# Supplementary materials

## Diurnal, seasonal and long-term behaviours of high Arctic tundra heath ecosystem dynamics inferred from model ensembles constrained by the time-integrated $CO_2$ fluxes

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**Table S1 List of relevant equations and parameters used in this modelling study. The parameters denoted as bold type are selected for the Monte-Carlo multi-runs. A detailed description of all biotic and abiotic processes in the CoupModel can be referred to Jansson and Karlberg (2011) or the CoupModel home page (http:/www.coupmodel.com).**

***Snow dynamics***

(1) Snow densification as a function of ice and liquid water content

$$\rho_{oldsnow} = \rho_{newsnow} + s_{dl} \frac{S_{wl}}{S_{wl\max}} + s_{dw} S_{res} \qquad \text{Eqn. S1}$$

$\rho_{oldsnow}$ : *DensityOfOldSnow* – Density of old snow (kg m$^{-3}$)

$\rho_{newsnow}$ : ***DensityOfNewSnow*** – Density of new snow (kg m$^{-3}$)

$s_{dl}$ : ***DensityCoefWater*** – Liquid water coefficient (kg m$^{-3}$)

$S_{wl\max}$ : *WROfSnowMax* – Maximum water retention capacity in the snowpack (m)

$S_{wl}$ : *WROfSnow* – Water retention in the snowpack (m)

$s_{dw}$ : ***DensityCoefMass*** – Mass coefficient (m$^{-1}$)

$S_{res}$ : *SWE* – Snow water equivalent (kg m$^{-2}$)

(2) The upper limit of the aerodynamic resistance at extreme stable conditions

$$r_{aa,snow} = \left(\frac{1}{r_{aa,snow}} + r_{a,\max,snow}{}^{-1}\right)^{-1} \qquad \text{Eqn. S2}$$

$r_{a,\max,snow}{}^{-1}$ : ***WindlessExSnow*** – Minimum turbulent exchange coefficient (inverse of the maximum allowed aerodynamic resistance) over snow (s$^{-1}$)

(3) The aerodynamic resistance at neutral conditions

$$r_{aa,snow} = \frac{1}{k^2 u} \ln\left(\frac{z_{ref} - d}{z_{OM,snow}}\right) \ln\left(\frac{z_{ref} - d}{z_{OH,snow}}\right) f(\mathrm{R_{ib}}) \qquad \text{Eqn. S3}$$

$u$ : *WindSpeed* – Wind speed at the reference height (m s$^{-1}$)

$k$ : The von Karman's constant (-)

$d$ : *DisplacementHeight* – Displacement height (m)

$z_{ref}$ : *ReferenceHeight* – Reference height (m)

$R_{ib}$ : The bulk Richardson number (-)

$z_{OM,snow}$ : ***RoughLMomSnow*** – Roughness length for momentum above snow (m)

$z_{OH,snow}$ : *RoughLHSnow* – Roughness length for heat above snow (m)

(4) Snow albedo

$$a_{snow} = a_{\min} + a_1 e^{a_2 S_{age} + a_3 \sum T_a} \quad \text{Eqn. S4}$$

$a_1, a_2, a_3$ : The parameters for calculating snow albedo (-)

$a_{\min}$ : ***AlbSnowMin*** – The lowest albedo in the albedo function, which accounts for snow age and positive sum of air temperature since latest new snow (-)

***Soil water and soil heat***

(5) The mixed composition of organic and mineral soil at the top layer

$$T_b = \frac{T_1 + aT_s}{1 + a} \quad \text{Eqn. S5}$$

$$a = \frac{k_{ho}(\Delta z_1 / 2 - \Delta z_{humus})}{k_{hm} \Delta z_{humus}} \quad \text{Eqn. S6}$$

$T_b$ : Boundary temperature between organic soil and mineral soil (°C)

$T_s$ : Soil surface temperature (°C)

$T_1$ : Top soil layer temperature (°C)

$k_{ho}$ : Conductivity of the organic soil (K m $W^{-1}$)

$k_{hm}$ : Conductivity of the mineral soil (K m $W^{-1}$)

$\Delta z_{humus}$ : ***OrganicLayerThick*** – Thickness of the organic layer (m)

$\Delta z_1$ : Thickness of the first soil layer (m)

$q_{h,low}$ : ***GeothermalFlow*** – Constant heat flow (J $m^{-2}$ $d^{-1}$)

(6) The vapor pressure at the soil surface

$$e_{corr} = 10^{-\delta_{surf} \psi_{eg}} \quad \text{Eqn. S7}$$

$\psi_{eq}$ : ***EquilAdjustPsi*** – The factor to account for the differences between water tension in the middle of top layer and vapor pressure at soil surface

$\delta_{surf}$ : *SurfmoistureBalance* – The flow variable for mass balance of water at the soil surface (mm)

(7) The correction of thermal conductivity in the upper soil layer due to freezing

$$R_f = e^{c_f T_s} c_{md} + (1 - c_{md}) \quad \text{Eqn. S8}$$

$R_f$ : *CorrFrozen* – The correction factor of thermal conductivity in the upper frozen soil

$c_{md}$ : *CFrozenMaxDamp* – The parameter for frozen surface damping function

$c_f$ : ***CFrozenSurCorr*** – The parameter for frozen surface damping function

(8) Surface water

$$q_{surf} = a_{surf}(W_{\text{pool}} - w_{\text{pmax}}) \quad \text{Eqn. S9}$$

$q_{surf}$ : *Surfrunoff* – Surface runoff (mm)

$a_{surf}$ : ***SurfCoef*** – First order rate coefficient used when calculating the surface runoff from the surface pool exceeding the residual storage.

$W_{pool}$ : ***SurfPool*** – The total amount of water in the surface pool (mm)

$w_{p\max}$ : ***SurfPoolMax*** – The maximal amount of water stored on the soil surface without causing any surface runoff (mm)

(9) Precipitation

$$P = (c_{rain} + Qc_{snow})P_m \qquad \text{Eqn. S10}$$

$c_{rain}$ : ***PreA0Corr*** – Wind correction for rain precipitation

$c_{snow}$ : ***PreA1Corr*** **–** Wind correction for snow precipitation

$P_m$ : *MeasPre* – Measured precipitation (mm)

(10) Altitude of simulated site

$e_{lesim}$ : ***AltSimPosition*** – The parameter used in the function of "air temperature affected by altitude"

(11) The fraction factor to calculate a residual unfrozen amount of water

$$\theta_{lf} = d_1\theta_{wilt} \qquad \text{Eqn. S11}$$

$d_1$ : ***FreezepointFWi*** – The fraction of wilting point ( $\theta_{wilt}$ ) remaining as unfrozen water ( $\theta_{lf}$ ) at -5 °C (-)

***Plan growth***

(12) Light use efficiency approach

$$C_{Atm\to a} = f(T_1)\, f(CN, p_{fixedN})\, f(E_{ta} / E_{tp})\, \mathrm{R}_{\mathrm{s,pl}} \qquad \text{Eqn. S12}$$

$C_{Atm\to a}$ :*CAtmNewMobile* – Carbon assimilation (g C m$^{-2}$ d$^{-1}$)

$p_{fixedN}$ : ***FixNsupply*** – Nitrogen supply capacity for photosynthesis

$\mathrm{R}_{\mathrm{s,pl}}$ : *RadSPl* – Radiation absorbed by canopy (J m$^{-2}$ d$^{-1}$)

$E_{ta}$ : *Eatr* – Actual transpiration (mm)

$E_{tp}$ : *Eptr* – Potential transpiration (mm)

(13) Leaf temperature response

$$f(T_1) = \begin{matrix} 0 & \\ (T_1 - p_{mm})/(p_{o1} - p_{mm}) & p_{mm} \le T_1 \le p_{o1} \\ 1 & p_{mm} \le T_1 \le p_{o1} \\ 1-(T_1 - p_{o2})/(p_{mx} - p_{o2}) & p_{mm} \le T_1 \le p_{o1} \\ 0 & \end{matrix} \qquad \text{Eqn. S13}$$

$p_{mn}$ :***TLMin***– Minimum mean air temperature for photosynthesis (°C)

$P_{o1}$ :***TLOpt1*** – Low limit mean air temperature for optimum photosynthesis (°C)

$p_{o2}$ :***TLOpt2*** – High limit mean air temperature for optimum photosynthesis (°C)

$P_{mx}$ :***TLMax*** – Maximum mean air temperature for photosynthesis (°C)

(14) Litter fall from leaves or roots

$$C_{Leaf\to LitterSurface} = f(\mathrm{l}_{(leaf/root)})\mathrm{C}_{Leaf} \qquad \text{Eqn. S14}$$

$l_{leave}$ : ***LeafRate*** – Rate coefficient for the litter fall from leaves before the first threshold temperature sum is reached.

$l_{root}$ : ***RootRate*** – Rate coefficient for the litter fall from roots before the first threshold temperature sum is reached.

(15) The maintenance respiration of leaves or roots

$$C_{respleaf} = k_{mrespleaf} f(\mathrm{T_a}) \mathrm{C}_{Leaf} + k_{gresp} C_{a \to leaf}$$ Eqn. S15

$k_{mresproot}$ : ***MCoefRoot*** – Maintenance respiration coefficient for roots ($d^{-1}$)

$k_{mresleaf}$ :***MCoefLeave*** – Maintenance respiration coefficient for leaves ($d^{-1}$)

(16) The temperature response function for plant respiration

$$f(\mathrm{T}) = t_{Q10}^{(\mathrm{T} - t_{Q10bas})/10}$$ Eqn. S16

$t_{Q10}$ : ***TempQ10*** – Response to a 10 °C of soil temperature change on microbial activity, mineralization, immobilization, nitrification and denitrification (-)

(17) $r_{Optimum}$ : ***rOptimum*** – Optimum root depth (m)

(18) $N_{leaf}$ : ***INleaf*** – Initial nitrogen mass in leaves (g)

***Soil decomposition***

(19) Decomposition of soil organic matter

$$C_{Decompl} = k_l f(T) f(\theta) C_{Litter}$$ Eqn. S17

$k_l$ : ***RateCoefLitter*** – Rate coefficient for the decay of litter ($d^{-1}$)

$$C_{Decompl} = k_h f(T) f(\theta) C_{Humus}$$ Eqn. S18

$k_h$ :***RateCoefHumus*** - Rate coefficient for the decay of humus ($d^{-1}$)

(20) Soil temperature response for soil decomposition using the Ratkowsky function

$$f(\theta) = 1 \qquad T > t_{\max}$$
$$f(T) = \left( \frac{T - t_{\min}}{t_{\max} - t_{\min}} \right)^2 \qquad t_{\min} < T < t_{\max}$$ Eqn. S19
$$f(\theta) = 0 \qquad T < t_{\min}$$

$t_{\min}$ :***TempMin***- Minimum temperature for microbial activity, mineralization-immobilization, nitrification and denitrification in the Ratkowsky function (°C). Below this temperature, the response function is 0.

$t_{\max}$ :***TempMax*** – Maximum temperature at which the response on microbial activity, mineralization-immobilization, nitrification and denitrification in the Ratkowsky function (°C). Above this temperature, the response function is 1.

(21) Soil moisture response function for soil decomposition

$$f(\theta) = p_{\theta satact} \qquad \theta = \theta_s$$
$$f(\theta) = \min\left( \left( \frac{\theta_s - \theta}{p_{\theta Upp}} \right)^{p_{\theta p}} (1 - p_{\theta satact}) + p_{\theta satact}, \left( \frac{\theta - \theta_{wilt}}{p_{\theta Low}} \right)^{p_{\theta p}} \right) \qquad \theta_{wilt} < \theta < \theta_s$$ Eqn. S20
$$f(\theta) = 0 \qquad \theta < \theta_{wilt}$$

$p_{\theta Low}$ :***ThetaLowerRange*** - Water content interval in the soil moisture response function for microbial activity, mineralization-immobilization, nitrification and denitrification (vol %). The response increases from 0 at the wilting point to optimum at the end of the interval.

$\theta$ : Actual soil moisture content (vol %)

$\theta_{wilt}$ : Soil moisture content at the wilting point (vol %)

$\theta_s$ : Soil moisture at saturation (vol %)

$P_{\theta Upp}$ : The water content interval in the soil moisture response function for microbial activity, mineralization-immobilization, nitrification and denitrification. The response decreases from optimum at the beginning of the interval to saturation activity at saturation.

$P_{\theta Satact}, P_{\theta p}$ : Coefficient in the soil moisture response function (-)

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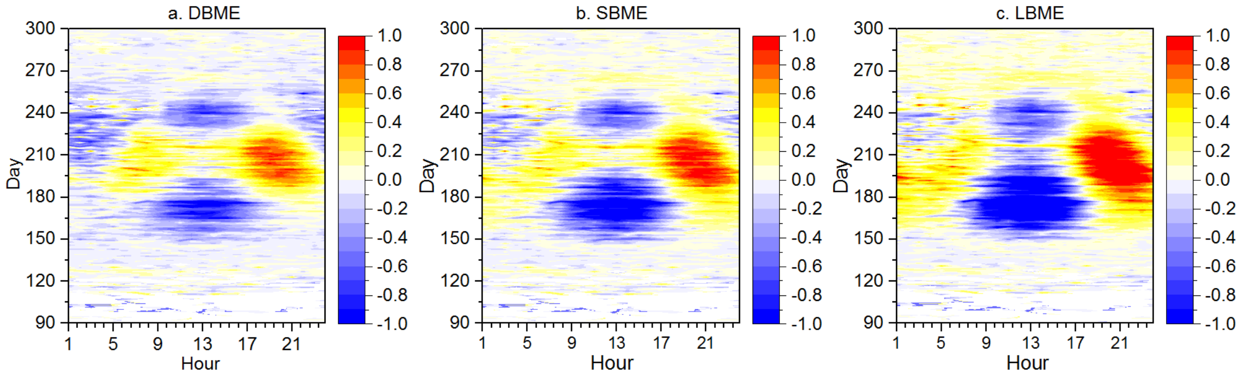


**Figure S1. Hourly residuals (the model minus measurements, g C m$^{-2}$) of the days in three behavior model ensembles. a. DBME, the diurnal behavior model ensemble. b. SBME, the seasonal behavior model ensemble. c. LBME, the long-term behavior model ensemble**.

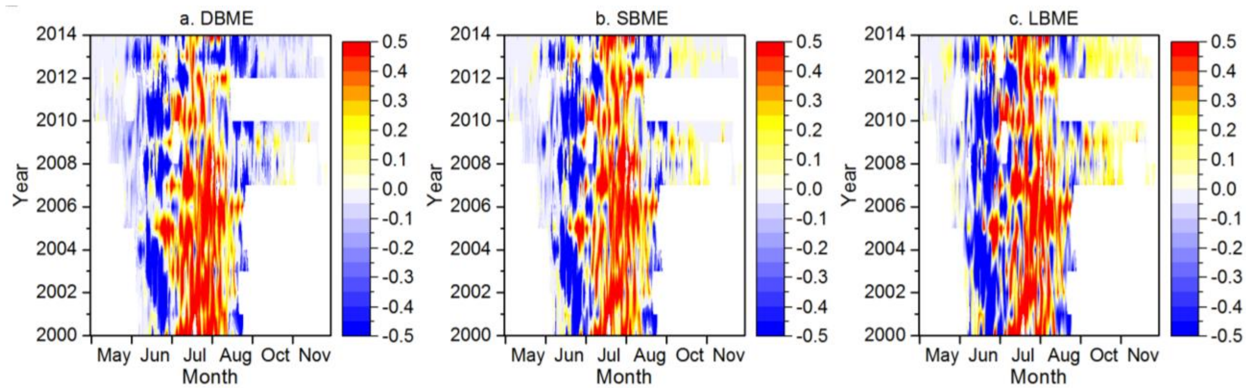


**Figure S2. Daily residuals (the model minus measurements, g C m$^{-2}$) of the years in three behavior model ensembles. a. DBME, the diurnal behavior model ensemble. b. SBME, the seasonal behavior model ensemble. c. LBME, the long-term behavior model ensemble.**

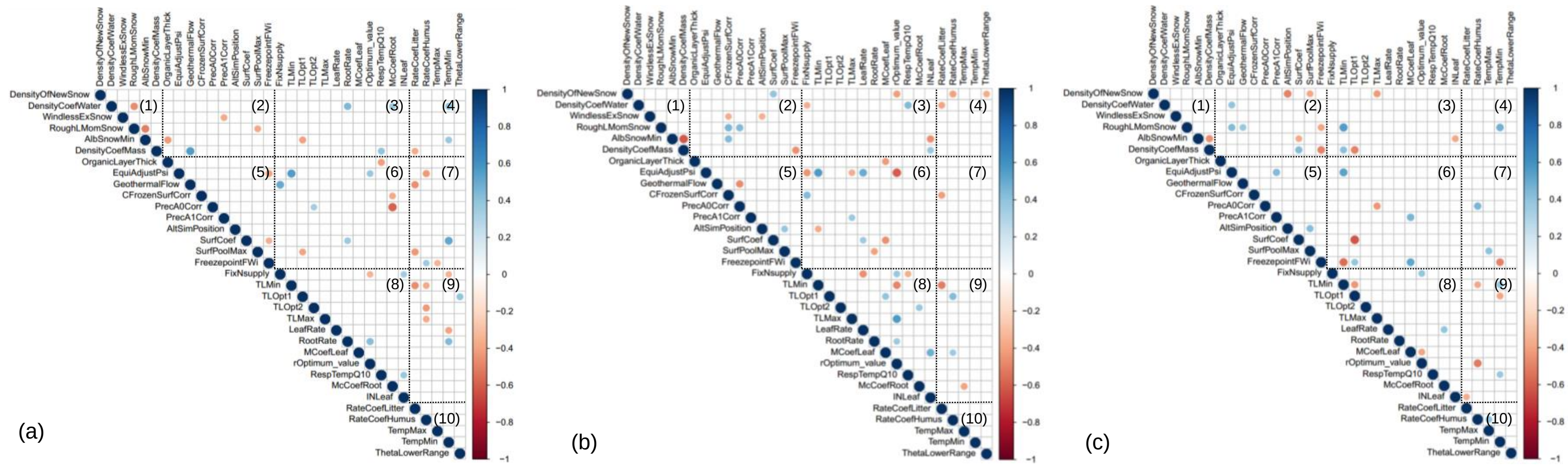


**Figure S3. The inter-correlations (-1, 1) for posterior parameters in the behavior model ensembles. a. DBME; b. SBME; c. LBME. The dots represent the Pearson correlation coefficients with P-value < 0.05. The correlation matrix has been divided into 10 areas according to different ecosystem processes. Snow dynamics: 1, 2, 3 and 4; Soil heat and soil water: 2, 5, 6 and 7; Plant growth: 3, 6, 8 and 9; Soil decomposition: 4, 7, 9 and 10.**

## References


Jansson, P.-E. and Karlberg, L.: Coupled heat and mass transfer model for soil–plant–atmosphere systems, Royal Institute of Technology, Stockholm, 484 pp., available at: http://www.coupmodel.com/documentation, 2010.