

## Supplementary Material

### 1 Global Averaging of Martin's $b$ parameter

Here we describe the calculation of a global mean from samples of spatially variable  $b$ . Figure 6 in the manuscript shows that when  $\text{CO}_2$  is plotted against the global mean  $b$  of the Latin hypercube samples, there is a constant offset versus runs where  $b$  is varied uniformly. We suggest that this offset is a function of a non-linear relationship between  $b$  and the amount of organic carbon reaching the ocean interior. This is demonstrated by expressing  $b$  as an  $e$ -folding depth (the depth at which the proportion of organic matter that has remineralisation =  $\frac{1}{e} \approx 0.63$ ). This is achieved by rearranging the Martin curve equation:

$$F_z = F_{z_0} \left( \frac{z}{z_0} \right)^{-b} \quad (1)$$

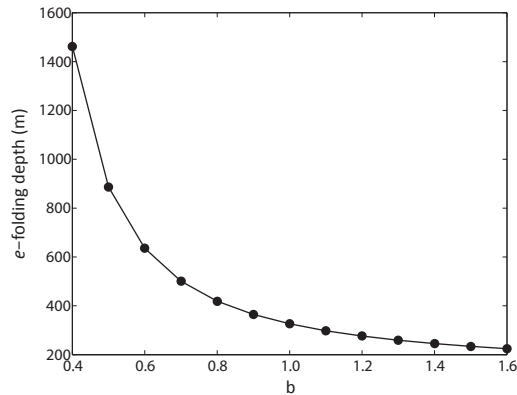
for depth ( $z$ ) when  $\frac{F_z}{F_{z_0}} = \frac{1}{e}$ :

$$z = \sqrt[b]{\frac{F_z}{F_{z_0}}} z_0 \quad (2)$$

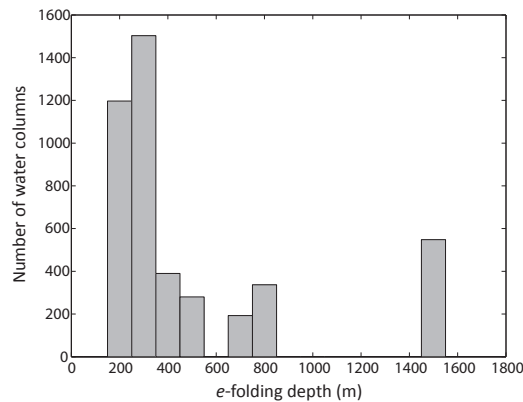
where  $z_0$  is the depth of the euphotic zone (here  $z_0=120$  m),  $F_z$  is the flux of POC at depth  $z$  and  $F_{z_0}$  is the flux of POC out of the euphotic zone ( $z_0$ ).  $e$ -folding depths get increasingly deeper with decreasing values of  $b$  (Figure 1). For example, a change in  $b$  from 1.4 to 1.3 results in a decrease in  $e$ -folding depth of 14 m whereas a change in  $b$  from 0.4 to 0.3 results in a 1902 m. This follows from the fact that the Martin curve represents the scenario of a fixed remineralisation rate and an increasing sinking rate (Kriest and Oschlies, 2008). Therefore, when averaging spatially variable  $b$  values, larger values will have disproportionately more weight in comparison to their impact on ocean biogeochemistry.

To account for this non-linear effect on biogeochemistry, we alternatively calculate the global mean of each Latin hypercube ensemble experiment when converted to  $e$ -folding depths. Because of the non-linear relationship between  $b$  and  $e$ -folding depths, an arithmetic mean is unsuitable (e.g., Figure 3). We alternatively calculate an area-weighted geometric-weighted mean and convert this back to a value of  $b$ :

An alternative approach to calculating a global mean is to calculate a set of weights based on  $e$ -folding depths. However, the calculation of these weights is not straightforward. Alternatively, a Latin hypercube ensemble could be sampled from a uniform distribution of  $e$ -folding depths rather than  $b$  and parameterising remineralisation with an exponential function. However, power-law based parameterisations are commonly used in biogeochemical models (e.g., Hülse et al., 2017) as well as in interpreting observations (Honjo et al., 2008). We note that a number of different mathematical functions, each with different mechanistic interpretations and including a power-law and exponential function, are not statistically distinguishable from each other based on existing observations (Cael and Bisson, 2018).



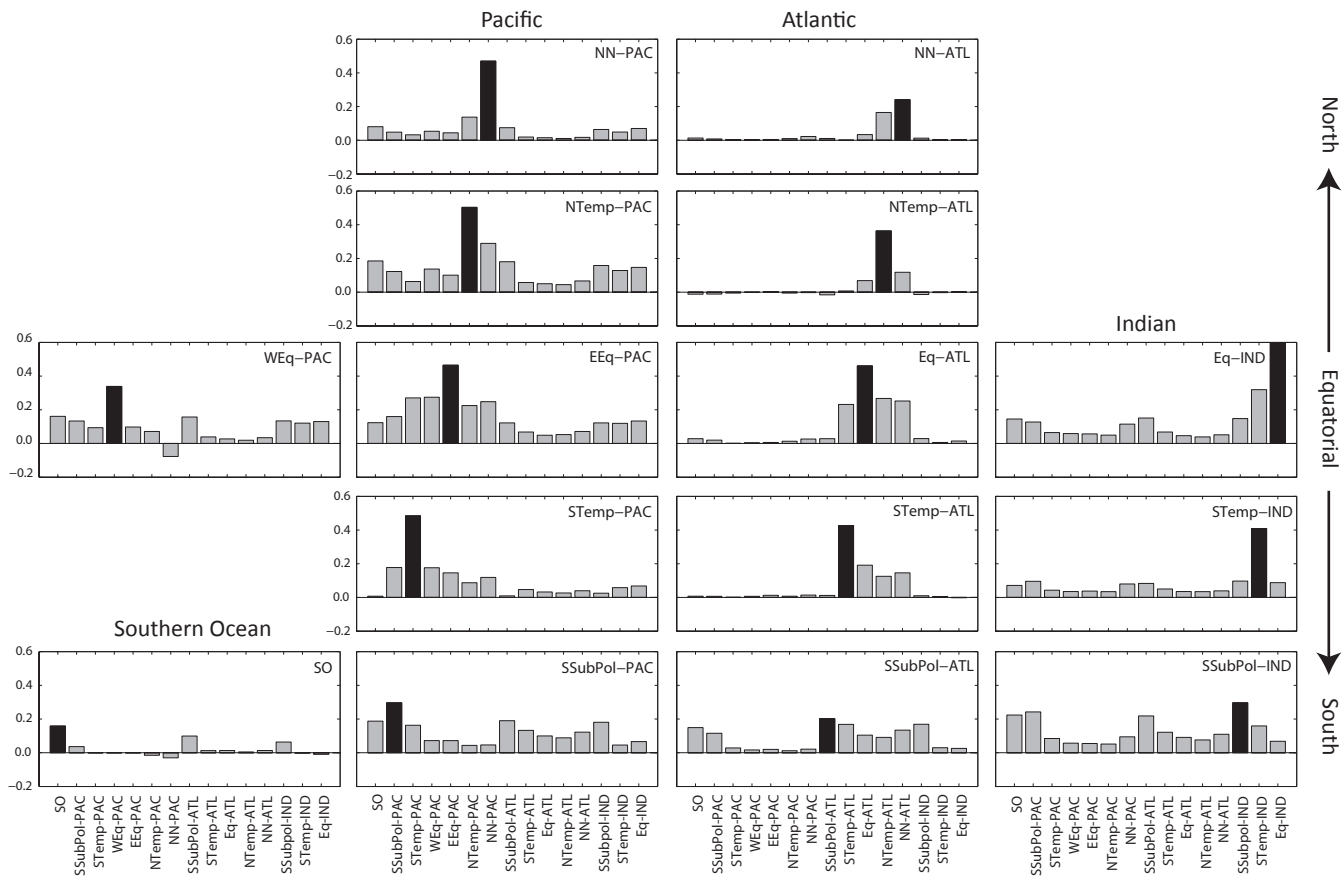
**Figure 1.** The  $e$ -folding depths (depth at which 0.63% of organic matter has been remineralised) corresponding to various Martin curve exponents.  $e$ -folding depths are calculated using eqn. 2.



**Figure 2.** Histogram of  $e$ -folding depths for all water columns in the model, calculated from  $b$  values from one Latin hypercube sample using equation 2.

## References

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- 10 Kriest, I. and Oschlies, A.: On the treatment of particulate organic matter sinking in large-scale models of marine biogeochemical cycles, *Biogeosciences*, 5, 55–72, <https://doi.org/10.5194/bg-5-55-2008>, <http://www.biogeosciences.net/5/55/2008/>, 2008.



**Figure 3.** Normalised sensitivity of steady-state mean preformed  $[PO_4]$  from each region to regional changes in  $b$  calculated with the restoring-export scheme. Sensitivity is calculated using the same linear regression method as in Figure 3 except that preformed  $[PO_4]$  is normalised before so that sensitivity can be compared on the same scale. Bars show the magnitude and sign of sensitivity to changes in  $b$  that are local to that region (black) and in other regions (grey). Panels are arranged by basins and by latitude. The equivalent plot for the constant-uptake scheme is found in Figure 5 of the main text.