Trend analysis of the airborne fraction and sink rate of anthropogenically released CO_2

Author response, Review 1

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Sections 1 and 2 contain the original comments from referee 1 and referee 2, respectively. Sections 3 and 4 present our responses. Section 5 is a change-log, documenting the changes made to the main paper. At the end of this document, the revised version of the paper is reproduced with the changes indicated.

1 Referee report number 1

Below are the comments from the first referee.

1.1 Comment 1

First, this study comes after almost a decade-long research (Knorr (2009), Gloor et al. (2010), and Ballantyne et al. (2015)) on the detection of the changes in AF or sink efficiency and does not provide new findings (e.g., results are in the line of Raupach 2014). Yet this work merits to be acknowledged because it is the first to my knowledge to investigate this long debate on the stationarity of the AF or SF variations. Here the authors confirm that there is no non-stationnarity in AF and SF using GCP2018 data (from 1959 to 2017). Therefore, I am wondering if it is not the real outcomes of the study ? I mean once the stationarity of the variance is proved, the state space system loses some interest. The potential caveats as suggested by Gloor et al 2010 are removed and thus a simple linear model can be used to estimate trends in AF and SF. Standard statistics can be then used to detect if the signal (the trends) is larger than the noise (the variability).

1.2 Comment 2

The second major comment concerns the attribution of the decreasing sink to the land carbon sink. Regarding the shape of the land C sink, we may be interested to test since how many years the land sink has started to decrease. To further this comment, *I think that several test of the length of the data and the influence of the sampling are missing in the manuscript. We need to see how far this approach is robust when using, for example, 5-year average data (removing ENSO and volcanoes influence).*

1.3 Comment 3

My last major comment relates to the use of the "balanced" C budget whereas Le Quere et al. 2018 provides the Bim terms that could be used as a third entry in you model. I mean does the variance of Bim is steady in time or does it vary ? How far this terms correlates with AF and SF ? Do you fin a trends in Bim that could explain why the sink rate declines whereas the AF does ? I think all these discussions might consolidate the study.

1.4 Specific comment: 1

P1 L4 what do you mean by "balanced carbon budget" ?

1.5 Specific comment: 2

P1 L4 please clarify this sentence. It is unclear to me what object are you talking about

1.6 Specific comment: 3

P1 L6 please explain a bit further because a decrease in the sink should end up ultimately by a change in the AF

1.7 Specific comment: 4

P1 L13 please add the reference period over which this % are estimated + the reference publication

1.8 Specific comment: 5

P1 L18 you could acknowledge more recent studies here

1.9 Specific comment: 6

P2 L5 anthropic = anthropogenic

1.10 Specific comment: 7

P2 L7 you can remove "which we argue is well designed for the problem at hand"

1.11 Specific comment: 8

P3 L12-16 I think paragraph should be move above and better explain why you are working on the "balanced" hypothesis. The Bim remains small compared to the other terms for example ?

1.12 Specific comment: 9

P4 L2 could you further explain the meaning of "Using a simplifying linear specification ?

1.13 Specific comment: 10

P6 L12-15 what about for a lower confidence threshold e.g., 90% do you get a better agreement ? why such a different in Beta estimates (one order of magnitude) ?

1.14 Specific comment: 11

1.14.1 Referee comment

P7 L14 please give the estimate of TtA ? besides I think there is a error in Eq 13 with the random noise epsilon. I read it as independent of time.

1.15 Specific comment: 12

P10 L9-10 the last sentence requires further explanations.

1.16 Specific comment: 13

Figure 3 I don't know what these two panels show. They show the two metrics, correct ? Why giving the confidence interval for 1 sigma whereas most of the statistical test were conducted with a 95% confidence threshold ?

1.17 Specific comment: 14

P12 L15 this looks like trivial. I guess that a simple correlation between the SF and LF should lead to the same conclusion. . .

2 Referee report number 2

Below are the comments from the second referee.

2.1 Comment 1

Firstly in the introduction the authors state: “ a key question is whether the airborne fraction is increasing ” but they do not say why. It would be good if they would add why this is so.

2.2 Comment 2

Sentence just above - 24% and 31% - I would add a reference here - and possibly uncertainties - just for completeness.

2.3 Comment 3

When applying the Kalman filter the authors will need to initialize it. I may have missed it but if not it would be good if the authors would add this in the main text.

2.4 Comment 4

Finally my comment - while all the results are sound - what the paper does not explain is the true reason for the decrease in sink rates - and thus it is not clear whether a decreasing sink rate is alarming or not. It would be nice if the authors could comment on that - but it is not a necessary condition.

Some earlier papers actually give a clue what the real reason may be.

3 Response to referee report number 1

3.1 Response to Comment 1 (Section 1.1)

Thank you for raising these important points. In our approach, stationarity or non-stationarity is a *finding* rather than an *assumption*. We agree with the referee that stationarity of the AF and negative linear trending in the sink rate are the main results, *ex post*. However, *a priori*, we have formulated a statistical dynamic model that allows for both, non-stationary and stationary processes for y_t and for linear as well as stochastic trending behavior. In the paper, below equation (7), we show that the solution to the difference equation (7) leads to a deterministic time trend when the iid Gaussian random variable η_t is zero (effectively when its variance is zero, that is $\sigma_\eta^2 = 0$). In this case, the time series is *trend-stationary* and the dynamic process for y_t does not exhibit a unit-root (which is the case for a *difference-stationary* time series). Since σ_η^2 is estimated using the observations for y_t, we can only conclude *ex post* that the time series is trend-stationary. Without the estimation of our state-space model, we could not have arrived at this conclusion.

Our main findings can be summarized as follows. (a) We find no statistical evidence of an increasing airborne fraction, while we do find statistical evidence for a decreasing sink rate. (b) While the findings (a) have also been reported elsewhere, most notably in Raupach et al. (2014), our statistical model does not make any a priori assumptions regarding the stationarity or non-stationary of the series and regarding deterministic or stochastic behaviour of the trends. These findings are thus *results*, as opposed to assumptions, of our approach. Furthermore, we find that we need to estimate our model on *all* the data from the global carbon budget jointly to reach the findings (a).

In the joint estimation, we take all data into account, that is the time series for AF and SR as defined on page 3 of the paper, but also the additional data obtained by assuming that the carbon budget is balanced, which we explain on page 4 of the paper. (Note that we follow the sink rate definition of Raupach (2013) and Raupach et al. (2014), with concentrations in the denominator, not emissions. The sink rate (SR) in our paper is thus not the complement of the airborne fraction.) More specifically, when analyzing the sink fraction, for example, we can opt for either one or both of the two time series:

$$k_S^{(1)} = \frac{S_t^O + S_t^L}{C_t},$$
$$k_S^{(2)} = \frac{E_t - G_t}{C_t},$$

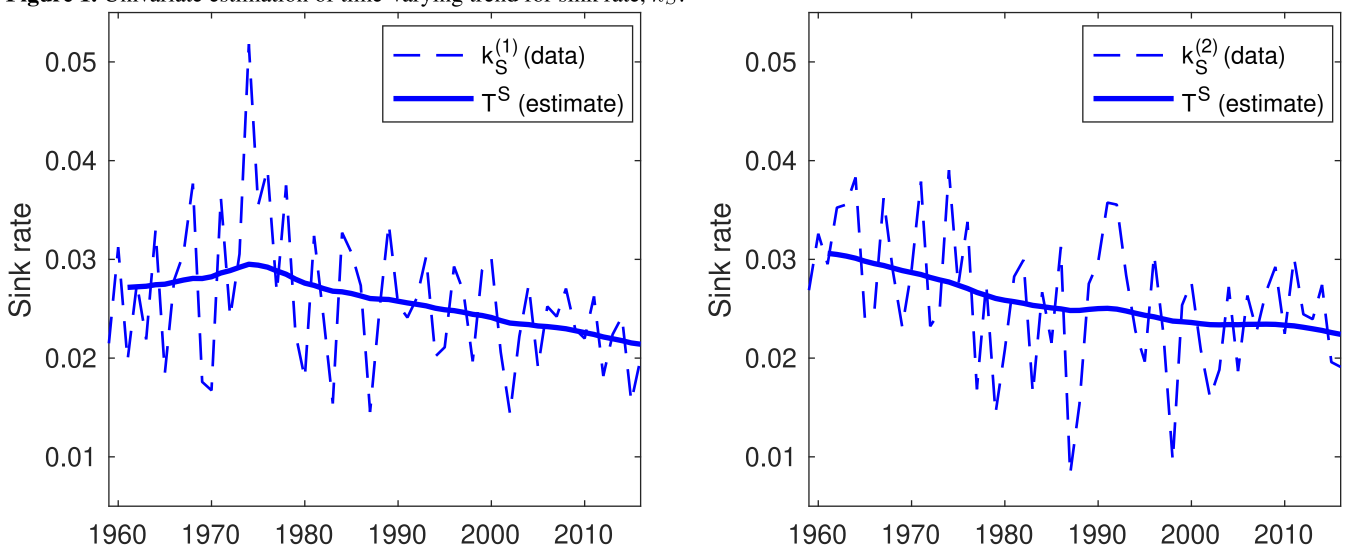
where we exploit the carbon budget equation to obtain $k_S^{(2)}$:

$$E_t = G_t + S_t^O + S_t^L + B_t^{IM}.$$

The budget imbalance B_t^{IM} is a zero-mean noise sequence that represents the measurement errors in the other variables of the carbon budget (Le Quéré et al., 2018). In Table 3 of the main paper we report the results when we estimate our state-space model on $k_S^{(1)}$ and $k_S^{(2)}$ separately. In Table 4 and Figure 2 of the main paper we report the results when we estimate the state-space model on $k_S^{(1)}$ and $k_S^{(2)}$ jointly. It is a feature of the state-space model that it allows for alternative measurements for the same object of interest.

In the Supplemental Material file, we have included Figure 1 that presents the extracted the latent trends, T_t in equation (7), from the separate analysis, and Figure 2 that presents the extracted comment trend T_t from the joint analysis. (This is a replication of Figure 2 in the main paper.) The extraction method is based on the filtering and smoothing approach as discussed in the main paper. The various extracted trends illustrate our points as argued above: in the separate analysis, we obtain (slightly) time-varying stochastic trends, whereas in the joint analysis, we obtain a deterministic linear trend with a significantly negative slope (compare Panel B of Table 4 in the main paper). We have found similar results for the AF series, which we include in the Supplemental Material file.

Figure 1. Univariate estimation of time-varying trend for sink rate, k_S.



3.1.1 Changes made to the paper in response

We have re-written the explanations of the AF and SR definitions in Sect. 2 to show the contribution of the state-space model in the context more crisply.

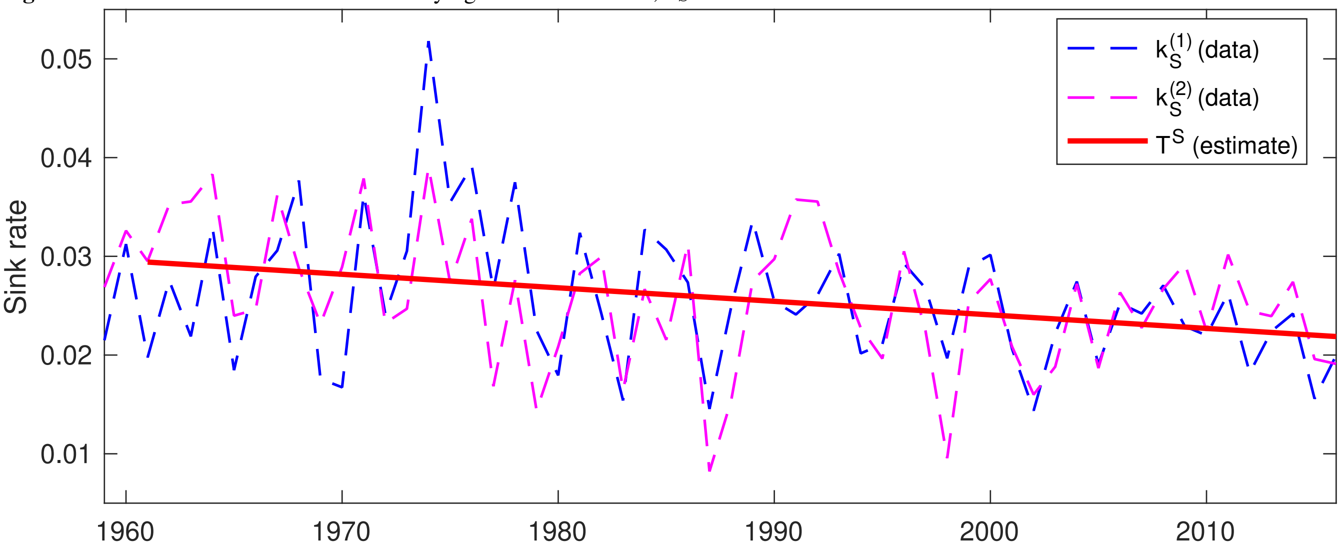
We have included Figure 1 and the corresponding results for the AF in a Supplementary Material file.

3.2 Response to Comment 2 (Section 1.2)

The referee raises two interesting questions; we treat them one-by-one.

(i) With respect to the land sink rate, we are treating it as a fraction: the ratio of flux in land sink over CO_2 concentration in the atmosphere. The land sink flux itself, S_t^L, is increasing over time but the land sink rate, $k_{L,t} = S_t^L/C_t$, shows evidence of a decreasing trend. Our paper shows (Figure 2 in this reply) that if we sum up ocean and land sink and use both time series for this object, $k_S^{(1)}$ and $k_S^{(2)}$, jointly, we obtain a significantly negatively sloping deterministic trend. We can of course also

Figure 2. Multivariate estimation of time-varying trend for sink rate, k_S.



consider $k_S^{(1)}$ and $k_S^{(2)}$ separately. The result is shown in Figure 1 of this reply. From the left panel of this figure, one might argue that the negative trend started in the mid 1970s. The right panel, which shows $k_S^{(2)}$, however, does not display such a kink. Finally, we can consider the land sink rate, k_L, individually. The result is shown in Figure 3 of this reply. We obtain a deterministic trend with an insignificant negative slope, cf. Table 5 of the main paper.

(ii) We have estimated the state-space model on 5-year average data in order to reduce the impact of effects such as ENSO, volcanic eruptions, and the like. The state-space model estimated on annual data is also capable of accounting for these effects, since it treats them as additive noise in the measurement equation. Repeating the analysis based on 5-year average data, however, provides a way to verify our estimation results and conclusions. (We also considered 2-, 3-, and 4-year averages, with similar results.)

We calculate 5-year non-overlapping averages in order to avoid introducing serial correlation into the time series. Running (i.e., overlapping) averages would necessitate specifying a model to capture this serial correlation, and we think this is a relatively bigger disadvantage than the reduction in the sample size that we incur from non-overlapping averages. Since we have 58 years of data, we calculate an average of the first three years followed by 5-year averages, resulting in 12 observations. The findings from estimating the state-space model on these time series of averages confirm those reported in the main paper: In the joint estimation, we find no statistical evidence of a trend in the airborne fraction (with a p-value of 0.32138), and we do find statistical evidence of a decreasing trend in the sink rate (with a p-value of 0.00064). Of course, the residual diagnostics for these short time series are not as convincing as those presented in the main paper. The extracted trends from these joint analyses are presented in Figures 4 (airborne fraction) and 5 (sink rate) in this reply. Incidentally, some analyses in earlier studies were based on running averages; we discuss these briefly in the Discussion section of the main paper (P13 L10).

We emphasize two points in this context: (1) The state-space model is advantageous in this exercise, since it allows to incorporate the alternative time series for both, AF and SR, which is particularly useful when the sample period is short. (2) The main finding from annual data prevails: In the separate analyses, the trends are estimated as stochastic. Only in the joint analysis do we obtain a deterministic trend and statistical significance for the sink rate.

Figure 3. Univariate estimation of time-varying trend for land sink rate, k_L.

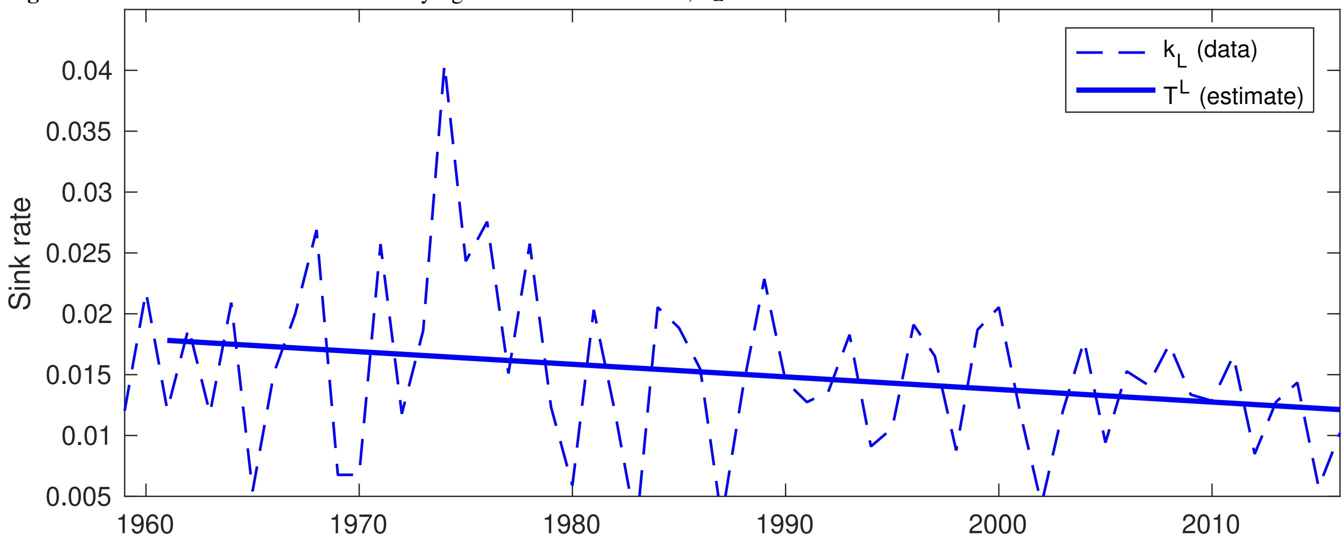


Figure 4. Multivariate estimation of time-varying trend for AF. Data: 5-year averages (no overlap)

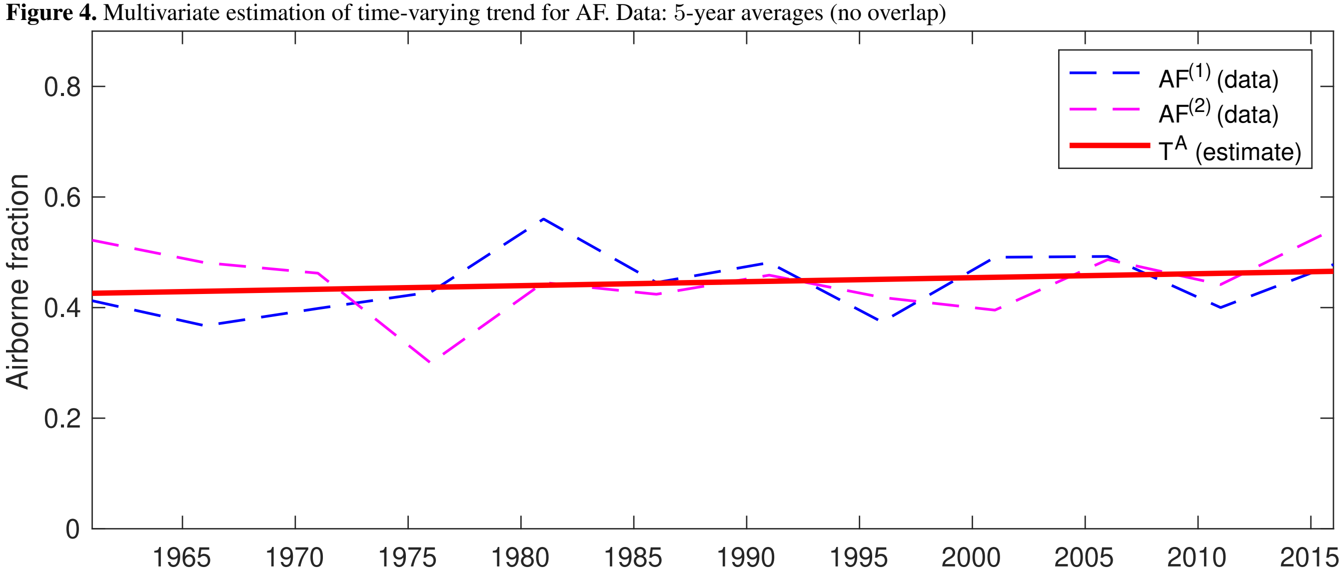
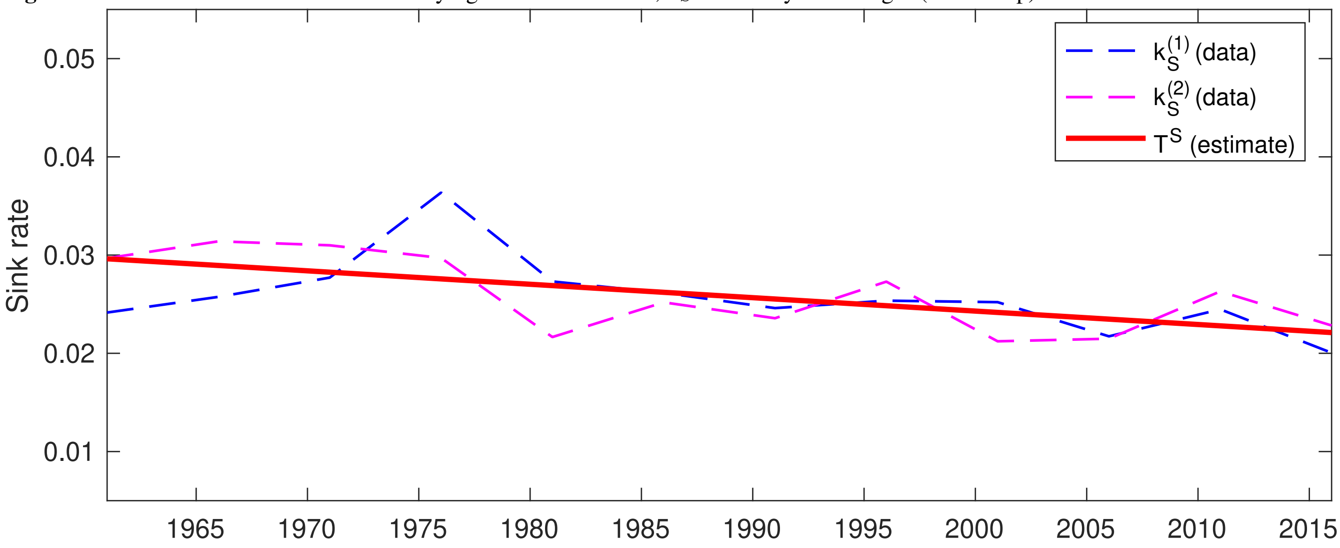


Figure 5. Multivariate estimation of time-varying trend for sink rate, k_S. Data: 5-year averages (no overlap)



3.2.1 Changes made to the paper in response

- (i) We include the separate analysis of the land sink rate (k_L) in the Supplementary Material. For completeness, we also submit the ocean sink rate (k_O) to the same analysis in the Supplementary Material.

- (ii) We include the estimation results based on 5-year averages in the Supplementary Material and briefly discuss these findings in the main paper as well, with a reference to the Supplementary Material for further details (cf. the Discussion Sect.). We emphasize that the findings of the paper are robust to averaging of the data.

3.3 Response to Comment 3 (Section 1.3)

Thank you for raising this point. By assuming that the carbon budget is balanced, we already include the B_t^{IM} data in the analysis. Specifically, the data on the budget imbalance enters as follows. For the case of the sink rate, the two time series employed are:

$$k_S^{(1)} = \frac{S_t^O + S_t^L}{C_t},$$
$$k_S^{(2)} = \frac{E_t - G_t}{C_t}.$$

Given the carbon budget equation, the latter expression can be written as

$$k_S^{(2)} = \frac{E_t - G_t}{C_t} = \frac{S_t^O + S_t^L + B_t^{IM}}{C_t} = k_S^{(1)} + \xi_t,$$

where

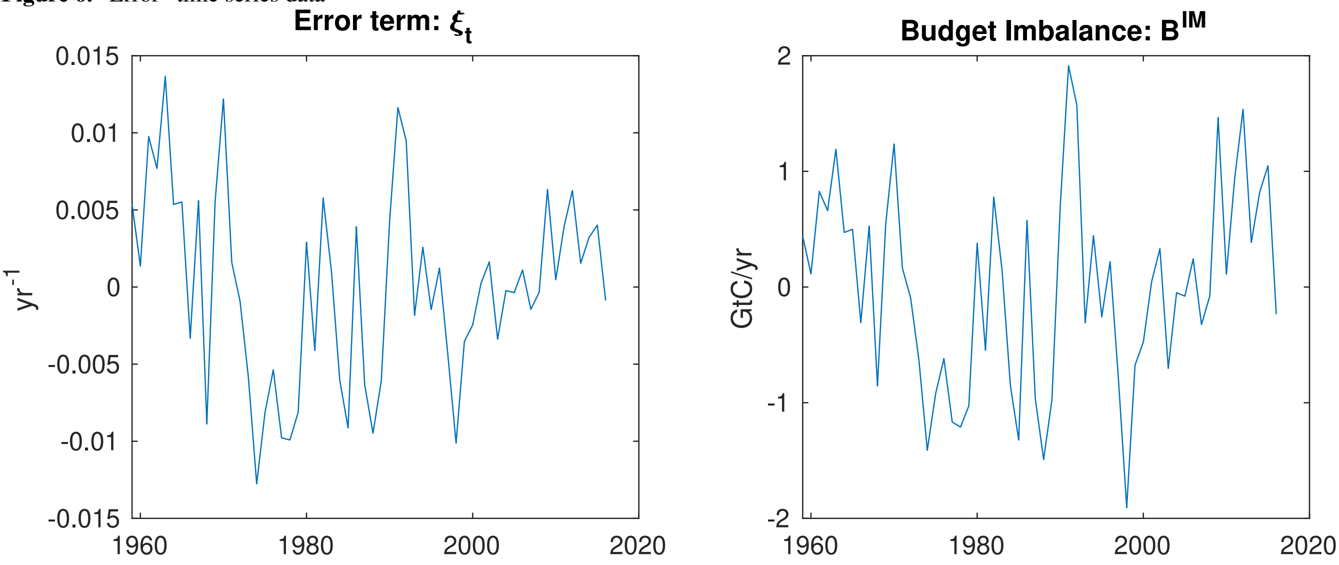
$$\xi_t = \frac{B_t^{IM}}{C_t},$$

can be regarded as an error term. This is the motivation for using the two time series $k_S^{(1)}$ and $k_S^{(2)}$ as data for the same underlying quantity, that is, the sink rate.

Figure 6 plots the time series of ξ_t (left plot) and B_t^{IM} (right plot). Both have a mean that is not significantly different from zero and follow stationary dynamics, albeit with some serial correlation.

In the joint state-space model, both $k_S^{(1)}$ and $k_S^{(2)}$ enter the measurement equation with an error term, and the residual diagnostics reported in the paper show that these error terms are well-behaved to such a degree that the statistical inference reported in the paper is valid.

Figure 6. "Error" time series data



3.3.1 Changes made to the paper in response

We include a discussion that relates the alternative measurements of the sink rate and AF to the budget imbalance, as explained above. When discussing residual diagnostics, we point out the connection with the time series properties of the budget imbalance.

3.4 Response to Specific Comment 1 (Section 1.4)

We mean that the sources of CO_2 should equal the sinks of CO_2, i.e., that the budget equation $E_t = G_t + S_t^L + S_t^O$ should hold (be "balanced") at all times and that any departures from this equation are due to measurement errors in the data. Departures from the equation are captured by the budget imbalance term, B_t^{IM}. Hence, what we mean is that this term is, on average, zero. (This has indeed been the case historically, see Le Quéré et al., 2018).

3.4.1 Changes made to the paper in response

We have rephrased the sentence in the paper to better capture our intended meaning.

3.5 Response to Specific Comment 2 (Section 1.5)

We are specifically referring to the airborne fraction and the sink rate. Also notice that we give an example in parentheses at the end of the sentence: "(for example, the airborne fraction)".

3.5.1 Changes made to the paper in response

We have rephrased the sentence in the paper to clearly identify the object we are referring to.

3.6 Response to Specific Comment 3 (Section 1.6)

As explained in Section 8 of Gloor et al. (2010), it is not necessarily the case that a decrease in the sink rate implies an increase in the airborne fraction. We touch briefly on this in the Discussion section of the initial submission (P13 L 15). See also Raupach (2013).

The main point is that the airborne fraction is defined as $AF_t = G_t/E_t$, while the sink rate is defined as $k_{S,t} = S_t/C_t$. In other words, the normalizations of these time series are different, and they are not complements. Raupach et al. (2014) argue that the latter quantity is more appropriate as an object of study. However, due to the interest in the literature in both the sink rate as well as the airborne fraction, we have analyzed both quantities in the main paper. In the Discussion section we give some further arguments as to why the sink rate may be an easier object to analyze statistically than the airborne fraction (P13 L18).

3.6.1 Changes made to the paper in response

We have rewritten much of Sect. 2 of the paper to make things more clear.

3.7 Response to Specific Comment 4 (Section 1.7)

Thank you. We have done this in the revised version of the paper.

3.8 Response to Specific Comment 5 (Section 1.8)

Thanks for pointing this out, we have done so in the revised version of the paper.

3.9 Response to Specific Comment 6 (Section 1.9)

Thank you. Corrected.

3.10 Response to Specific Comment 7 (Section 1.10)

Thank you. Removed.

3.11 Response to Specific Comment 8 (Section 1.11)

Thanks for pointing this out. We have clarified this in the revised version of the paper.

3.12 Response to Specific Comment 9 (Section 1.12)

In Section 3 of Gloor et al. (2010), the variable $k_{S,t}$ is interpreted as a "sink efficiency". To see why this is, note that we can write (cf. Equation (3) in the main paper)

$$S_t^O + S_t^L = k_{S,t} \cdot C_t.$$

In other words, $k_{S,t}$ is the amount of CO_2 transferred into the sinks, for every unit of CO_2 in the atmosphere above pre-industrial levels (C_t). In this way, k_{S_t} gives an indication of the efficiency with which the carbon system transfers CO_2 to the sinks. See also Raupach (2013) Section 3.1 for a discussion of this "efficiency" interpretation.

3.12.1 Changes made to the paper in response

We have changed the paper to better reflect the above discussion and make our statement clearer.

3.13 Response to Specific Comment 10 (Section 1.13)

In Table 1 of the main paper, we indeed get two different estimates of β, namely 0.00109 and 0.00049. However, we notice that the standard deviations of the estimates are given by 0.00179 and 0.00203, respectively. It indicates that although the estimates are very different (by an order of magnitude, as pointed out by the referee), this difference is not statistically significant. The p-values are 0.5423 and 0.8084 respectively.

The p-values also give an answer to the second question: the estimates are not significant at a 90% level.

3.13.1 Changes made to the paper in response

We have added the p-values to the main paper.

3.14 Response to Specific Comment 11 (Section 1.14)

The estimate of T_t^A is shown in Figure 1 in the main paper (page 8, initial version). The estimates of the accompanying parameters are given in Table 2 (page 7 in the initial version of the main paper).

We have indeed missed the subscript in Equation (13). Thank you for pointing this out.

3.14.1 Changes made to the paper in response

Subscript “t”s have been added in the equations where they were missing.

3.15 Response to Specific Comment 12 (Section 1.15)

Agreed. The forecasts we provide are implied by the model and can be computed within our state space approach. The forecasts for the next 25 years are displayed in Fig. 3 of the main paper and the downward trend is the result of a negative estimate of β as reported in Table 4. Under the current conditions, our forecast implies that it takes more than 25 years before the sink rate is below the value of 0.02.

3.15.1 Changes made to the paper in response

We have added some additional comments on the forecasting exercise.

3.16 Response to Specific Comment 13 (Section 1.16)

Correct and agreed. We now explain this more carefully. We have also given 95% thresholds instead of 68% thresholds.

3.17 Response to Specific Comment 14 (Section 1.17)

The wording of the sentence in the paper is somewhat unclear. What we meant to say is that the variation in the combined sink rate is mostly driven by the variation in the land sink rate. We have rephrased the paragraph to make the point clear.

4 Response to referee report number 2

4.1 Response to Comment 1 (Section 2.1)

We have added some text in the paper explaining the importance and added some references to the literature (e.g., Gloor et al. (2010), Raupach et al. (2014), Bacastow and Keeling (1979)).

4.2 Response to Comment 2 (Section 2.2)

We calculated these numbers from the GCB data. We have included this information, along with a reference where similar numbers can be found.

4.3 Response to Comment 3 (Section 2.3)

We use a diffuse initialisation of the Kalman Filter as outlined in Chapter 5 of Durbin and Koopman (2012). We added a comment on this in the main paper.

4.4 Response to Comment 4 (Section 2.4)

Thank you for pointing this out. Raupach (2013) argue that a necessary condition for a constant sink rate is that emissions (E_t) grow exponentially. Hence, a decreasing sink rate could be the result of less-than-exponential growth in emissions.

Another explanation can be fertilisation/saturation of the sinks. To illustrate this, we focus on the land sink rate, since we find some evidence in the paper for a decreasing land sink rate. Recall that (Equation (5) in the main paper)

$$S_t^L = k_{L,t} \cdot C_t,$$

where $k_{L,t}$ is the land sink rate, S_t^L the land sink CO_2 flux, and C_t the amount of CO_2 in the atmosphere above pre-industrial levels. If the flux of CO_2 to the land sink was linear in C_t, then $k_{L,t}$ would be constant. Conversely, a decreasing $k_{L,t}$ implies that the efficiency with which the land sink absorbs CO_2 is decreasing. That is, the flux of CO_2 to the land sink is non-linear in C_t and this non-linearity is such that the efficiency is decreasing. This is in line with simulation results from climate cycle models (Friedlingstein et al., 2006).

We can illustrate how such non-linearities can arise. The precise relationship between S_t^L and C_t still alludes us but Bacastow and Keeling (1973) (p. 94) suggest that (in our notation):

$$S_t^L \approx \beta \log(1 + C_t/\mathcal{C}^0),$$

wherre $\mathcal{C}^0 = 591.30$ GtC is the amount of CO_2 in the atmosphere in pre-industrial times. Using this, we can deduce

$$\begin{aligned} S_t^L &\approx \beta \log(1 + C_t/\mathcal{C}^0) \\ &\approx \beta \frac{C_t}{\mathcal{C}^0} - \frac{1}{2}\beta \left(\frac{C_t}{\mathcal{C}^0}\right)^2. \end{aligned}$$

Now, if $\mathcal{C}^0$ is large as compared to C_t, this shows how a linear specification between S_t^L and C_t might be reasonable. However, once C_t becomes large as compared to $\mathcal{C}^0$, this shows how the the estimated sink rate can be found to be decreasing. To see this, use the above to write

$$S_t^L \approx k_{L,t} C_t,$$

where

$$k_{L,t} = \frac{\beta}{\mathcal{C}^0} - \frac{1}{2}\frac{\beta}{\mathcal{C}^0}\frac{C_t}{\mathcal{C}^0}$$

is decreasing in C_t. For example, we have $C_{1959} \approx 80$ GtC and $C_{2016} \approx 267$ GtC, resulting in $C_{1959}/\mathcal{C}^0 \approx 14\%$ and $C_{2016}/\mathcal{C}^0 \approx 45\%$.

4.4.1 Changes made to the paper in response

We have changed the paper in several places to better explore these important questions (cf. the change-log below). In particular, we have added the mathematical derivations above to Appendix A.

5 Change-log

Below we list the changes made in the revised version of the paper. This version is reproduced at the end of this document with changes indicated by colour; red for deletions and blue for additions. All page and line numbers refer to this new version reproduced below.

5.1 Changes

The following reported changes are organized in the order of the questions posed by the referees, reproduced above. Each change is given a number, which we refer to as a "Point". For instance, if we want to refer back to the first change described, we would refer to "Point 1".

Points 1–3 report changes of a general nature; Point 4 is in response to a comment from the Associate Editor; Points 5–21 are in response to the comments from referee 1; and Points 22–25 are in response to the comments from referee 2.

1. We have included a Supplementary Material containing additional statistical analyses. We reference this in the Introduction (P2, L34) and the Discussion Sect. 7 (P17, L16).
2. We have added p-values for the different hypothesis tests conducted, see P8, L1 for an example.
3. We have corrected the wording and typos in several places, as highlighted by the tracked changes.
4. We provide a more in-depth discussion of why the topic of this article is an important area of study (P1, first paragraph of introduction). We have also sought to contextualise the paper better: We emphasize more that our approach addresses the methodological criticism of earlier studies (P2, L22-28); we work out better that our findings of deterministic trends are *results*, as opposed to being a priori assumptions as in earlier studies (P15, L13-15); we have developed further our investigation of the apparent decreasing land sink and proposed possible explanations for this (P2, L27; P16, L29; Appendix A, P19). This latter point connects to a large literature on the behavior of the terrestrial land sink. (P17, L1; P18, L10-13; P19, L3-4, P19, L11) This is in response to comments from the Associate Editor.
5. P3-P5: We have cleared up the definitions of the airborne fraction and the sink rate in Sect. 2. This is in response to the comment in Section 1.1.
6. P17, L3: In the Discussion section, we present the results from the analysis using 5-year average data and reference the Supplementary Material, where the details of this analysis can be found. This is in response to the comment in Section 1.2.
7. In response to the comment in Section 1.3, we have changed the following:
   (a) P7, L10, Eq. (8): We have changed the notation slightly, when introducing the two different data series for the AF.
   (b) P10, L14, Eq. (13): We have changed the notation slightly, when introducing the two different data series for the SR.

(c) P8, L13-15: We briefly discuss the statistical properties of the error term $\xi_t = B_t^{IM}/E_t^{ANT}$ which are implied by the diagnostics in Table 1.

(d) P11, L5-7: We briefly discuss the statistical properties of the error term $\xi_t = B_t^{IM}/C_t$ which are implied by the diagnostics in Table 1.

8. P1, L4: We changed "balanced carbon budget" to capture our intended meaning. This is in response to the comment in Section 1.4.

9. P1, L5: We re-phrased this part to make things more clear. This is in response to the comment in Section 1.5.

10. P5, L4: We comment on the differences between the airborne fraction and the sink rate. This is in response to the comment in Section 1.6.

11. P1. L15: We added the period over which the numbers were calculated as well as references. For some formatting reason, this addition can not be read in the version which tracks the changes (it reads fine in the revised version of the paper). The sentence reads: "These percentages are calculated over the period 1959 to 2016 using the data described below, see e.g. Raupach et al. (2014) for similar estimates.". This is in response to the comment in Section 1.7.

12. P2, L6: We added the reference "Rayner et al. (2015)". This is in response to the comment in Section 1.8.

13. P2, L13: anthropic $\rightarrow$ anthropogenic (although this discussion has been changed). In response to the comment in Section 1.9.

14. P2, L16: Removed "which we argue is well designed for the problem at hand". In response to the comment in Section 1.10.

15. P3, L15-17: We discuss the budget imbalance term B_t^{IM} a bit more in-depth, motivating it's later role as part of an error term. The justification for this is strengthened by the diagnostics coming from the state-space analysis, cf. the changes discussed above in points 7c and 7d. This is in response to the comment in Section 1.11.

16. P5, L8: We have deleted the part starting with "Using a simplifying linear specification...". Indeed, the whole of Sect. has been re-worked to make our intended meaning more clear, cf. also point 10 above. This is in response to the comment in Section 1.12.

17. As mentioned above in point 2, we have added p-values to the estimates of the slope parameters. This is partly in response to the comment in Section 1.13.

18. A subscript t has been added in Eq. (13) (which is now Eq. (12) on P9) and others where it was missing. This is response to the comment in Section 1.14.

19. p. 12, L10: We elaborate on the forecasting exercise. This is in response to the comment in Section 1.15.

20. P13, Figure 3. We explained this figure more in-depth in the text, cf. the comment in Point 19. We also substituted the 68% confidence bands for 95% confidence bands in Figure 3 in the main paper. This is in response to the comment in Section 1.16.

21. P14, L20: The wording of last sentence in the last paragraph of Sect. 6 has been changed slightly. This is in response to the comment in Section 1.17.

22. P1 (first paragraph of introduction): We elaborate on why the topic of this paper is important, cf. also Point 4 above. This is in response to the comment in Section 2.1. Cf. also Point 4.

23. P1, L15: We added a comment on how we arrived at those numbers $(45\%, 24\%, 31\%)$ plus a reference. For some formatting reason, this addition can not be read in the version which tracks the changes (it reads fine in the revised version of the paper). The sentence reads: "These percentages are calculated over the period 1959 to 2016 using the data described below, see e.g. Raupach et al. (2014) for similar estimates.". This is in response to the comment in Section 2.2. Cf. also Point 11.

24. P6, L7-8: We have added a comment on the initialization of the Kalman filter. This is in response to the comment in Section 2.3.

25. P16, L29: We have added a paragraph in the Discussion section, where we offer possible explanations for the finding of the decreasing sink rate, see also Appendix A (P19). Similarly, we now briefly comment on this in the Conclusion (P18, L10-13). This is in response to the comment in Section 2.4.

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Trend analysis of the airborne fraction and sink rate of anthropogenically released CO_2

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Abstract. Is the fraction of anthropogenically released CO_2 that remains in the atmosphere (the airborne fraction) increasing? Is the rate at which the ocean and land sinks take up CO_2 from the atmosphere decreasing? We analyze these questions by means of a statistical dynamic multivariate model, from which we estimate the unobserved trend processes together with the parameters that govern them. ~~By assuming a balanced~~ We show how the concept of a global carbon budget ~~, we obtain more than one data series to measure the same object (for example,~~ can be used to obtain two separate data series measuring the same physical object of interest, such as the airborne fraction~~)~~. Incorporating these additional data into the dynamic multivariate model ~~in effect~~ increases the number of available observations, thus improving the reliability of trend and parameter estimates. We find no statistical evidence of an increasing airborne fraction but we do find statistical evidence of a decreasing sink rate. We infer that the efficiency of the sinks ~~to absorb~~ in absorbing CO_2 from the atmosphere is decreasing at approximately 0.54% per year.

1 Introduction

A part of the anthropogenically released CO_2 emitted to the atmosphere flows to the oceans (the ocean sink) and the terrestrial biosphere (the land sink). Approximately 45% of released CO_2 stays in the atmosphere (the airborne fraction), while the two sinks take up approximately 24% and 31% of the CO_2, respectively. (These percentages are calculated over the period 1959 to 2016 using t A key question is whether the airborne fraction is increasing or if it remains constant at around 45%. An increasing airborne fraction implies that the share of anthropogenically released CO_2 that ultimately remains in the atmosphere increases, and projections of future atmospheric CO_2 levels need to take this into account (Gloor et al., 2010). Closely related is the question whether the sinks will continue taking up CO_2 at the same rate (the sink rate) or if this rate is decreasing. A decreasing sink rate implies that the efficiency with which ocean and land sinks are absorbing CO_2 from the atmosphere is decreasing.

Thus, analyzing the behavior of the sink rate can help predict the future uptake of CO_2 through the ocean and the land sink. The answers to ~~these questions~~ the questions posed above are important for our understanding of the global carbon cycle and ~~consequently~~ are relevant for policy makers and the public in general.

A series of papers argue that the airborne fraction of anthropogenically released CO_2 (mainly through fossil fuel emissions, cement production, and land-use change) is increasing ~~(Canadell et al., 2007a; Le Quéré et al., 2009; Raupach et al., 2008)~~ (Canadell et al., 2007a; Le Quéré et al., 2009; Raupach et al., 2008; Rayner et al., 2015). Similarly, in Raupach et al. (2014) it is argued that, although the statistical evidence of an increasing airborne fraction is relatively weak, the evidence of a decreasing CO_2 sink rate is clearer. However, the methods in these studies have been criticized in, for example, Knorr (2009), Gloor et al. (2010), and Ballantyne et al. (2015). Indeed, by considering a longer data set and incorporating uncertainties into the data, Knorr (2009) found that the conclusion of an increasing airborne fraction was not warranted. Similarly, Ballantyne et al. (2015) argues that errors in the data can lead to erroneous conclusions regarding possible trends in the airborne fraction and in the sink rate.

In this paper, we ~~conduct a statistical analysis of the dynamics and interactions of anthropic emissions of CO_2 and its uptake in the atmosphere, the oceans, and the terrestrial biosphere. We study both the airborne fraction and the CO_2 sink rate. The statistical problem is cast in a~~ address these statistical issues within the framework of a ~~*state space*~~ *state-space system*~~*, which we argue is well designed for the problem at hand. The state space framework*~~. It allows us to conduct statistical inference by taking explicit account of stochastic and deterministic trends in the data, transient shocks to the data (coming from, e.g., volcanic eruptions or strong El Niño events), and (potential) measurement errors. ~~The state space system~~ It also allows for the simultaneous incorporation of multiple data sets for the same object, which can improve ~~estimation~~ the estimation of trends and increase reliability of parameter estimates. ~~By assuming a balanced carbon budget (Le Quéré et al., 2018), we obtain more than one data series of the same physical object of interest (e.g., the airborne fraction or the CO_2 sink rate). This seems particularly important in the context of the global carbon budget data considered here, which goes back only to 1959.~~ We find strong evidence for purely deterministic trends when we incorporate multiple measurements for the airborne fraction and the sink rate. These deterministic trends have a statistically significantly negative slope in the case of the sink rate and an insignificant slope in the case of the airborne fraction. These findings corroborate earlier findings in the literature, especially Raupach et al. (2014), but address the statistical concerns raised by Knorr (2009) and Ballantyne et al. (2015), among others. Finally, by analyzing the ocean and land sink rates separately, we find no evidence of a decreasing ocean sink rate but we do find evidence that the land sink rate is decreasing.

The paper is organized as follows. In Sect. 2 we state the fundamental equations of the global carbon budget, the definitions of the airborne fraction of anthropogenically released CO_2, and the CO_2 sink rate, which will motivate the specification of the ~~state space model~~state-space system. Sect. 3 introduces the ~~state space models~~ state-space system used in the paper. In Sect. 4 we conduct a trend analysis of the airborne fraction within the proposed statistical framework. In Sect. 5 we carry out the corresponding analysis of the CO_2 sink rate, and in Sect. 6 of the land and ocean sink rates separately. Sect. 7 discusses the results and Sect. 8 concludes. A Supplementary Material file is available online.

2 The global carbon budget

The so-called *global carbon budget* is defined as

$$E_t^{ANT} = G_t + S_t^O + S_t^L, \tag{1}$$

where E_t^{ANT} is anthropogenically released CO_2 into the atmosphere, G_t is growth of atmospheric CO_2 concentration, S_t^O is the flux of CO_2 from the atmosphere to the oceans (the ocean sink), and S_t^L is the flux of CO_2 from the atmosphere to the terrestrial biosphere (the land sink). In words, Eq. (1) states that emissions of CO_2 should equal the fluxes of CO_2 to the atmosphere, the ocean sink, and the land sink. We use the data set provided by The Global Carbon Project (Le Quéré et al., 2018).[1] All data are measured in gigatonnes of carbon (GtC) and are recorded at a yearly frequency, beginning in 1959 and ending in 2016, resulting in 58 observations for each quantity in (1).

While the carbon budget is in principle always balanced for the physical quantities, in the sense that Eq. (1) always holds, this might not be the case when inserting actual data for emissions and sinks, due to measurement errors in the data. For this reason, Le Quéré et al. (2018) introduce a residual term into the budget Eq. (1) to capture measurement error. It is denoted B_t^{IM} for *budget imbalance*. Therefore, when considering actual data, the carbon budget is defined as

$$E_t^{ANT} = G_t + S_t^O + S_t^L + B_t^{IM}. \tag{2}$$

The sample mean of the budget imbalance over the observation period is not significantly different from zero and shows no sign of a trend (Le Quéré et al., 2018). These facts are important in the developments below, since they motivate treating B_t^{IM} as part of an error term.

The growth rate in atmospheric CO_2 data, G_t, is ~~thus~~ from Dlugokencky and Tans (2018), while the sink data, S_t^O and S_t^L, are averages over several independent model-based estimates, constructed as explained in Le Quéré et al. (2018). ~~All data are given in gigatonnes of carbon (GtC) and are recorded at a yearly frequency, beginning in 1959 and ending in 2016, resulting in 58 observations for each quantity in .~~

The anthropogenic emissions of CO_2 ~~are defined as~~ can be decomposed in two parts:

$$E_t^{ANT} = E_t^{FF} + E_t^{LUC},$$

~~based~~ where E_t^{FF} ~~is~~ are emissions from fossil fuel burning, cement production, and gas flaring, while E_t^{LUC} ~~is~~ are emissions from land-use change. ~~The former data~~Fossil fuel emissions, E_t^{FF}, are from Boden et al. (2018), while ~~the latter data~~land-use change emissions, E_t^{LUC}, are averages over the model-based estimates of Hansis et al. (2015) and Houghton and Nassikas (2017), updated as in Le Quéré et al. (2018). The time series of concentrations (above preindustrial levels) of CO_2 in the atmosphere is constructed as

$$C_t = 2.127 \cdot ([CO_2]_{1959} - [CO_2]_{1750}) + \sum_{\tau=1}^{t} G_\tau,$$

[1] The data are available at http://www.globalcarbonproject.org/ and were downloaded on June 1st, 2018.

where $[CO_2]_{1750} = 279$ ppmv (parts per million volume) and $[CO_2]_{1959} = 315.39$ ppmv are the concentrations of CO_2 in the atmosphere in 1750 and 1959, respectively; see Raupach et al. (2014). The number 2.127 is the conversion factor from ppmv to GtC.

In words, ~~Eq. states that emissions of CO~~the atmospheric concentration C_t above pre-industrial levels is given by the initial value in 1959 plus the cumulative sum of the growth in atmospheric CO_2 ~~should equal the fluxes of CO_2 to the atmosphere, the ocean sink, and the land sink. The term G_t is a growth rate per unit time, and sometimes it is written in the continuous time version as~~

$$G_t = \frac{dC_t}{dt}.$$

~~While the carbon budget is in principle always balanced, in the sense that Eq.~~ concentrations G_t, which result from the budget equation (1)~~always holds, this might not be the case when inserting actual data for emissions and sinks, due to measurement errors in the data. The residual term is referred to as the *budget imbalance* by Le Quéré et al. (2018) and is denoted by B_t^{IM}. Therefore, when considering actual data, the carbon budget is defined as~~

$$E_t^{ANT} = G_t + S_t^O + S_t^L + B_t^{IM}.$$

.

We follow Raupach (2013) and Raupach et al. (2014) and define the airborne fraction as

$$AF_t = \frac{G_t}{E_t^{ANT}}$$

and the ~~sink fraction~~

$$SF_t = \frac{S_t^O + S_t^L}{E_t^{ANT}} = 1 - AF_t,$$

~~where the second equality assumes that B_t^{IM} is equal to zero. These fractions are for the anthropogenically released~~ CO_2 sink rate as

$$k_{S,t} = \frac{S_t^O + S_t^L}{C_t}, \qquad (3)$$

which is the flux of CO_2 ~~that stays in the atmosphere (AF_t) and in the combined sink of~~ from the atmosphere to the sinks (ocean plus land~~(SF_t) . One~~), normalized by the amount of CO_2 (above preindustrial levels) currently in the atmosphere. We can also consider the ~~ocean and land sinks separately and define the~~ individual components of the sink rate for ocean and land~~fractions as~~

$$OF_t = \frac{S_t^O}{E_t^{ANT}}, \qquad LF_t = \frac{S_t^L}{E_t^{ANT}},$$

, which are given by

$$k_{O,t} = \frac{S_t^O}{C_t}, \qquad k_{L,t} = \frac{S_t^L}{C_t}, \qquad (4)$$

respectively, with ~~$SF_t = OF_t + LF_t$~~ $k_{S,t} = k_{O,t} + k_{L,t}$.

~~Following Raupach (2013) and Raupach et al. (2014), we further consider the CO_2 sink rate which is defined at time t by~~

$$k_{S,t} = \frac{S_t^O + S_t^L}{C_t},$$

~~which is~~ The airborne fraction and the ~~flux of~~ sink rate are fundamentally different quantities. The airborne fraction $AF_t = G_t / E_t^{ANT}$ is the ratio of the growth of atmospheric CO_2 in period t to the amount of CO_2 emitted in period t. It is thus a measure of the fraction of emitted CO_2 ~~from the atmosphereinto the sinks (ocean plus land), normalized by the~~ that stays in the atmosphere. In contrast, the sink rate $k_{S,t} = (S_t^O + S_t^L)/C_t$ is the ratio of the CO_2 flux in the sinks in period t to the total amount of CO_2 ~~(above preindustrial levels)currently in the atmosphere. Using a simplifying linear specification, Gloor et al. (2010) interprets the variable $k_{S,t}$ as a “sink efficiency”~~ in the atmosphere (above pre-industrial levels).

~~From the global carbon budget , it follows that the sink efficiency~~ By writing $S_t^O + S_t^L = k_{S,t} C_t$, we can interpret the sink rate $k_{S,t}$ ~~can alternatively be written as~~

$$k_{S,t} = \frac{E_t^{ANT} - G_t}{C_t}.$$

~~We can also consider the individual components of~~ as the “efficiency”, with which CO_2 flows from the atmosphere to the sinks, i.e. as the ~~sink rate for ocean and land which are given by~~

$$k_{O,t} = \frac{S_t^O}{C_t}, \qquad k_{L,t} = \frac{S_t^L}{C_t},$$

~~respectively, with $k_{S,t} = k_{O,t} + k_{L,t}$.~~ amount of CO_2 going into the sinks for an extra unit of CO_2 added to the atmosphere (Gloor et al., 2010; Raupach, 2013). We discuss the relationship between the airborne fraction and the sink rate further in Sect. 7.

3 Trend model specification

In this section, we consider several models for the data generating process behind observations of the objects of interest defined in Sect. 2. Common to all models is that they can be cast in a ~~state space~~ state-space system of the form:

$$\begin{aligned} y_t &= A x_t + \xi_t, \\ x_{t+1} &= B x_t + \kappa_t, \end{aligned} \qquad t = 1, \ldots, n, \tag{5}$$

where y_t is a vector of observations at time $t = 1, \ldots, n$ with time series length n, and the system matrices A and B have appropriate dimensions~~, the~~. The vector x_t is usually referred to as the state vector, which can include deterministic and stochastic trends, and the error terms ξ_t and κ_t are both independent and identically distributed (iid) random vectors of appropriate dimension and with mean zero. For example, when we need to model the airborne fraction alone, we have $y_t = AF_t$ and the ~~state space~~ state-space system represents a univariate dynamic model for the airborne fraction. When modelling the ocean and land

~~fractions~~ sink rates jointly, we have ~~$y_t = (OF_t, LF_t)'$ and the state space system is for~~ $y_t = (k_{O,t}, k_{L,t})'$, and the state-space system is a bivariate dynamic model. For given matrices A and B, and under the assumption of mutually and serially uncorrelated Gaussian errors ξ_t and κ_t (with their respective variance matrices Σ_ξ and Σ_κ), the ~~state space~~ state-space system is a linear Gaussian model. In such regular cases, an analytic formulation for the likelihood function is ~~avaiable~~ available and relies on the prediction error decomposition. Hence the parameters (variances and possibly covariances in Σ_ξ and Σ_κ) can be estimated by the maximum likelihood method. It requires the numerical optimization of the log-likelihood function that is evaluated via the Kalman filter. The resulting algorithm is initialized with specific starting values; we use a diffuse initialization as outlined in Chapter 5 of Durbin and Koopman (2012). The smooth estimate of the state process x_t can also be obtained by means of the Kalman filter together with a smoothing algorithm. The extracted state is effectively the conditional mean $\mathbb{E}(x_t|y_1,\ldots,y_n; A, B, \Sigma_\xi, \Sigma_\kappa)$, for $t = 1,\ldots,n$. Details of the ~~state space approach to time series modeling, including the statistical treatment of the initial state x_1,~~ state-space approach are given by Durbin and Koopman (2012), where both signal extraction and maximum likelihood estimation are discussed.

Our baseline model is the local linear trend (LLT) model~~with a fixed and unknown growth (or slope) coefficient.~~[2] . For a univariate time series y_t, we treat the underlying trend T_t as a stochastic process given by

$$T_{t+1} = T_t + \beta + \eta_t, \tag{6}$$

where $\beta \in \mathbb{R}$ is a fixed and unknown coefficient and η_t is an iid Gaussian random variable with mean zero and variance σ^2_η. The solution to the difference equation (6) is given as

$$T_{t+1} = T_1 + t\beta + \sum_{i=0}^{t-1} \eta_{t-i}, \qquad t = 1,2,\ldots,n-1,$$

where T_1 can be treated as a fixed unknown coefficient (intercept or constant) or as a random variable. ~~It~~ The solution shows that the trend component is made up of the starting value T_1, a deterministic linear term with slope β, and ~~the~~ a random walk component $\sum_{i=0}^{t-1} \eta_{t-i}$. ~~In this way~~Thus, T_t can be interpreted as a long-term trend in the time series and β as the slope of the deterministic part of the trend. We also considered a time-varying slope, β_t, but found no evidence supporting this generalization in either the airborne fraction or the sink rate. The observation equation for y_t is given by

$$y_t = T_t + \epsilon_t, \tag{7}$$

where T_t is given by (6) and ϵ_t captures deviations of the observed time series from the unobserved trend component. The deviations ϵ_t can be viewed as (i) actual (transient) disturbances of the physical systems arising from, for example, volcanic eruptions and El Niño events, and/or (ii) measurement errors arising from the way the data are collected. [2] The random variable ϵ_t is assumed to be iid Gaussian with mean zero and variance σ^2_ϵ.

[2] ~~We also considered a time-varying slope but found no evidence supporting this generalization in either the airborne fraction or the sink rate.~~

[2] ~~See Ballantyne et al. (2015) for the importance of accounting for measurement errors in the data.~~

The local linear trend model can be cast in the ~~state space~~ state-space system (5) where vectors and matrices are defined as

$$x_t = \begin{pmatrix} T_t \\ \beta \end{pmatrix}, \quad A = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad \xi_t = \epsilon_t, \quad \kappa_t = \begin{pmatrix} \eta_t \\ 0 \end{pmatrix},$$

for $t = 1, \ldots, n$. The state vector x_t consists of the two variables of interest: stochastic trend variable T_t and deterministic slope variable β. The ~~state space~~ state-space methods as discussed above can treat such mixed compositions of the state vector. We have illustrated how the ~~state space~~ state-space system can be used for a univariate time series. In the next sections, we also consider trend ~~analyes~~ analyses based on multivariate time series models.

4 Trend analysis of the airborne fraction

~~When we assume that the carbon budget is balanced, see the discussion in Sect. 2, for all time periods $t = 1, \ldots, n$,~~ It follows immediately from Eq. (2) that we can measure the airborne fraction AF_t in two alternative ways:

$$AF_t^{(1)} = \frac{G_t^{ATM}}{E_t^{ANT}}, \qquad AF_t^{(2)} = \cancel{1 - SF_t} \frac{E_t^{ANT} - S_t^O - S_t^L}{E_t^{ANT}} = AF_t^{(1)} + \xi_t, \cancel{1 - \frac{S_t^O + S_t^L}{E_t^{ANT}}}. \tag{8}$$

where $\xi_t = B_t^{IM}/E_t^{ANT}$, since $E_t^{ANT} - S_t^O - S_t^L = G_t + B_t^{IM}$. Although the two quantities in (8) measure the same underlying object (the airborne fraction AF_t), they ~~may differ when we consider the actual data; see also Eq.~~ differ in practice, because of a non-zero budget imbalance, i.e. $\xi_t \neq 0$. Our statistical analysis implies that ξ_t is a well-behaved zero-mean and covariance stationary error process.

We consider our baseline local linear trend model of Sect. 3 for each of the objects, that is,

$$y_t = AF_t^{(i)} = T_t^{(i)} + \epsilon_t^{(i)},$$

for $i = 1, 2$, where the trend $T_t^{(i)}$ is specified in (6) and with error $\epsilon_t^{(i)}$. Table 1 reports the output of the estimation, using the ~~state space~~ state-space system and the Kalman filter. The first part of Table 1 presents estimates of the standard deviations of the ~~error terms ϵ and η~~ observation error term $\epsilon_t^{(i)}$ and the trend error term $\eta_t^{(i)}$, as well as the estimate of the slope parameter β, including the estimated standard ~~deviation~~ error (s.e.) of $\hat{\beta}$ and the resulting t-statistic, ~~t-stat $= \frac{\hat{\beta}}{\text{s.d.}(\hat{\beta})}$~~ t-stat $= \hat{\beta}\,/\,\text{s.e.}(\hat{\beta})$. Based on these estimation results, we can formally test hypotheses of the type

$$H_0 : \beta = 0 \quad \text{against} \quad H_1 : \beta \neq 0, \tag{9}$$

or, more relevantly,

$$H_0 : \beta = 0 \quad \text{against} \quad H_1 : \beta > 0. \tag{10}$$

By using the normal approximation to the t-distribution and for a 95% confidence level, the critical value for the test (9) is 1.96, and for (10) it is 1.645. In case of the airborne fraction, we are interested in testing (10). It is evident from Table 1 that we cannot reject H_0 in this case (p-values 0.2711 and 0.4042, respectively). In other words, although the estimate $\widehat{\beta}$ is positive, we cannot conclude, statistically at 95% confidence, that the airborne fraction is increasing over time.

Table 1 also contains diagnostic statistics for the standardized prediction residual u_t based on ~~$y_t - \mathbb{E}(y_t|y_1,\ldots,y_{t-1};A,B,\Sigma_\xi,\Sigma_\kappa)$,~~

$$y_t - \mathbb{E}(y_t|y_1,\ldots,y_{t-1};A,B,\Sigma_\xi,\Sigma_\kappa),$$

for $t = 1,\ldots,n$, and where Σ_ξ and Σ_κ are replaced by their respective maximum likelihood estimates. Under the assumption that the local linear trend model is correctly specified for the time series y_t, the residuals u_t are Gaussian iid; see ~~(Durbin and Koopman, 2012, p.38).~~ Durbin and Koopman (2012), p. 38. To verify these properties of u_t empirically, we consider two residual diagnostic statistics: the normality test statistic N of Jarque and Bera (1987) and the serial correlation test statistic DW of Durbin and Watson (1971). As a goodness-of-fit statistic, we consider the R_d^2 which is a relative measure of model fit against a random walk model. The statistic is defined in a similar way as the standard regression fit measure R^2, we have

$$R_d^2 = 1 - \frac{\sum_{t=2}^{n} u_t^2}{\sum_{t=2}^{n}[(y_t - y_{t-1}) - m]^2}, \qquad m = (n-1)^{-1}\sum_{t=2}^{n}(y_t - y_{t-1}).$$

The reported diagnostic statistics and goodness-of-fit in Table 1 are satisfactory for the time series $AF_t^{(1)}$ and $AF_t^{(2)}$. We may conclude from these results that the local linear trend model (6)-(7) provides an adequate description of the dynamic features in the time series. Since the $AF_t^{(2)}$ is well-described within our state-space framework, the extra error term $\xi_t = B_t^{IM}/E_t^{ANT}$ in $AF_t^{(2)}$, as introduced by the budget imbalance term in Eq. (8), is well-behaved. Hence the assumptions underlying the state-space system appear to be valid.

Table 1. Univariate analysis of the airborne fraction

	Parameter estimates					Diagnostics		
	$\widehat{\sigma}_\epsilon$	$\widehat{\sigma}_\eta$	$\widehat{\beta}$	s.e.($\widehat{\beta}$)	t-stat($\widehat{\beta}$)	N	R_d^2	DW
$AF_t^{(1)}$	0.1357	0.0101	0.00109	0.00179	0.60934	0.274	0.442	1.829
$AF_t^{(2)}$	0.1353	0.0122	0.00049	0.00203	0.24246	2.324	0.489	~~1.9905~~1.991

We report parameter estimates for the standard deviations σ_ϵ and σ_η, and slope coefficient β together with its standard error (s.e.) and t-statistic (t-stat). We further report the normality (N) test, the goodness-of-fit statistic R_D^2 and the Durbin-Watson (DW) test statistic for serial correlation; all computed for the standardized prediction errors u_t which are obtained from the Kalman filter. The normality test N is the χ^2 distributed, with 2 degrees of freedom, statistic of Jarque and Bera (1987) with its 95% critical value of 5.99; the statistic relies on the sample estimates of skewness and kurtosis of u_t. The goodness-of-fit statistic R_d^2 is defined as $1 - ESS/DSS$ where $ESS = \sum_{t=2}^{n} u_t^2$ and $DSS = \sum_{t=2}^{n}[(y_t - y_{t-1}) - m]^2$ with $m = (n-2)^{-1}\sum_{t=2}^{n}(y_t - y_{t-1})$. The Durbin-Watson DW test statistic is developed by Durbin and Watson (1971), where also its critical values are tabulated. If $DW = 2$ the sequence u_t is serially uncorrelated; if $DW < 2$ there is evidence that the errors u_t are positively autocorrelated; if $DW > 2$ there is evidence that the errors u_t are negatively autocorrelated.

The ~~state space~~ state-space system allows both measures for the airborne fraction, $AF_t^{(1)}$ and $AF_t^{(2)}$, to be included in a single model with the purpose to improve the quality of the trend estimation and inference. We begin with an "uninformed" system using two different trend components, $T_t^{(1)}$ and $T_t^{(2)}$, both specified as (6), for the two time series~~, we have~~. We have

$$y_t = \begin{bmatrix} AF_t^{(1)} \\ AF_t^{(2)} \end{bmatrix} = \begin{bmatrix} G_t^{ATM}/E_t^{ANT} \\ 1-(S_t^{OCEAN}+S_t^{LAND})/E_t^{ANT} \end{bmatrix} = \begin{bmatrix} T_t^{(1)} \\ T_t^{(2)} \end{bmatrix} + \begin{bmatrix} \epsilon_t^{(1)} \\ \epsilon_t^{(2)} \end{bmatrix}, \tag{11}$$

where the error terms $\epsilon_t^{(i)}$, for $i = 1, 2$, are correlated and ~~its~~ their correlation coefficient can be estimated by the method of maximum likelihood together with the other parameters. The estimation results for this model are presented in Panel A of Table 2. The main difference to Table 1 is the inclusion of the estimated correlation matrix for $(\epsilon_t^{(1)}, \epsilon_t^{(2)})$. The diagnostic test statistics are reasonable. In comparison with the univariate analysis, the goodness-of-fit values for R_d^2 are slightly higher for the multivariate model. Hence we trust the model to be a good representation of the data. Furthermore, the slope is estimated to be positive in both cases (that is $\hat{\beta} > 0$)~~, indicating an increasing airborne fraction~~. However, when testing the null hypothesis given in (10), we cannot reject the hypothesis that the slopes are zero (p-values 0.3753 and 0.4895, respectively).

Table 2. Multivariate analysis of the airborne fraction

	Parameter estimates					Correlation matrix (ϵ)		Diagnostics		
Panel A: Two individual trends as in Eq. (11).										
	$\widehat{\sigma}_\epsilon$	$\widehat{\sigma}_\eta$	$\widehat{\beta}$	s.e.$(\widehat{\beta})$	t-stat$(\widehat{\beta})$	$AF^{(1)}$	$AF^{(2)}$	N	R_d^2	DW
$AF^{(1)}$	0.1268	0.0333	0.00146	0.00459	0.31797	1.0000	0.7612	0.603	0.484	2.0152
$AF^{(2)}$	0.1307	0.0274	0.00010	0.00383	0.02629	0.7612	1.0000	1.469	0.525	2.0853
Panel B: One common trend as in Eq. (12).										
	$\widehat{\sigma}_\epsilon$	$\widehat{\sigma}_\eta$	$\widehat{\beta}$	s.e.$(\widehat{\beta})$	t-stat$(\widehat{\beta})$	$AF^{(1)}$	$AF^{(2)}$	N	R_d^2	DW
$AF^{(1)}$	0.1370	7.2e-09	0.00073	0.00095	0.77258	1.0000	0.5518	0.245	0.470	1.8722
$AF^{(2)}$	0.1375	–	–	–	–	0.5518	1.0000	2.573	0.516	1.9820

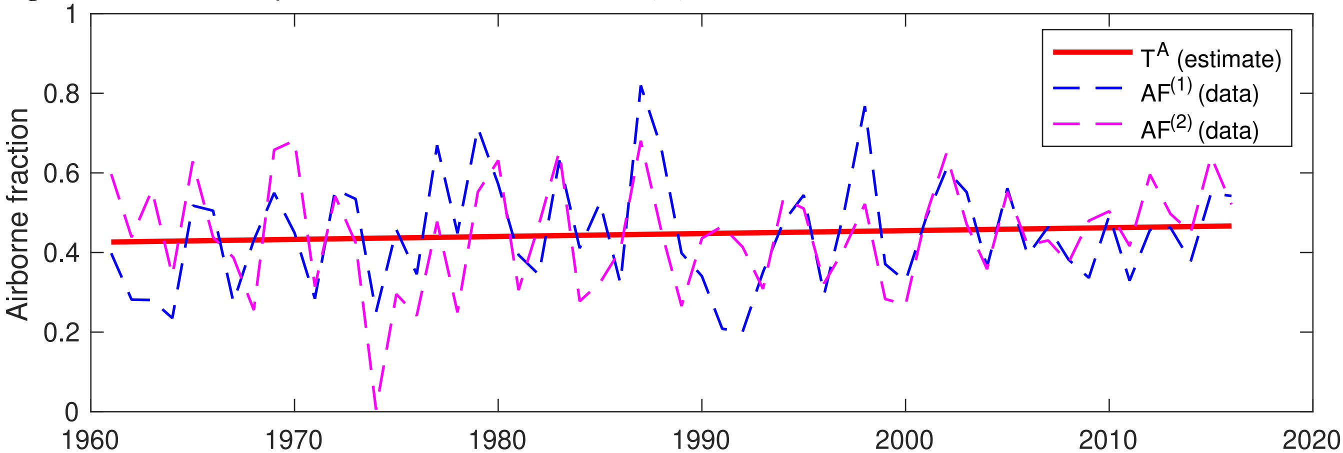
We report parameter estimates for the standard deviations $\sigma_\epsilon^{(i)}$ and $\sigma_\eta^{(i)}$, for $i = 1, 2$, correlation matrix for ϵ_t, and slope coefficient β together with its standard error (s.e.) and t-statistic (t-stat). We further report the normality (N) test, the goodness-of-fit statistic R_D^2 and the Durbin-Watson (DW) test statistic for serial correlation; for details see Table 1. In Panel B, the trend cofficients (σ_η and β) for $AF^{(2)}$ are the same as for $AF^{(1)}$ given the construction of model (12).

Since the two quantities in (8) are measuring the same object, the airborne fraction, we now force the ~~state space~~ state-space system to recognize that these data are driven by the same underlying common trend, T_t^A ~~say~~, but with possibly different error terms $\epsilon_t^{(1)}$ and $\epsilon_t^{(2)}$. In other words, we consider

$$y_t = \begin{bmatrix} AF_t^{(1)} \\ AF_t^{(2)} \end{bmatrix} = \begin{bmatrix} G_t^{ATM}/E_t^{ANT} \\ 1-(S_t^{OCEAN}+S_t^{LAND})/E_t^{ANT} \end{bmatrix} = \begin{bmatrix} T_t^{A} \\ T_t^{A} \end{bmatrix} + \begin{bmatrix} \epsilon_t^{(1)} \\ \epsilon_t^{(2)} \end{bmatrix}. \tag{12}$$

The output of the estimation of this system is shown in Panel B of Table 2; the estimated common trend and the data are plotted in Fig. 1. A slight deterioration of the diagnostic statistics is to be expected when introducing a common trend into the system, but the diagnostic statistics are still such that we can accept (12) as a plausible model. For the estimate of the slope $\widehat{\beta}$, we find a larger t-statistic in absolute value than in the uninformed model, indicating that the restriction to the common trend increases the precision of the estimates. An explanation of this finding is that the informed system ~~in effect~~ has used twice as many observations for estimating the trend ~~, when~~ compared to the uninformed system. The hypothesis test (10) reveals that the estimate of the slope parameter, although again positive, is still not statistically different from zero (p-value 0.2199).

Figure 1. Estimated trend T_t^A of the airborne fraction from Model (12).



5 Trend analysis of the CO_2 sink rate

In this section, we analyse the CO_2 sink rate in the same way as the airborne fraction above. ~~The definition~~ Analogously we can define two alternative versions of the sink rate~~is given in Eq. . The assumption of a balanced carbon budget provides the alternative definition . As a result we can now define two alternative versions of the sink rate~~:

$$k_{S,t}^{(1)} = \frac{S_t^O + S_t^L}{C_t}, \qquad k_{S,t}^{(2)} = \frac{E_t^{ANT} - G_t}{C_t}.$$

~~Our~~

$$k_{S,t}^{(1)} = \frac{S_t^O + S_t^L}{C_t}, \qquad k_{S,t}^{(2)} = \frac{E_t^{ANT} - G_t}{C_t} = k_{S,t}^{(1)} + \xi_t, \tag{13}$$

where now $\xi_t = B_t^{IM}/C_t$ and where we have used Eq. (2). As was the case for the airborne fraction, these two quantities are measuring the same underlying object (the sink rate, $k_{S,t}$) but differ in practice because of a non-zero budget imbalance, i.e. $\xi_t \neq 0$.

The basic (univariate) local linear trend model for each of these objects is then given by

$$y_t = k_{S,t}^{(i)} = T_t^{(i)} + \epsilon_t^{(i)},$$

for $i = 1, 2$, where $T_t^{(i)}$ is specified as in (6). When the model is cast in the ~~state space~~ state-space system, the parameters can be estimated for each of the data series individually. The estimation results are presented in Table 3. The diagnostic statistics are ~~again satisfactory. Although we do have negative~~ satisfactory, and we conclude again that the error term $\xi_t = B_t^{IM}/C_t$ is well-behaved, in the sense that the assumptions underlying the state-space system appear to be valid, also for the alternative sink rate data, $k_{S,t}^{(2)}$. Even though the estimates of the slopes are negative, we cannot reject the null hypothesis of $\beta = 0$ (p-values 0.2233 and 0.0761, respectively). We still consider a one-sided test as in (10) but now the relevant alternative hypothesis is $H_1 : \beta < 0$.

Table 3. Univariate analysis of the CO_2 sink rate

	Parameter estimates					Diagnostics		
	$\widehat{\sigma}_\epsilon$	$\widehat{\sigma}_\eta$	$\widehat{\beta}$	s.e.($\widehat{\beta}$)	t-stat($\widehat{\beta}$)	N	R_d^2	DW
$k_S^{(1)}$	0.0066	8.8077e-04	-0.00010	0.00013	-0.76117	4.880	0.464	1.968
$k_S^{(2)}$	0.0063	6.3982e-04	-0.00015	0.00010	-1.43179	0.967	0.442	1.875

We report parameter estimates for the standard deviations σ_ϵ and σ_η, and slope coefficient β together with its standard error (s.e.) and t-statistic (t-stat). We further report the normality (N) test, the goodness-of-fit statistic R_D^2 and the Durbin-Watson (DW) test statistic for serial correlation; all computed for the standardized prediction errors u_t which are obtained from the Kalman filter; for details see Table 1.

~~Similar~~ Analogously to the airborne fraction above, these data can be put in a joint "uninformed" system with two different trend components, and we have

$$y_t = \begin{bmatrix} k_{S,t}^{(1)} \\ k_{S,t}^{(2)} \end{bmatrix} = \begin{bmatrix} (S_t^O + S_t^L)/C_t \\ (E_t^{ANT} - G_t)/C_t \end{bmatrix} = \begin{bmatrix} T_t^{(1)} \\ T_t^{(2)} \end{bmatrix} + \begin{bmatrix} \epsilon_t^{(1)} \\ \epsilon_t^{(2)} \end{bmatrix}, \tag{14}$$

which can be compared with model (11). The estimation results for this model are reported in Panel A of Table 4. Although the slope estimates are negative, they are not ~~significantly negative~~significant (p-values 0.3106 and 0.1947, respectively).

Finally, we consider the ~~state space~~ state-space system that imposes a common trend for both time series, T_t^S ~~say~~, that is

$$y_t = \begin{bmatrix} k_{S,t}^{(1)} \\ k_{S,t}^{(2)} \end{bmatrix} = \begin{bmatrix} (S_t^O + S_t^L)/C_t \\ (E_t^{ANT} - G_t)/C_t \end{bmatrix} = \begin{bmatrix} T_t^S \\ T_t^S \end{bmatrix} + \begin{bmatrix} \epsilon_t^{(1)} \\ \epsilon_t^{(2)} \end{bmatrix}, \tag{15}$$

which can be compared with model (12). The estimation results are presented in Panel B of Table 4. Similar to ~~our~~ the analysis of the airborne fraction in the previous section, the diagnostic statistics are somewhat worse for ~~our~~ the less flexible system

Table 4. Multivariate analysis of the CO_2 sink rate

	Parameter estimates					Correlation matrix (ϵ)		Diagnostics		
Panel A: Two individual trends as in Eq. (14).										
	$\widehat{\sigma}_\epsilon$	$\widehat{\sigma}_\eta$	$\widehat{\beta}$	s.e.($\widehat{\beta}$)	t-stat($\widehat{\beta}$)	$AF^{(1)}$	$AF^{(2)}$	N	R^2_d	DW
$k_S^{(1)}$	0.0064	0.0015	-0.00010	0.00020	-0.49406	1.0000	0.7733	3.348	0.511	2.0233
$k_S^{(2)}$	0.0060	0.0014	-0.00017	0.00020	-0.86071	0.7733	1.0000	1.365	0.488	2.0185
Panel B: One common trend as in Eq. (15).										
	$\widehat{\sigma}_\epsilon$	$\widehat{\sigma}_\eta$	$\widehat{\beta}$	s.~~d~~e.($\widehat{\beta}$)	t-stat($\widehat{\beta}$)	$k_S^{(1)}$	$k_S^{(2)}$	N	R^2_d	DW
$k_S^{(1)}$	0.0068	4.1762e-09	-0.00014	0.00005	-2.99145	1.0000	0.5621	4.012	0.499	2.0276
$k_S^{(2)}$	0.0065	–	–	–	–	0.5621	1.0000	0.090	0.474	1.7967

We report parameter estimates for the standard deviations $\sigma_\epsilon^{(i)}$ and $\sigma_\eta^{(i)}$, for $i = 1, 2$, correlation matrix for ϵ_t, and slope coefficient β together with its standard error (s.e.) and t-statistic (t-stat). We further report the normality (N) test, the goodness-of-fit statistic R^2_D and the Durbin-Watson (DW) test statistic for serial correlation; for details see Table 1. In Panel B, the trend cofficients (σ_η and β) for $k_S^{(2)}$ are the same as for $k_S^{(1)}$ given the construction of model (15).

with a common trend. However, the diagnostics are still satisfactory while the goodness-of-fit statistics have improved overall. ~~Our~~ The estimate of the slope is

$$\widehat{\beta} = -0.00014,$$

and this estimate is statistically significant: we reject the hypothesis $H_0 : \beta = 0$ in favor of $H_1 : \beta < 0$ at a 95% confidence level (p-value 0.0014). The mean of the sink rate (calculated using either data set $k_S^{(1)}$ or $k_S^{(2)}$) is 0.0258. It follows that we estimate the sink rate to be decreasing with approximately $0.00014/0.0258 = 0.54\%$ every year. The estimated trend and the data are plotted in Fig. 2.

The ~~state space~~ state-space system is also well-suited for forecasting; see Durbin and Koopman (2012). ~~The output of the forecasting exercise for the sink rate is presented in Fig. 3 where~~ Using model (15), we forecast the sink rate 25 years ahead in time. The ~~decreasing nature of~~ output is presented in Fig. 3. The downward trend in the forecasts is ~~clearly visible.~~ the result of the negative estimate of β. Under current conditions, the forecast implies that in approximately 15 years, the sink rate will have declined to below 2%.

6 Trend analysis of the ocean and land sink rates

We may conclude from the analysis in the previous section that the combined (land plus ocean) sink rate appears to be decreasing. To ~~verify~~ investigate this finding in more detail, we ~~can~~ study two alternative models, which consider the two sink variables separately. The ~~analysis can be done simultaneously based on the model~~ first model specifies local linear trends for

Figure 2. Estimated trend T_t^S of the CO_2 sink rate from Model (15).

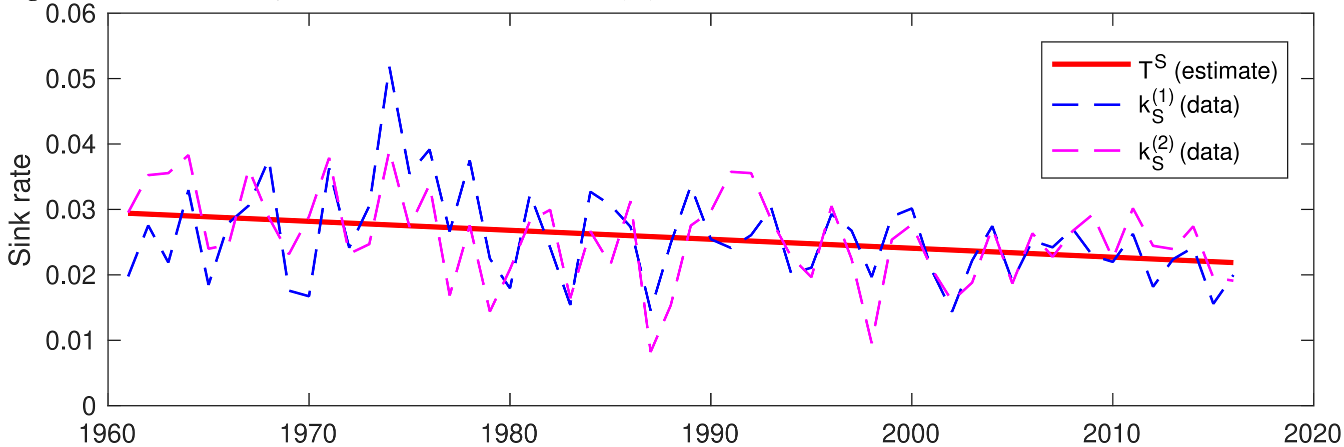


Figure 3. Forecasting the CO_2 sink rate based on Model (15).

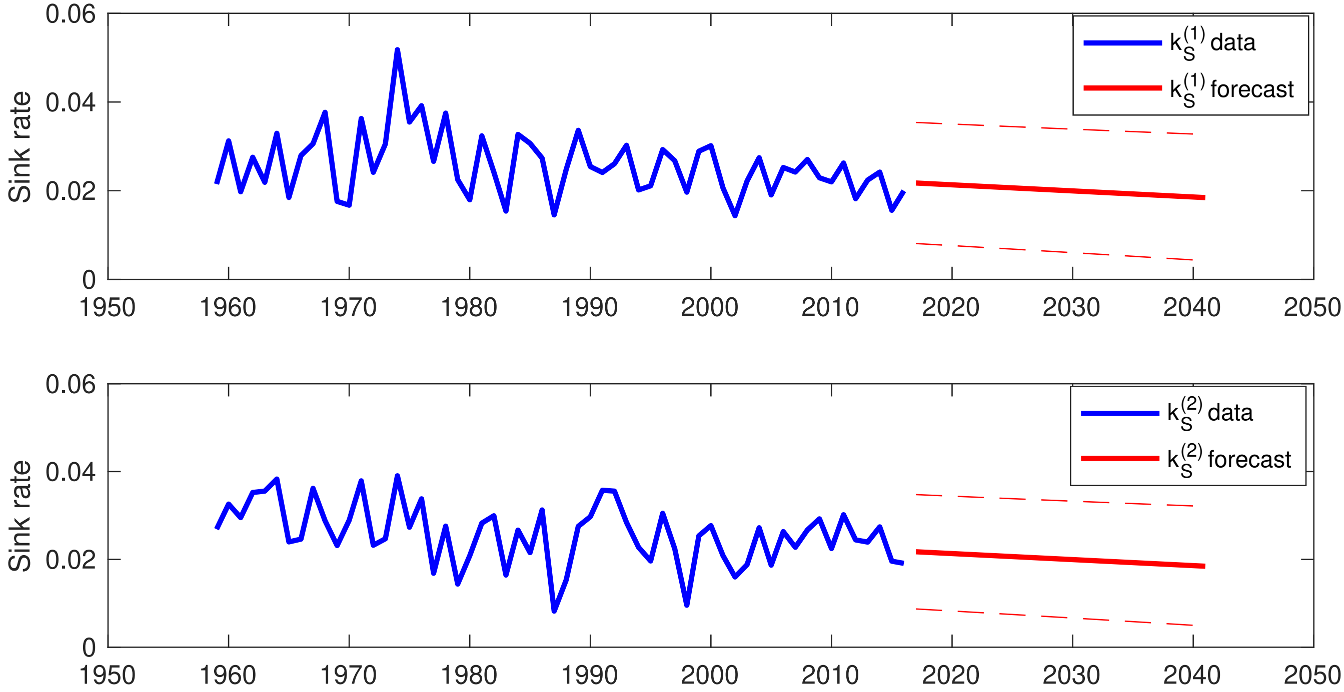


Figure 3. The blue solid line represents the data, while the red solid line represents the point forecasts from the Kalman filter with the unknown parameters estimated by maximum likelihood. The dashed red lines are 95% confidence bands (± 1 standard deviation) for the forecasts.

ocean and land sink rates, i.e.,

$$y_t = \begin{bmatrix} k_{O,t} \\ k_{L,t} \end{bmatrix} = \begin{bmatrix} S_t^O/C_t \\ S_t^L/C_t \end{bmatrix} = \begin{bmatrix} T_t^O \\ T_t^L \end{bmatrix} + \begin{bmatrix} \epsilon_t^{(1)} \\ \epsilon_t^{(2)} \end{bmatrix}, \tag{16}$$

where the time series $k_{O,t}$ and $k_{L,t}$ are defined in (4) while the trend variables T_t^O and T_t^L are specified as in (6). To inform the ~~state space~~ state-space system of the structure of the carbon budget, we also consider the model equations

$$y_t = \begin{bmatrix} k_{O,t} \\ k_{L,t} \\ k_{S,t} \end{bmatrix} = \begin{bmatrix} S_t^O/C_t \\ S_t^L/C_t \\ (E_t^{ANT} - G_t)/C_t \end{bmatrix} = \begin{bmatrix} T_t^O \\ T_t^L \\ T_t^O + T_t^L \end{bmatrix} + \begin{bmatrix} \epsilon_t^{(1)} \\ \epsilon_t^{(2)} \\ \epsilon_t^{(3)} \end{bmatrix}. \tag{17}$$

~~Also this~~ This trivariate model equation can be cast in the ~~state space~~ state-space system (5) as well. The model specification has two independent trend processes of the form (6) for land and ocean sinks. ~~The $k_{S,t}$ time series~~ Since $k_{S,t} = k_{O,t} + k_{L,t}$, the time series $k_{S,t}$ of combined ocean and land sinks must ~~therefore~~ feature the sum of the two trend processes for the individual sinks as its trend process.

The estimation results for these two model specifications are presented in Table 5. The residual diagnostic statistics N and DW are satisfactory, but we are particularly interested in the estimates of the slope parameters. It seems that most of the decrease in the sink rate can be attributed to the land sink. The slope estimates of the trend driving the ocean sink rate are very close to zero and not statistically ~~different from zero~~significant (p-values 0.5227 and 0.5168, respectively). On the other hand, the slope estimates of the trend driving the land sink rate are negative for both specifications. In the first model (16), we can reject the hypothesis that the slope of the trend driving the land sink rate is zero, in favor of the one-sided alternative $H_1 : \beta < 0$ at a 95% confidence level (p-value of 0.0420). For the more informed model specification (17), the estimation results are reported in Panel B of Table 5. ~~We learn from this analysis~~ Here we can reject H_0 at a 90% confidence level (p-value of 0.0882). Further, the results show that the estimate of the slope parameter from the land sink rate is equal to the estimate of the slope parameter from the combined sink rate as ~~we have found it~~ in Sect. 5, that is, $\hat{\beta} = -0.00014$. In other words, it appears that the ~~slope in the land sink rate explains all of the slope in the~~ decrease in the combined sink rate studied in the previous section ~~.~~

~~In summary, the statistical evidence presented for the trivariate model is not as strong as we have presented for the model of the combined sink rate in the previous section. For instance, if we would have conducted the two-sided test , as opposed to the one-sided test in , on the basis of model specification , with the results presented in Panel A of Table 5, we could not have rejected~~ ~~$H_0 : \beta = 0$~~ ~~in favor of~~ ~~$H_1 : \beta \neq 0$~~~~. Nevertheless, the findings of this section provide some evidence that the~~ is entirely explained by the decrease in the ~~sink rate, as found in Sect. 5 above, is mainly driven by a decrease in the~~ land sink rate.

Table 5. Analysis of ocean and land sink rates

	Parameter estimates					Correlation matrix (ϵ)			Diagnostics		
Panel A: Two trends, two observation series as in Eq. (16).											
	$\widehat{\sigma}_\epsilon$	$\widehat{\sigma}_\eta$	$\widehat{\beta}$	s.~~d~~e.($\widehat{\beta}$)	t-stat($\widehat{\beta}$)	$k_{O,t}$	$k_{L,t}$		N	R_d^2	DW
$k_{O,t}$	0.0001	0.00081	0.00001	0.00011	0.057	1.00	-1.00		4.839	0.0343	1.847
$k_{L,t}$	0.0067	0.00015	-0.00010	0.00006	-1.728	-1.00	1.00		5.332	0.513	1.908
Panel B: Two trends, three observation series as in Eq. (17).											
	$\widehat{\sigma}_\epsilon$	$\widehat{\sigma}_\eta$	$\widehat{\beta}$	s.~~d~~e.($\widehat{\beta}$)	t-stat($\widehat{\beta}$)	$k_{O,t}$	$k_{L,t}$	$k_{S,t}$	N	R_d^2	DW
$k_{O,t}$	0.0001	0.00081	0.00000	0.0001	0.0422	1.00	-0.122	-0.884	4.839	0.0343	1.916
$k_{L,t}$	0.0068	0.00068	-0.00014	0.0001	-1.352	-0.122	1.00	0.572	4.054	0.494	1.989
$k_{S,t}$	0.0065	-	-	-	-	-0.884	0.572	1.00	1.114	0.477	1.801

We report parameter estimates for standard deviations $\sigma_\epsilon^{(i)}$ and $\sigma_\eta^{(i)}$, for $i = 1,2,3$, correlation matrix for ϵ_t, and slope coefficient β together with its standard error (s.e.) and t-statistic (t-stat). We further report the normality (N) test, the goodness-of-fit statistic R_D^2 and the Durbin-Watson (DW) test statistic for serial correlation; for details see Table 1. In Panel B, we have two trends and two sets of trend cofficients (σ_η and β) for $k_{O,t}$ and $k_{L,t}$, the trend for $k_{S,t}$ is a combination of the two, given the construction of model (17).

7 Discussion

Previous studies of the airborne fraction and the CO_2 sink rate have focused on detecting a single linear and deterministic trend in the data of the form $a_0 + a_1 t$, where a_0, a_1 are constants (Canadell et al., 2007a; Le Quéré et al., 2009; Knorr, 2009; Raupach et al., 2008, 2014). However, possible statistical difficulties in such analyses have been pointed out in Knorr (2009). For instance, a linear regression analysis of two or more non-stationary variables can yield invalid inference ~~if the data are non-stationary, e.g., if they contain trends~~ (Granger and Newbold, 1974). The approach of this paper is to consider the data in a ~~state space~~ state-space system. In this way, non-stationary components are explicitly modelled as unobserved trend components and inference is valid (e.g., Durbin and Koopman, 2012). Furthermore, the trend specification of the ~~state space~~ state-space system allows for both deterministic and stochastic trend components.

~~Further, several~~ In some of the "un-informed" models, cf. Table 1, Panel A of Table 2, and Panel A of Table 4, we estimate $\hat{\sigma}_{Slp} > 0$ and, thus, in these cases, we find evidence of the trend component varying in time. However, in our "informed" models with a single trend object for two alternative time series, the extracted trends are practically deterministic, that is, the estimates of σ_{Slp} in Panel B of Tables 2 and 4 are near zero, cf. also Fig. 1 and 2. In conclusion, there is evidence that a simple deterministic trend fits both the airborne fraction and the sink rate data well, although this only becomes evident when incorporating two data sets for each of these objects.

Several studies have highlighted the need for accounting for noise in measurements of climate-related data (Knorr, 2009; Ballantyne et al., 2015). The ~~state space~~ state-space approach explicitly incorporates such noise in the framework as well. Ballantyne et al. (2015) argue that errors in E_t^{ANT} might be autocorrelated. As shown in Tables 1 through 5, the diagnostic statistics do not indicate that autocorrelated errors pose a serious problem in our specifications. Nevertheless, the ~~state space~~ state-space framework can incorporate autocorrelated errors in the measurement equation.

~~This paper considers data recorded at a yearly frequency, while many of the previous studies of the airborne fraction and the sink rate use monthly data. The advantage of using monthly data is obvious: more observations. However, there are also some downsides. For instance, while the CO_2 concentration C_t (and therefore also the growth rate G_t) are recorded every month, these data contain a strong seasonal component induced by the photosynthesis/respiration cycle of terrestrial vegetation. This seasonality needs to be accounted for in some way; for instance, Raupach et al. (2014) smooth the data using a 15-month running mean. Conversely, some of the other data are recorded only yearly; for instance, the emissions data available to us, E_t^{ANT}, are reported at a yearly frequency. In this case Raupach et al. (2014) use linear interpolation to get monthly estimates of emissions. Such transformations of the data, i.e., smoothing or interpolation, might introduce new and complicated errors into the transformed data, possibly invalidating the analyses. For these reasons, we prefer to work with yearly data.~~

Why do we find statistical evidence of a decreasing CO_2 sink rate but no evidence of an increasing airborne fraction when these two quantities are closely linked and the data ~~going into~~ entering the analyses are the same? It was noted in Gloor et al. (2010) that the airborne fraction and the sink rate are actually not as closely linked as they may seem *prima facie*. In particular, an increasing airborne fraction does not necessarily imply a decreasing sink rate (Gloor et al., 2010, Section 8). Secondly, ~~we believe that the way the two quantities are defined makes the sink rate an easier object to study statistically. The idea~~ the concept of an airborne fraction ~~(and a sink fraction) appears to be a~~ is that of a long-term quantity: the airborne fraction should represent the amount of anthropogenically released CO_2 that *eventually* stays in the atmosphere, *after* other fluxes have been ~~taking~~ taken into account. However, the ratio of the concurrent fluxes, i.e., G_t/E_t^{ANT}, is likely a very noisy measurement of this object. Also, as we saw above, it is reasonable to ~~think that~~ consider sink fluxes, and therefore indirectly G_t, ~~will depend~~ as dependent on the *level* of CO_2 in the atmosphere (i.e., $C_t = \sum G_t$), which is not captured by the concurrent ratio G_t/E_t^{ANT}. When studying the airborne fraction, it would perhaps be more reasonable to study an object taking this cumulative nature into account, e.g. $\sum G_t / \sum E_t^{ANT} = C_t / \sum E_t^{ANT}$ (in fact, such specifications were often considered in earlier parts of the literature, e.g. Keeling, 1973; B . However, cumulative statistics of this type would present other difficulties. The dominance of the long-term history may mask sudden changes, for example. ~~These difficulties are even more pronounced when studying the sink fractions SF, OF, and LF: observations such as S_t^O/E_t^{ANT} are very noisy and since, as just discussed, S_t actually depends on C_t and generally not directly on E_t^{ANT}, this makes it difficult to interpret the results directly.~~ In contrast, the sink rate S_t/C_t, as a flow-to-stock ratio, is immediately compatible with the underlying theory, at least as long as the linear approximation of ~~Gloor et al. (2010)~~ the relationship between S_t and C_t, such as was made in e.g. Gloor et al. (2010) and Rayner et al. (2015), is adequate.

~~In our "informed" models with a single trend object for two alternative time series, the extracted trends are practically deterministic,~~ What are possible physical reasons for the apparent decrease in the sink rate? Raupach (2013) argues that a necessary condition for a constant sink rate is that the so-called "LinExp" assumption holds, i.e., that the sink fluxes S_t^O and S_t^L are linear in concentrations C_t ("Lin") and that emissions (E_t^{ANT}) grow exponentially ("Exp"). Constancy of the airborne fraction rests on a similar "LinExp" argument. Since we find no statistical evidence that the airborne fraction, AF_t, and the ocean sink rate, $k_{O,t}$, are non-constant in time, it is unlikely that the "Exp" assumption is violated over the observation period considered in this paper. In contrast, it was found above that the efficiency of the land sink, $k_{L,t}$, is decreasing. A plausible explanation of these findings is that ~~is~~the "Lin" assumption no longer holds for the land sink, for instance because the terrestrial sink could be slowly saturating (Canadell et al., 2007b). In Appendix A we give a formal argument for how this could lead to the findings documented above.

It is possible that the analyses conducted above are influenced by external natural events such as ENSO, volcanic eruptions, and the like (Frölicher et al., 2013). The state-space system used in this paper can explicitly account for such effects through the additive error terms ϵ_t, ~~the estimates of σ_{Slp} in Panel B of Tables 2 and 4 are near zero, cf. also Fig. 1 and 2. It is important to stress that our modeling framework for the trend component, as specified in , allows for the trend to vary stochastically over time. However, we have not found strong evidence for a stochastic trend in our analyses. In contrast, in some of the univariate models,~~ cf. ~~Table 1, Panel A of Table 2, and Panel A of Table 4, we estimate $\hat{\sigma}_{Slp} > 0$ and, thus, in these cases~~Eq. (5). To verify that the approach is indeed robust to such external and transitory events, we have also conducted our analyses using 5-year average data. The findings from the estimated state-space system for these time series of averages confirm those reported above: In the joint estimation, we find ~~evidence~~ no statistical evidence of a trend in the airborne fraction (p-value of ~~the trend component varying in time. This variability disappears, however, once we impose a common~~ 0.3214), and we do find statistical evidence of a decreasing trend in the ~~models. In other words, there is evidence that a simple deterministic trend fits the data well (both the~~ sink rate (p-value of 0.00064). We conclude that the findings of this paper are not likely to be driven by external natural events such as ENSO and volcanic eruptions. We also considered 2-, 3-, and 4-year averages with similar results. We present details of this analysis in the Supplementary Material file.

This paper considers data recorded at a yearly frequency, while many of the previous studies of the airborne fraction and the sink rate ~~), and therefore that allowing for time-variation in the trend is redundant.~~ use monthly data. The advantage of using monthly data is obvious: more observations. However, there are also some disadvantages. For instance, while the CO_2 concentration C_t (and therefore also the growth rate G_t) are recorded every month, these data contain a strong seasonal component induced by the photosynthesis/respiration cycle of terrestrial vegetation. This seasonality needs to be accounted for in some way; for instance, Raupach et al. (2014) smooth the data using a 15-month running mean. In contrast, some of the other data are recorded only yearly; for instance, emissions data available to us, E_t^{ANT}. In this case, Raupach et al. (2014) use linear interpolation to get monthly estimates of emissions. Such transformations of the data, i.e., smoothing or interpolation, might introduce new and complicated errors, possibly invalidating the analyses. For these reasons, we prefer to work with yearly data.

8 Conclusions

We have argued that the ~~state space~~ state-space system can be a useful approach to analyze possible trends in the airborne fraction of anthropogenically released CO_2 and in the CO_2 sink rate. We have shown that deterministic and stochastic trend processes can be explicitly and jointly incorporated as unobserved components, allowing for ~~a~~ valid inference, even when the observed time series are non-stationary. The ~~state space~~ state-space framework also allows for the incorporation of multiple data sets for the same object, which ~~can increase~~ increases reliability of the resulting estimates.

We estimate a positive, yet statistically insignificant, slope in the data for the airborne fraction. ~~The sink rate exhibits some evidence of a decreasing trend.~~ Using two alternative time series ~~as data~~ for the sink rate and imposing a common trend~~component for both~~, we obtain a significantly negative deterministic trend~~slope in the sink rate.~~

. Our analyses support the conclusions as set out by Raupach et al. (2014): the rate at which the combined (ocean plus land) sink takes up CO_2 from the atmosphere seems to be decreasing. The best estimate resulting from our ~~state space model~~ state-space system is that the CO_2 sink rate, and therefore the efficiency with which the combined land and ocean sink is absorbing carbon from the atmosphere, is decreasing ~~with~~ by 0.54% per year. We do not find evidence of this rate itself changing over time.

Finally, there is tentative evidence that the decrease in the sink rate is mainly driven by a weakening uptake in the land sink. This could be the result of non-linearities in the response of the terrestrial carbon sink to the level of atmospheric concentrations of CO_2. That is, although the land sink is itself increasing and thus continuing to take up a large part of anthropogenically emitted CO_2, as also noted recently by e.g. Rayner et al. (2015), Keenan et al. (2016), and Fernández-Martínez et al. (2019), the *rate* of this uptake appears to be decreasing. The statistical evidence for this is not strong, however, and we suggest that additional research must be conducted to further investigate this question.

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Appendix A: Linear approximation of the relation of land sink and concentrations

In this appendix, we argue that the levels of atmospheric concentrations of CO_2 may have risen to a point where a linear expansion of the logarithmic Bacastow and Keeling (1973) formula, describing the flux of CO_2 into the land sink, is no longer sufficient. Consequently, the "Lin" assumption of Raupach (2013) might be violated for the land sink, implying that second-order effects may be driving the negative slope of the sink rate that we document in this paper.

From Eq. (4) we obtain the relation

$$S_t^L = k_{L,t} \cdot C_t,$$

which implies that the flux of CO_2 to the land sink is linear in C_t, where $k_{L,t}$ would then be treated as a constant. On the other hand, a decreasing $k_{L,t}$ implies that the efficiency with which the land sink absorbs CO_2 is decreasing, i.e., that the flux of CO_2 to the land sink is non-linear in C_t and this non-linearity is such that the efficiency is decreasing. These statements are consistent with simulation results from climate cycle models (Friedlingstein et al., 2006). Here we illustrate mathematically how such non-linearities can arise.

The precise relationship between S_t^L and C_t still alludes us but Bacastow and Keeling (1973), p. 94, suggest that (in our notation):

$$S_t^L \approx \alpha \log(1 + C_t/\mathcal{C}^0),$$

where α is a constant and $\mathcal{C}^0 = 591.30$ GtC is the amount of CO_2 in the atmosphere in pre-industrial times. Using this function, we can write a second-order Taylor expansion

$$S_t^L \approx \alpha \log(1 + C_t/\mathcal{C}^0) \approx \alpha \frac{C_t}{\mathcal{C}^0} - \frac{1}{2}\alpha \left(\frac{C_t}{\mathcal{C}^0}\right)^2.$$

Thus, if $\mathcal{C}^0$ is large compared to C_t, this implies that a linear specification between S_t^L and C_t is reasonable. However, once C_t becomes large compared to $\mathcal{C}^0$, this shows that the estimated sink rate will be found to be decreasing. To see this, use the Taylor expansion to write

$$S_t^L \approx k_{L,t} C_t,$$

where

$$k_{L,t} = \frac{\alpha}{\mathcal{C}^0} - \frac{1}{2}\frac{\alpha}{\mathcal{C}^0}\frac{C_t}{\mathcal{C}^0}$$

is decreasing in C_t. In our data, we have $C_{1959} \approx 80$ GtC and $C_{2016} \approx 267$ GtC, resulting in $C_{1959}/\mathcal{C}^0 \approx 14\%$ and $C_{2016}/\mathcal{C}^0 \approx 45\%$. In other words, the linear approximation to the Bacastow and Keeling model of the land sink flux might have been reasonable in the past, since $C_{1959}/\mathcal{C}^0 \approx 14\%$, but is likely misspecified in the present, since $C_{2016}/\mathcal{C}^0 \approx 45\%$. That is, if this model is accurate, then a decreasing (land) sink rate indicates that we have entered a regime of atmospheric CO_2 concentrations, where the linear approximation breaks down and higher order terms should be taken into account.

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