Reply to comments

This is an interesting study seeking to solve the so-called Kuroshio Paradox. As a physical oceanographer with expertise in small-scale ocean physics I am not in a position to comment on the biological part of this paper, but I do have fundamental concerns on the physics the authors employed in this study.

>We appreciate your point of view to our findings. Our point-by-point responses are as follow. Hopefully, these are enough responses to your comments.

First of all, turbulent diffusivity was not "measured", but rather estimated involving important physical assumptions, such as isotropy of small-scale (3D) turbulence for the estimation of the turbulent kinetic energy (TKE) dissipation rate from microscale velocity shear measurements, and the Osborn formula (i.e., a local energy balance assuming constant mixing efficiency) for the estimation of diffusivity from the TKE dissipation rate. These and the procedures of data processing should be explained at least briefly in the manuscript. This is in particular necessary given the interdisciplinary nature of the work; the readers with different backgrounds should be able to well appreciate the foundations of the numbers that the authors use to support their points.

>We appreciate your kind suggestions and agree with you. As you mentioned, we add some descriptions at the Materials and Methods section in the revised manuscript as follows.

“The nitrate profiles were measured by a nitrate sensor (Deep SUNA V2) attached on a SBE-9plus CTD system. The turbulence diffusivity was estimated from microstructure measurements by TurboMAP-L (JFE Advantech Co. Ltd.) based on Osborn (1980)'s formula, which were deployed instantly after each CTD cast for the same stations. The nitrate sensor was calibrated with the observed nitrate concentrations (accuracy: 0.37 mmol m^{-3}, Hasegawa et al. 2019). Total of sixteen nitrate and the turbulence diffusivity profiles obtained among the stations at KG1515 cruise by T/S Kagoshima-maru across the Kuroshio path were averaged, then the profiles of the gradient of the averaged nitrate, and the averaged turbulence diffusivity were multiplied for each depth to get the averaged turbulent nitrate fluxes. Both parameters were binned and averaged within 10-meter intervals. The vertical gradient of the averaged nitrate profile (C_{NO3}) and the averaged vertical diffusivity profile (K_z) were then multiplied at each depth (z) to estimate the area-averaged vertical turbulent nitrate flux (F_{NO3}) with the following equation:”

Moreover, and more crucially, although it has been customary (in the biogeochemical literature particularly) to estimate diapycnal turbulent fluxes considering only the diffusive flux (i.e., equation (1) in the manuscript), it is now well recognized that this is fundamentally improper, because there is always a diapycnal advective flux associated with the diffusive flux. The physical reason is in fact quite straightforward, that is, diapycnal mixing induces fluxes not only of passive properties such as nutrients, but also of the buoyancy, so that the density of the water parcel is changed due to mixing, and thus a diapycnal advective velocity is induced. These ideas have in fact been rigorously elaborated by Trevor McDougall in 1980s (albeit apparently with insufficient attentions), and the biogeochemical implications have recently been explained by Du et al. (2017). It would be very interesting to see how the refined estimate would affect the authors’ results.

>Thank you for the useful comment on the diapycnal advection contribution. We think that the important nutrient flux in the present study is the one across the euphotic depth, not through the density layer, which is

transformed by the turbulent mixing. In addition, as our studied regions are frontal regions unlike the SCS, where the Kuroshio flows over the seamounts, density fluctuations should be caused not only by turbulent mixing but also by advection and the movement of the fronts. Accordingly, we focus our discussions on the vertical turbulent nutrient flux using cartesian coordinate, rather than diapycnal flux using isopycnal coordinate.