

After selecting a value for ‘ re ’ for a certain grid cell and PFT we first calculated the respiration rate of the surface soil layer (k_0) when all SOC pools are in an equilibrium state, with the following equation:

$$SOC_{orchidee} = \sum_{z=0}^{z=n} \frac{L(z)}{k_0 * e^{-r_e * z}} \quad (1)$$

Where, $SOC_{orchidee}$ is the total equilibrium SOC stock derived from ORCHIDEE for a certain grid cell and PFT. $L(z)$ is the total litter input to the soil for a certain soil layer discretized according to the root profile.

Then we derived the equilibrium SOC stocks per soil layer as:

$$SOC(z) = \frac{L(z)}{k_0 * e^{-r_e * z}} \quad (2)$$

Assuming that the ratios between the active, slow and passive SOC pools do not change with depth and are equal to the ratios derived from ORCHIDEE, we can calculate the SOC stocks of each pool with the following equation:

$$1 + \frac{soil_s(z)}{soil_a(z)} + \frac{soil_p(z)}{soil_a(z)} = \frac{SOC(z)}{soil_a(z)} \quad (3)$$

Where, $soil_a(z)$, $soil_s(z)$, $soil_p(z)$ are the emulator derived active, slow and passive SOC stock per soil layer, grid cell and PFT. Now, for the equilibrium state the input is equal to the output, so we can derive k_{0a} , k_{0s} and k_{0p} from the following equations:

$$\sum_{z=0}^{z=n} \left(\frac{L_a(z) + k_{sa} * soil_s(z) + k_{pa} * soil_p(z)}{k_{0a} * e^{-r_e * z} + k_{as} + k_{ap}} \right) = SOC_a \quad (4a)$$

$$\sum_{z=0}^{z=n} \left(\frac{L_s(z) + k_{as} * soil_a(z)}{k_{0s} * e^{-r_e * z} + k_{sa} + k_{sp}} \right) = SOC_s \quad (4b)$$

$$\sum_{z=0}^{z=n} \left(\frac{k_{sp} * soil_s(z) + k_{ap} * soil_a(z)}{k_{0p} * e^{-r_e * z} + k_{pa}} \right) = SOC_p \quad (4c)$$

Where, L_a is the total litter input to the active SOC pool, L_s is the total litter input to the slow SOC pool. SOC_a , SOC_s , SOC_p are the total active, slow and passive SOC per grid cell and PFT, respectively, derived from ORCHIDEE. k_{as} , k_{ap} , k_{sa} , k_{sp} , k_{pa} are the coefficients determining the fluxes between the SOC pools.

In the transient period (no land use change or erosion) we assume a time-constant ‘ re ’ fixed to the equilibrium state. Using the mass-balance approach we can find the daily values for k_{0a} , k_{0s} , k_{0p} per grid cell and PFT with:

$$\frac{dSOC_a}{dt} = \sum_{z=0}^{z=n} (L_a(z, t) + k_{sa} * soil_s(z, t - 1) + k_{pa} * soil_p(z, t - 1) - (k_{0a}(t) * e^{-re*z} + k_{as} + k_{ap}) * soil_a(z, t - 1)) \quad (8a)$$

$$\frac{dSOC_s}{dt} = \sum_{z=0}^{z=n} (L_s(z, t) + k_{as} * soil_a(z, t - 1) - (k_{0s}(t) * e^{-re*z} + k_{sa} + k_{sp}) * soil_s(z, t - 1)) \quad (8b)$$

$$\frac{dSOC_p}{dt} = \sum_{z=0}^{z=n} (k_{sp} * soil_s(z, t - 1) + k_{ap} * soil_a(z, t - 1) - (k_{0p}(t) * e^{-re*z} + k_{pa}) * soil_p(z, t - 1)) \quad (8c)$$

In case there was no solution for the ‘ k_{0i} ’ at a certain time-step we took the values from the previous time-step.