Decomposition of the change in ecosystem C transit time

Mean C transit time ($\bar{\tau}_R$) can be expressed as the weighted mean age of respired C with fraction of respired C flux from each pool:

$$\bar{\tau}_R(t) = \sum_{i=1}^{d} a_i\,(t) f_{\mathrm{hr},i}(t) \quad \text{(S1)}$$

where a_i is compartment C mean age and the $f_{\mathrm{hr},i}$ is the fractional composition of respired C.

We define the change of a variable (F) is the difference between the variable at t and the variable at t_0:

$$\Delta F(t) = F(t) - F(t_0) \quad \text{(S2)}$$

The change of mean C transit time can be expressed as:

$$\Delta\bar{\tau}_R(t) = \sum_{i=1}^{d} \Delta\left(a_i(t) f_{\mathrm{hr},i}(t)\right)$$

$$= \sum_{i=1}^{d} \left(a_i(t) f_{\mathrm{hr},i}(t)\right) - \left(a_i(t_0) f_{\mathrm{hr},i}(t_0)\right) \quad \text{(S3)}$$

For the i^{th} compartment, the compartment C mean age $a_i(t)$ and fractional composition of respired C $f_{\mathrm{hr},i}$ equal to initial add the change:

$$a_i(t) = a_i(t_0) + \Delta a_i(t) \quad \text{(S4)}$$

$$f_{hr,i}(t) = f_{hr,i}(t_0) + \Delta f_{hr,i}(t) \quad \text{(S5)}$$

Combine equation (S3), (S4), and (S5), the change of mean C transit time is:

$$\Delta\bar{\tau}_R(t) = \sum_{i=1}^{d} \left(\left(a_i(t_0) + \Delta a_i(t)\right)\left(f_{\mathrm{hr},i}(t_0) + \Delta f_{hr,i}(t)\right)\right) - \left(a_i(t_0) f_{\mathrm{hr},i}(t_0)\right)$$

$$= \sum_{i=1}^{d} a_i(t_0)\Delta f_{hr,i}(t) + \sum_{i=1}^{d} \Delta a_i(t) f_{\mathrm{hr},i}(t_0) + \sum_{i=1}^{d} \Delta a_i(t)\Delta f_{hr,i}(t) \quad \text{(S6)}$$

When $\Delta a_i(t) \ll a_i(t_0)$ and $\Delta f_{hr,i}(t) \ll f_{hr,i}(t_0)$, the last term $\Delta a_i(t)\Delta f_{hr,i}(t)$, residual, is in much smaller magnitude than first two terms (See Figure 3). $\sum_{i=1}^{d} a_i(t_0)\Delta f_{hr,i}(t)$ and $\sum_{i=1}^{d} \Delta a_i(t) f_{\mathrm{hr},i}(t_0)$ respecitvely represent C transit time change due to change in composition of respired C and C age structure.