SUPPLEMENT

Biogeochemical Model Equation and Parameters

Variables:

NH_4: Ammonium

NO_3: Nitrate

PS: Phytoplankton small (nanophytoplankton)

PL: Phytoplankton large (diatom)

ZS: Zooplankton small (microzooplankton)

ZL: Zooplankton large (mesozooplankton)

DS: Small detritus (slow sinking detritus)

DL: Fast detritus (fast sinking detritus)

DON: Dissolved organic nitrogen

$SiOH_4$: Silicate

Opal: Particulate silica

O_2: Dissolved oxygen

CHL_S: Chlorophyll PS

CHL_L: Chlorophyll PL

I: Photosynthetic Available Radiation

T: Temperature

Processes:

μ_{NO3}: phytoplankton growth fueled by NO_3

μ_{NH4}: phytoplankton growth fueled by NH_4

exud: phytoplankton exudation

$graz_{ps}$: zooplankton grazing upon PS

$graz_{pl}$: zooplankton grazing upon PL

pred: ZL predation upon ZS

excr: zooplankton excretion

egest: zooplankton egestion

mort: mortality

decomp: decomposition of organic nitrogen and opal

nitr: nitrification

$uptake_{Si}$: PL uptake of SiOH4

$Prod_{O2}$: O_2 production

$Cons_{O2}$: O_2 consumption

μ_{CHL}: chlorophyll production

$graz_{CHL}$: chlorophyll loss due to zooplankton grazing

$mort_{CHL}$: chlorophyll loss due to phytoplankton mortality

Dynamic equations

$$\frac{\delta PS}{\delta t} = \mu_{NO3}(PS) + \mu_{NH4}(PS) - graz_{ps}(ZS) - graz_{ps}(ZL) - mort(PS) - exud(PS)$$

$$\frac{\delta PL}{\delta t} = \mu_{NO3}(PL) + \mu_{NH4}(PL) - graz_{pl}(ZS) - graz_{pl}(ZL) - mort(PL) - exud(PL) - w_p \frac{\delta PL}{\delta z}$$

$$\frac{\delta ZS}{\delta t} = graz_{ps}(ZS) + graz_{pl}(ZS) - pred(ZL) - mort(ZS) - excr(ZS) - eges(ZS)$$

$$\frac{\delta ZL}{\delta t} = graz_{ps}(ZL) + graz_{pl}(ZL) + pred(ZL) - mort(ZL) - excr(ZL) - eges(ZL)$$

$$\frac{\delta DS}{\delta t} = mort(PS) + mort(ZS) + egest(ZS) - decomp_{NH4}(DS) - decomp_{DON}(DS) - w_{DS} \frac{\delta DS}{\delta z}$$

$$\frac{\delta DL}{\delta t} = mort(PL) + mort(ZL) + egest(ZL) - decomp_{NH4}(DL) - decomp_{DON}(DL) - w_{DL} \frac{\delta DL}{\delta z}$$

$$\frac{\delta NO_3}{\delta t} = -\mu_{NO3}(PS) - \mu_{NO3}(PL) + nitr$$

$$\frac{\delta NH_4}{\delta t} = -\mu_{NH4}(PS) - \mu_{NH4}(PL) + decomp_{NH4}(DS) + decomp_{NH4}(DL) + decomp_{NH4}(DON)$$

$$\frac{\delta DON}{\delta t} = exud(PS) + exud(PL) + decomp_{DON}(DS) - decomp_{DON}(DL) - decomp_{NH4}(DON)$$

$$\frac{\delta Si(OH)_4}{\delta t} = -uptake_{Si}(PL) + exud_{Si}(PL) + decomp_{Si}(Opal)$$

$$\frac{\delta opal}{\delta t} = mort_{Si}(PL, ZL) + egest_{Si}(ZS, ZL) - decomp_{Si}(Opal) - w_{opal}\frac{\delta opal}{\delta z}$$

$$\frac{\delta O_2}{\delta t} = Prod_{O_2} - Cons_{O_2}$$

$$\frac{\delta CHL_S}{\delta t} = \mu_{CHL_S} - graz_{CHL_S}(ZS, ZL) - mort_{CHL_S}$$

$$\frac{\delta CHL_L}{\delta t} = \mu_{CHL_L} - graz_{CHL_L}(ZS, ZL) - mort_{CHL_L} - w_p\frac{\delta CHL_L}{\delta z}$$

Processes equations

1. Growth Phytoplankton Small

1.1 $\mu_{NO3}(PS) = V_S \cdot \left(\frac{NO_3}{K_{NO3S}+NO_3}\right)\left(\frac{1}{1+NH_4/K_{NH4S}}\right) \cdot PS$

1.2 $\mu_{NH4}(PS) = V_S \cdot \left(\frac{NH_4}{K_{NH4S}+NH_4}\right) \cdot PS$

1.3 $V_S = V_{pS} \cdot f_{PS}(I)$

1.4 $V_{pS} = V_{maxS} \cdot e^{k_{Gpp} \cdot T}$

1.5 $NL_{PS} = \left(\frac{NO_3}{K_{NO3S}+NO_3}\right)\left(\frac{1}{1+NH_4/K_{NH4S}}\right) + \left(\frac{NH_4}{K_{NH4S}+NH_4}\right)$

1.6 $f_{PS}(I) = \frac{\alpha_{PS} I}{\sqrt{(\alpha_{PS} I)^2 + V_{pS}^2}}$

2. Growth Phytoplankton Large

2.1 $\mu_{NO3}(PL) = V_L\left(\frac{NO_3}{K_{NO3L}+NO_3}\right) \cdot \left(\frac{1}{1+NH_4/K_{NH4L}}\right) \cdot min\left\{1, \left(\frac{L_{Si}}{L_N}\right)\right\} \cdot PL$

2.2 $\mu_{NH4}(PL) = V_L\left(\frac{NH_4}{K_{NH4L}+NH_4}\right) \cdot min\left\{1, \left(\frac{L_{Si}}{L_N}\right)\right\} \cdot PL$

2.3 $V_{pL} = V_{maxL} \cdot e^{k_{Gpp} \cdot T}$

2.4 $V_L = V_{pL} \cdot f_{PL}(I)$

2.5 $NLF = \left(\frac{NO_3}{K_{NO3L}+NO_3}\right) \cdot \left(\frac{1}{1+\frac{NH_4}{K_{NH4L}}}\right) + \left(\frac{NH_4}{K_{NH4L}+NH_4}\right)$

2.6 $SLM = \left(\frac{SiOH_4}{K_{Si}+SiOH_4}\right)$

2.7 $NL_{PL} = min\{NLF, SLF\}$

2.8 $$f_{PL}(I) = \frac{\alpha_{PL} I}{\sqrt{(\alpha_{PL} I)^2 + V_{pL}^2}}$$

3. Phytoplankton Exudation

3.1 $$exud(PS) = \varphi_{PS} \cdot \left(\mu_{NH4}(PS) + \mu_{NO3}(PS)\right)$$

3.2 $$exud(PL) = \varphi_{PL} \cdot \left(\mu_{NH4}(PL) + \mu_{NO3}(PL)\right)$$

4. Grazing Zooplankton Small

4.1 $$graz_{PS}(ZS) = GR_{mPZS} \cdot e^{k_{ZMor} \cdot T} \left(\frac{PS^2}{PS^2 + K_{PSZS}}\right) \cdot ZS$$

4.2 $$graz_{PL}(ZL) = GR_{mPLZS} \cdot e^{k_{ZMor} \cdot T} \left(\frac{PL^2}{PL^2 + K_{PLZS}}\right) \cdot ZS$$

5. Grazing-Predation Zooplankton Large

5.1 $$graz_{PS}(ZL) = GR_{mPSZ} \cdot e^{k_{ZMor} \cdot T} \left(\frac{PS^2}{PS^2 + K_{PSZL}}\right) \cdot ZL$$

5.2 $$graz_{PL}(ZL) = GR_{mPLZL} \cdot e^{k_{ZMor} \cdot T} \left(\frac{PL^2}{PL^2 + K_{PLZL}}\right) \cdot ZL$$

5.3 $$pred_{ZS}(ZL) = GR_{mZSZL} \cdot e^{k_{ZMor} \cdot T} \left(\frac{ZS^2}{ZS^2 + K_{ZSZL}}\right) \cdot ZL$$

6. Zooplankton Egestion

6.1 $$egest(ZS) = (1 - \alpha_{ZS}) \cdot \left(graz_{PS}(ZS) + graz_{PL}(ZS)\right)$$

6.2 $$egest(ZL) = (1 - \alpha_{ZL}) \cdot \left(graz_{PS}(ZL) + graz_{PL}(ZL) + pred_{ZS}(ZL)\right)$$

7. Zooplankton Excretion

7.1 $$excr(ZS) = (\alpha_{ZS} - \beta_{ZS}) \cdot \left(graz_{PS}(ZS) + graz_{PL}(ZS)\right)$$

7.2 $$excr(ZL) = (\alpha_{ZS} - \beta_{ZS}) \cdot \left(graz_{PS}(ZL) + graz_{PL}(ZL) + pred_{ZS}(ZL)\right)$$

8. Plankton Mortality

8.1 $$mort(PS) = PMor_S \cdot e^{k_{PMor} \cdot T} \cdot PS$$

8.2 $$mort(PL) = PMor_L \cdot e^{k_{PMor} \cdot T} \cdot PL$$

8.3 $$mort(ZS) = ZMor_S \cdot e^{k_{ZMor} \cdot T} \cdot ZL$$

8.4 $$mort(ZL) = ZMor_L \cdot e^{k_{ZMor} \cdot T} \cdot ZL$$

9. Decomposition/Remineralization

9.1 $$decomp_{NH4}(DS) = \tau_{NH4_S} \cdot e^{k_D \cdot T} \cdot \left(\frac{O_2 - Ox_{th}}{K_{O_2} + (O_2 - Ox_{th})}\right) \cdot DS$$

9.2 $$decomp_{NH4}(DL) = \tau_{NH4_L} \cdot e^{k_D \cdot T} \cdot \left(\frac{O_2 - Ox_{th}}{K_{O_2} + (O_2 - Ox_{th})}\right) \cdot DL$$

9.3 $$decomp_{DON}(DS) = \tau_{DON_S} \cdot e^{k_D \cdot T} \cdot \left(\frac{O_2 - Ox_{th}}{K_{O_2} + (O_2 - Ox_{th})}\right) \cdot DS$$

9.4 $$decom_{DON}(DL) = \tau_{DON_L} \cdot e^{k_D \cdot T} \cdot \left(\frac{O_2 - Ox_{th}}{K_{O_2} + (O_2 - Ox_{th})}\right) \cdot DL$$

9.5 $$decomp_{NH4}(DON) = \gamma_{NH4} \cdot e^{k_D \cdot T} \cdot \left(\frac{O_2 - Ox_{th}}{K_{O_2} + (O_2 - Ox_{th})}\right) \cdot DON$$

9.6 $$nitr = Nit \cdot e^{k_{Nit} \cdot T} \cdot \left(\frac{O_2 - Ox_{th}}{K_{O_2} + (O_2 - Ox_{th})}\right) \cdot \left(1 - \frac{I - I_{th}}{D_p + I - 2 \cdot I_{th}}\right) \cdot NH_4$$

10. Oxygen

10.1 $$Prod_{O_2} = \mu_{NO3} \cdot R_{O_2:NO_3} + \mu_{NH4} \cdot R_{O_2:NH_4}$$

10.2 $$Cons_{O_2} = nitr \cdot R_{O_2:Nitr} + (excr + decomp) \cdot R_{O_2:NH_4}$$

11. Silica

11.1 $$uptake_{Si}(PL) = (\mu_{NH4}(PL) + \mu_{NH4}(PL)) \cdot Si{:}N$$

11.2 $$exud_{Si}(PL) = exud_{DON}(PL) \cdot Si{:}N$$

11.3 $$decom_{Si}(opal) = \tau_{Si} \cdot e^{k_{Si} \cdot T} \cdot opal$$

11.4 $$mort_{Si}(PL, ZL) = \big(mort(PL) + mort(PL)\big) \cdot Si{:}N$$

11.5 $$egest_{Si}(ZS, ZL) = \big((1 - \alpha_{ZS}) \cdot graz_{PL}(ZS) + (1 - \alpha_{ZL}) \cdot graz_{PL}(ZL)\big) \cdot Si{:}N$$

12. Chlorophyll Phytoplankton Small

12.1 $$\mu_{CHL_S} = \big(\mu_{NH4}(PS) + \mu_{NO3}(PS)\big) \cdot \rho_{CHL_S} \cdot CHL_S$$

12.2 $$\rho_{CHL_S} = \frac{\theta_{maxS} \cdot \mu_S \cdot PS}{\alpha_{PS} \cdot I \cdot CHL_S}$$

12.3 $$graz_{CHL_S}(ZS, ZL) = (graz_{PS}(ZS) + graz_{PS}(ZL)) \left(\frac{CHL_S}{PS}\right)$$

12.4 $$mort_{CHL_S} = PMor_s \cdot e^{k_{PMor} \cdot T} \cdot CHL_S$$

13. Chlorophyll Phytoplankton Large

13.1 $$\mu_{CHL_L} = \big(\mu_{NH4}(PL) + \mu_{NO3}(PL)\big) \cdot \rho_{CHL_L} \cdot CHL_L$$

13.2 $$\rho_{CHL_L} = \frac{\theta_{maxL} \cdot \mu_L \cdot PL}{\alpha_{PL} \cdot I \cdot CHL_L}$$

13.3 $graz_{CHL_L}(ZS, ZL) = (graz_{PL}(ZS) + graz_{PL}(ZL)) \left(\frac{CHL_L}{PL}\right)$

13.4 $mort_{CHL_L} = PMor_L \cdot e^{k_{PMor} \cdot T} \cdot CHL_L$

14. Light attenuation

14.1 $I_z = I_0 \cdot e^{Att \cdot z}$

14.2 $Att = Att_{sw} + Att_{PS} \cdot CHL_S + Att_{PL} \cdot CHL_L$

Sediment flux formulation

$$\frac{\delta NH_4}{\delta t} = \left[(w_P PL + w_{DS} DS + w_{DL} DL) \cdot \frac{4}{16\Delta z}\right]_{z=H}$$

$$\frac{\delta O_2}{\delta t} = -\left[(w_P PL + w_{DS} DS + w_{DL} DL) \cdot \frac{115}{16\Delta z}\right]_{z=H}$$

$$\frac{\delta SiOH_4}{\delta t} = \left[\frac{(w_{Si} \cdot opal)}{\Delta z} \cdot 0.9\right]_{z=H}$$