d^{13}G(t)/dt$ can then be determined using

$$p(0, {}^{13}E) = p(0, E + {}^{13-12}\Delta E).$$
 (46)

 $^{13}\mathrm{C}\text{-containing}$ molecules decay at rates governed by a pdf of E that is identical to p(0,E) but has been shifted by $1.8\times 10^{-3}\,\mathrm{kJ\,mol^{-1}}$. We then correct the measured $\delta^{13}\mathrm{C}$ so values of each RPO fraction f according to

$$\delta^{13} C_f^{\text{corrected}} = \frac{1}{^{13/12} r(t)_f^{\text{av}}} \left(\delta^{13} C_f + 1000 \left[^{13/12} r(t)_f^{\text{av}} - 1 \right] \right),$$

$$f = 1, \dots, n_f, \tag{47}$$

where $^{13/12}{\bf r}(t)_f^{\rm av}$ is the average $^{13/12}{\bf r}(t)_f$ value over the time of collection for RPO fraction f. For the samples analyzed here, $^{13/12}{\bf r}(t)$ is initially ≈ 0.999 , indicating slightly faster $^{12}{\bf C}$ decay at low temperatures, and gradually increases to ≈ 1.002 when $G(t) \ll 0.01G_0$, as has been described previously (Cramer, 2004; Hemingway et al., 2017). Resulting kinetic fractionation corrections are near or within analytical uncertainty, with absolute $\delta^{13}{\bf C}$ values for all RPO fractions shifted by <0.2%0.

5.3 Comparing E to δ^{13} C and Fm

Finally, we describe a framework to directly relate OC reactivity and isotope distributions by plotting \overline{E}_f for each RPO fraction vs. the corresponding measured δ^{13} C and Fm values (Table 2). Resulting relationships, as well as plant-wax fatty acid isotope values (Galy et al., 2011; Galy and Eglinton, 2011), are shown in Fig. 9. Within this framework, it can be seen that Narayani POC must contain at least two 70 end members with drastically different isotope compositions and unique yet overlapping E distributions. Previous studies have shown that $\approx (20 \pm 5) \%$ of OC contained in this sample is derived from the erosion of carbon-rich bedrock (Galy et al., 2008; Rosenheim and Galy, 2012). Rock-derived OC 75 is the likeliest source of high-E, low-Fm material, as this end member is 14 C-free by definition. Plant-wax FA δ^{13} C and Fm values are similar to those for low-E RPO fractions (Fig. 9), suggesting that vascular plant OC is the source of low-E material. Narayani POC isotope trends are thus consistent with 80 predominantly biospheric carbon below $\approx 150 \,\mathrm{kJ}\,\mathrm{mol}^{-1}$, a mixed region from $\approx 150 \,\mathrm{kJ}\,\mathrm{mol}^{-1}$ to $\approx 200 \,\mathrm{kJ}\,\mathrm{mol}^{-1}$, and exclusively rock-derived OC above $\approx 200 \,\mathrm{kJ}\,\mathrm{mol}^{-1}$. This result provides initial evidence for the utility of RPO E vs. isotope relationships to directly relate the distribution of OC sources, environmental turnover times, and chemical bonding environments.

6 Conclusions

In this study, we present a regularized, inverse method to determine the distribution of E, a measure of OC reactivity, 90

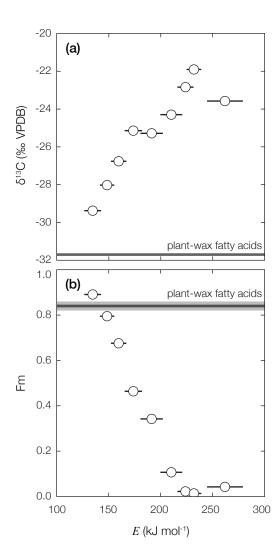


Figure 9. E vs. isotope relationships. (a) E vs. $\delta^{13}\mathrm{C}$ and (b) E vs. Fm for Narayani POC. All isotope values have been corrected for blank carbon contribution following Hemingway et al. (2017), and $\delta^{13}\mathrm{C}$ values have additionally been corrected for kinetic fractionation. Gray lines and shading are the plant-wax fatty acid biomarker isotope values (mean ± 1 std. dev. analytical uncertainty; Galy et al., 2011; Galy and Eglinton, 2011). Note that plant-wax fatty acids are known to contain less $^{13}\mathrm{C}$ (lower $\delta^{13}\mathrm{C}$ values) than corresponding bulk biospheric OC. Each point is plotted at $E = \overline{E}_f$. Error bars in E are equal to σ_f , while $\delta^{13}\mathrm{C}$ and Fm analytical uncertainty is always smaller than point marker and is therefore not shown.

when natural organic matter is exposed to serial oxidation. We show that OC decay follows parallel, first-order kinetics. In contrast, the kinetics of carbonate oxidation cannot be constrained due to matrix effects and we therefore recommend acidification to remove carbonates prior to RPO analysis. We propose that p(0,E), the distribution of E contained within a sample, is a useful proxy to describe the range of OC bonding environments. Importantly, our method does not require

a priori assumptions about the distributional form of p(0,E). Finally, we determine the subset of E contained within each RPO fraction in order to directly relate reaction energetics with the distribution of carbon isotope compositions within a complex OC mixture. We suggest that E vs. isotope relationships can provide new insight into understanding the compositional controls on OC source and residence time, although we note that further study is required in order to test the general applicability of this result. This manuscript is accompanied by an open-source Python package for performing all analyses.

Code and data availability. All thermogram data are available 20 in the supplementary material. The open-source 'rampedpyrox' package is accessible using the Python Package Index (http://pypi.python.org/pypi/rampedpyrox).

Author contributions. J.D.H., S.Z.R., and V.V.G. performed all laboratory measurements; J.D.H., V.V.G., and D.H.R. analyzed the data; J.D.H. and D.H.R. developed the inverse model; J.D.H. wrote the manuscript with input from all authors.

Competing interests. The authors declare that they have no conflict of interest.

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