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Dear Jens,

Thanks for the opportunity to submit a revised version of our manuscript. We made substantial changes to the manuscript based on the reviewers' comments and your overall recommendation of presenting the CBS metric as a novel concept rather than presenting specific simulations on Earth system feedbacks. This resulted in a major re-structuring of the examples and the use of a different model following the recommendations provided by reviewer 2.

Point-by-point answers to reviewer comments are provided below and in the discussion forum of the manuscript. A marked-up version with all the changes is also provided here.

In general, the main changes in this revised version are:

- We changed the main model used in the examples. Instead of using a biospheric model for the entire terrestrial carbon cycle, we use now an ecosystem-level model parameterized for the Duke Forest, USA. This model is still very simple, which we think is important for reproducibility and transparency, and avoids potential problems of a major disturbance of the atmospheric carbon pool as highlighted by reviewer 2.
- We eliminated the forestry and soil carbon examples also following recommendations by the reviewers. We realized that for these examples to be meaningful, we would need more complex models parameterized with better data, which would result in a more complex simulation study. Since our objective is not to present such detailed simulations, but rather to introduce the new CS and CBS concepts, we introduce new examples that present properties of the new concepts and can help with their implementation and interpretation.
- We give more emphasis in the introduction and discussion sections to the problem of treating all carbon removals by sinks equally. This is a problem implicit in the current guidelines for carbon inventories produced by the IPCC. We believe our new concepts can help to improve the quantification of climate benefits of carbon sequestration in climate policy and carbon

markets.

The manuscript has changed considerably, therefore we understand if you send the manuscript for another round of reviews. However, we believe it has improved tremendously compared to the previous version, and we are confident it will be well received by the reviewers.

Sincerely,

Carlos A. Sierra, PhD
On behalf of all authors

# Response to Reviewer 1

We thank the reviewer for taking the time to review and comment on our manuscript, and regret that he finds this manuscript 'not a meaningful contribution to the climate literature'. We understand that in the current version of the manuscript, the link to state-of-the-art Earth system models (ESMs) is not completely clear. However, we would like to clarify that the concepts we introduce are very general and can deal with models of any level of complexity, working at different scales, and developed for different purposes. The compartmental approach we use works for simple 'outdated' models as well as for highly complex models. We have written extensively in the past about this generalization approach (Sierra and Müller, 2015; Metzler et al., 2018; Sierra et al., 2018; Ceballos-Núñez et al., 2020), and similar ideas are also well developed in papers by Dr. Yiqi Luo's group (e.g Luo and Weng, 2011; Xia et al., 2013; Luo et al., 2017). Unfortunately, we do not have much space in this manuscript to demonstrate the generality of the compartmental framework, but we have done exactly that in previous publications. Similarly for the limitations of the concept 'residence time', for which the reviewer asks why we do not show the limitations of such a concept. We also have written a number of publications about issues with this concept and the common methods to compute it (ratio of stock over flux). We developed new mathematical approaches to advance on this subject (Metzler and Sierra, 2018; Metzler et al., 2018) and this manuscript is a step further in the application of the new methods. Again, we feel this manuscript is not the appropriate venue to elaborate on the 'residence time' issues and the new methods. Instead, we provide a presentation on these ideas, show with more detail some of the formulas in the appendix, and provide appropriate references, but can't go in more detail. Also, we want to point out that the problem on how to account for time in carbon sequestration has been a long-standing issue, with important debates in forestry (e.g. Fearnside, 1995; Fearnside et al., 2000; Sedjo and Sohngen, 2012), ecological economics (e.g. Moura Costa and Wilson, 2000), and ecosystem management (e.g. Neubauer and Megonigal, 2015). We think we provide here a relevant contribution to those previous debates. This may not be very obvious for researchers currently working on climate feedbacks, but it is a topic that touches on different disciplines and we think it is a meaningful contribution to the overall topic of carbon sequestration in natural and anthropogenic sinks.

Below, we provide specific answers to the main issues raised by the reviewer (in *italics*), with a description of corresponding changes in the manuscript to address those comments.

## Answers to specific comments

- *To justify the development of this metric, the authors would need to make a compelling case for where existing, widely-used related metrics of carbon sequestration (such as carbon residence time and net ecosystem productivity) fall short, and why this new metric is superior (or at least comple-*

*mentary). However, there is hardly any mention of these existing metrics in this manuscript.*

We discuss in our manuscript the limitations of the use of global warming potential (GWP), which is the most common metric to assess climate consequences of carbon management. Limitations of the concept of residence times have been already published in Sierra et al. (2017), where we elaborate on the need to distinguish between the concepts of system age and transit time. The present manuscript is a further development of the concept of transit time to show that it can be used to quantify the climate benefit of having carbon stored in ecosystems during the time it remains there. Therefore, we focus not in showing the limitations of the ambiguous concept of residence time, but rather in showing the power of the transit time/age framework. The manuscript did not elaborate on limitations of the concept of net ecosystem production (NEP) to assess carbon sequestration, although its limitations are somehow intuitive based on Fig 1 and the text in the introduction. NEP provides a net flux between the carbon exchanged between ecosystems and the atmosphere, without accounting for harvest exports. It does not tell you for how long the carbon in the output flux stayed in the ecosystem. Two different ecosystems with similar NEP values could have very different carbon storage values and transit times, so this concept is not very useful to assess carbon sequestration, particularly for long time scales.

To address these issues, we added text in the introduction briefly mentioning our previous work on the ambiguity and limitations of 'residence time'. We also elaborate more on the limitations of studying either gross (e.g. GPP) or net (NEP) fluxes to assess carbon sequestration.

We also would like to mention that our CBS concept just tells something different that other metrics do not tell. It combines in a single metric the amount of carbon that enters a sink and the time it remains there. Previous metrics simply do not provide this information in an integrated form.

- *For that matter, the manuscript's application of CBS to terrestrial C cycle models focuses only on dated and/or simplistic models and makes no mention of recent syntheses of terrestrial carbon cycling and associated climate feedbacks by modern land surface models—for instance, Friedlingstein et al. 2014 (https://doi.org/10.1175/JCLI-D-12-00579.1) or Heinze et al. 2019 (https://doi.org/10.5194/esd-10-379-2019)—which makes it difficult to draw meaningful conclusions from those results.*

  We chose a 'dated' model because, to introduce a new complex metric, we believe a simple model is more effective and transparent than a complex Earth system model. With an ESM, potential users of this framework would not have the opportunity to test results if they do not have access to the source code of the ESM and to a supercomputer. A simple model allows readers to test the framework with very simple code. We believe this is a more transparent approach for a paper that introduces a new concept. Future applications can of course be implemented in large ESMs, but that is beyond the scope of this manuscript.

However, a simple model like the one we used does not have any feedbacks with other components of the Earth system. Nevertheless, these feedbacks can be part of the atmospheric impulse response function. In fact, the IRFs of Joos et al. (2013) include all feedbacks that are part of the ESMs of the CMIP5 generation. If one uses these IRFs for the computation of the CBS, it is not necessary to include feedbacks in the terrestrial carbon model as long as simulations do not deviate much from the original simulations used to create the IRF. Alternatively, one can use an ESM and compute the CBS directly from the net biospheric fluxes and the particular IRF of the model. This would require a complex simulation setup, that again is beyond the scope of this manuscript. More importantly, we would like to emphasize that our aim is not to produce state-of-the-art calculations of the CBS for the terrestrial biosphere, but rather to introduce the concept and the mathematical theory behind it. For this reason, and also based on the comments from reviewer 2, we decided to remove some of the examples and give more emphasis to the development and explanation of the concepts. After all, this is a theoretical paper and not a state-of-the-art analysis of the actual climate benefit of sequestering carbon in the terrestrial biosphere.

- *Ultimately, I cannot recommend this manuscript for publication beyond the Discussion format. I would encourage the authors to carefully read a recent review of terrestrial C cycling and its climate implications and consider how CBS fits into that context*

  We will check the recent carbon cycle literature to see if we are missing something. We have coauthored a good number of papers on the carbon cycle, including reviews. But indeed there might be something we are missing. We would have hoped the reviewer to point out more clearly what exactly are we missing. Without a clear indication of what particular concepts are problematic in our framework, it is impossible for us to guess them.

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# Response to Reviewer 2

We appreciate the thoughtful comments from the Reviewer and the critical questions she/he poses. Based on this review, we decided to make a major restructuring of the manuscript. First, we decided to give more emphasis to the theoretical part of the manuscript and decrease the emphasis on the importance of the examples. Therefore, we show now very simple examples on the computation of the CBS, still using a simple model for clarity and transparency, and discussing potential applications in the Discussion section. Second, we use now a different model, slightly updated, focused on the ecosystem level and not on the biospheric level. It is still a very simple model, which has the advantage that it is tractable and the computations can be reproduced in a transparent way.

Below, we elaborate more on the proposed changes, and provide specific answers to the reviewer's comments (in *italics*).

# Answers to general comments

- *Overall: I think this is an interesting paper, and lays a simple and elegant formalism for understanding the importance of changes to ecosystem carbon flows from the perspective of atmospheric radiative balance. However, the applications of the method are more confusing than helpful, and don't really sell the utility of the method for answering policy-relevant questions.*

  This is an important comment and we took it seriously. We acknowledge that the examples we provided are not necessarily very useful in explaining the main CBS concept and showing its relevance for policy-related questions. In the new revised version, the examples were completely changed, they now illustrate how these quantities are computed and what are the meaning of the obtained results. Direct applications to policy are addressed in the Discussion section. We realized that our original examples can be topics of importance to be addressed in separate studies, but the use of our simple model creates an imbalance of addressing an important topic with a very simple model. Therefore, we think those particular questions can be addressed in subsequent manuscripts that use more appropriate models for the questions being asked.

- *So my overall suggestions here are major revisions along the lines of: (1) take some time in section 2 to explain a bit more what each of the terms here mean, (2) reformulate the subsequent examples to use more real-world numbers that describe specific sequestration activities: afforestation, deforestation, changed agricultural practices, etc, where the comparison is made between a perturbed and unperturbed ecosystem; (3) concentrate less on the long-term dynamics and more on the comparison of unperturbed vs perturbed ecosystem changes over discrete and policy-relevant time horizons.*

  We made major revisions following these recommendations. In particular,

1) we expanded section 2 to better explain the terms of the equations and their potential use. Also, we made an effort to better explain the special cases of the framework and how they can be used for particular problems. 2) We changed considerably the examples to avoid misunderstandings in that we do not provide definite answers to particular questions. Rather, we use examples that show how the framework is used for particular *computations* that could be helpful for some particular problems addressed in the Discussion section.

## Answers to specific comments

- *Section 2.3. I think more detail needs to be given here for how this method works when the linear and/or equilibrium assumptions are not justified*

  More details are giving here and also in section 2.4 on how to apply the concepts to the non-equilibrium case.

- *Sections 3-4. I think some detail is needed on what exactly the proposed sequestration that is being modeled here is. It seems like the sequestration proposed here is to create a new ecosystem where none existed previously, so that x(t=0) = 0 and u is being changed from zero to some global-mean value. But a typical sequestration plan is to afforest a given patch of ground, i.e. converting from grassland or crops to forest. How would such a transition, where the change is to both the u vector and B matrix when x(t=0) != 0, be calculated?*

  The analyses presented in sections 3 and 4 are steady-state analyses and not an afforestation case. The idea here is to show what is the climate benefit of an amount of carbon taken up by an ecosystem that is already in equilibrium. For example, it can be used to calculate carbon sequestration as an ecosystem service, say an old-growth tropical forest and how much warming is avoided over the long-term. We give now more details about this and explain cases in which these analyses might be justified.

- *Line 245: I disagree with the authors here and think that if they are going to go this route, then they need to justify their decision much more than they do. Of course carbon does return to geologic reservoirs, but the timescale of this is much longer than the 10-1000 year timescale discussed here. So I think, if anything, the Joos et al curves should be used and the Lashof & Aruha ones removed. I'm honestly confused about why the authors would suggest the opposite.*

  We agree with the reviewer and we are aware that this manuscript is not the best place to challenge the IRFs of Joos et al. (2013). However, it is still problematic to use this IRF for long-term and steady-state analyses. The decision of Joos et al. (2013) to introduce an intercept term with an infinity timescale in their IRFs is not well addressed in their manuscript. This is something not relevant for many analyses focused on policy-relevant timescales, but it is relevant for steady-state analyses as we showed in the previous version of the manuscript. More recently, Millar et al. (2017) addressed this issue by simply assigning a timescale of 1 million years to the proportion of carbon that in the IRFs of Joos et al. (2013) had an infinity timescale. We decided to follow the same approach as Millar et al. (2017), still using the same timescales of Joos et al. (2013), but avoiding entering in a discussion of appropriate timescales for geologically stabilized carbon. The IRF of Lashof and Ahuja (1990) was removed from the manuscript as suggested by the reviewer.

- *Line 265: I disagree with the idea that changing u will, in general, not lead to a change in B. I think there is quite a bit of evidence (forest self-thinning, soil carbon saturation) that B is highly sensitive to changes in u in real ecosystems. This comes up again a few paragraphs later. While it is mathematically convenient to separate these two things, I think in general it is not really possible to change without the other (nor a priori to assert what sign that change to u or B will necessarily be).*

  This is basically the difference between a linear and a nonlinear system, and this is why we make the distinction in our manuscript. In a linear system $u$ and $\mathbf{B}$ are independent by the same definition of linearity. However, in many systems they are not independent as the reviewer points out, and this is a strong indication for nonlinear behavior. However, our intention in this section is to show how CS and CBS behave in the linear case, so when someone makes this assumption is aware of the consequences. We added text in this section making this point clear. We try to show that these results only apply to linear systems in equilibrium, but for more realistic systems this is not the case. We also want to point out that even though the assumptions may be unrealistic, they are still made in many different analyses and for this reason it is important to know what are the consequences of the linearity assumption.

- *Line 293: This is a fairly obvious result and so I'm not sure why this formalism is needed to make that point?*

  Yes, this is an obvious result, but there are policy relevant cases in which this is not so obvious. For example, in discussions of the 4 per mil initiative it is commonly assumed that inputs of carbon to soils can be achieved by increasing C inputs by a proportion of 0.004 of the current soil carbon stocks. However, there is no distinction between increasing C sequestration by management inputs versus management rates, or a combination of both. Therefore, we believe it is still important to clearly show that carbon sequestration can be maximized by managing both. We do not show here any formal optimization analysis, but the idea is that this framework can be used to better pose the maximization problem on formal mathematical grounds. We elaborate now better on this idea in the new version.

- *Section 5. I am not sure I understand the point of this example, and I also think there is a conceptual error being made here when the method* *is applied to large (relative to the total biospheric fluxes) sequestration perturbations: as I understand the notation used here, the function h_a(t) represents the remaining pulse (positive or negative) of CO2 into the atmosphere. But much (âĹij50%) of the loss of that atmospheric concentration pulse is due to the beta effect of land carbon responding to the carbon that has been emitted. So it seems like you are double counting this biospheric response, as it appears in both h_a(t) and in r(t)? I suspect this whole approach only works for small perturbations to the biosphere, where h_a(t) and r(t) are approximately non-overlapping, and thus excludes this example here.*

The idea behind this example was simply to show that the CBS and the CS metrics can be computed for a time-dependent situation. The reviewer is correct in that for a large perturbation, there is potential for double counting because the atmospheric response already includes biospheric effects. For a correct computation of the time-dependent response of the atmosphere to large biospheric perturbations, a time-dependent response function $h_a(t_0, t - t_0)$ obtained directly from the particular simulation should be used. This function should exclude the effects of the biosphere and only include carbon removal from ocean sinks. However, since our aim is simply to show how to compute CBS for the time-dependent case, we decided to remove this example. We use now a different model that works at the ecosystem level and not at the biosphere level. With this model we show now how to compute the proposed metrics for systems out of equilibrium within the limit of a small perturbation, so we can still use a constant atmospheric response function.

- *Table 1. Where are these numbers coming from? It seems like the authors are just sort of making them up as heuristic examples. Is that the case, and if so, might it be more useful to use numbers based on real-world, even if highly simplified, examples? Similarly, the u and B numbers from Emanuel et al (1980) are for a globally-averaged ecosystem? If so, I think this wouldn't make sense for this example and you would have to use examples for a specific forest ecosystem instead. I understand the intention here is to be heuristic but more realistic numbers shouldn't be too hard to track down and it'd be infomrative to try to do something that corresponds more to reality when talking about concrete examples such as this.*

  These are indeed heuristic values, only for showing the consequences of different types of biospheric carbon management. However, for the reasons mentioned earlier, we decided to remove this example.

- *Line 395. Can they? I see how albedo could, but other surface energy terms or surface roughness imply a tradeoff of one or another type of energy, or redistributions of energy between the land and atmosphere, and so can't really be compared to this metric.*

  They cannot be compared directly, and we only claim that they are in 'units more comparable to those used to assess the overall effect of forest on climate'. The point is that values of CBS in units of W $m^{-2}$ yr may be easier to relate to energy balance terms than GWP values, which are reported in $CO_2$-equivalents.

- *Section 7: I'm not totally sure how comparing the CBS metric of two extremely vintage ecosystem models (one of which is a global-mean number and the other is a sort of reference-temperature number, so not really comparable even) is really of any importance to the argument being made in this manuscript, or anything else really. If the point is just that soils store a lot of carbon for a long time, don't we already know that? Suggest substantially revising or deleting this section.*

  This section/example was removed as recommended by the reviewer.

- *Line 476-484. The other (much larger, really) problem with GWP is that its utility completely depends on what time interval the metric is integrated over; hence the unending debates about how much policy should focus on CH4 as compared to CO2. This problem applies equally to the CBS, but is completely skipped over in this paper. How would the CBS be used in a policy-relevant context where we care about limiting peak temperature at some time period? Are there sequestration methods that are positive at a 50-year time horizon but negative on shorter or longer timescales? Is there a CBS analogue for GWP-star? Exploring these question would seem to be central to how this metric would actually be used in practice, but isn't actually touched on at all in this manuscript. I think this is a mistake and a major shortcoming of the current manuscript.*

  Thanks for this suggestion. There has been a lot of debate on the time horizon for integration in GWPs, and this is indeed problematic for the case of emissions. But for the case of sequestration, it is an advantage to consider a finite time period for assessing sequestration. This is basically the problem of Permanence in the carbon accounting literature, where it is clear that sequestrations of carbon cannot be considered as permanent. To address this topic we decided to add an example with differences in integration time to show that different conclusions could be obtained by comparing systems at different integration times. We also added a section in the Discussion on this topic.

- *Lines 501-509. These are really not trivial problems, and substantially degrade the utility of this metric. The criticism of the Joos et al model strikes me as wildly off base; the irreversibility of global warming on shorter than multi-millenial timescales is a core feature of the problem and so asserting it away as something that can be ignored is not a good idea. In principle, the uncertainty in the impulse response function would be the same if used for two separate treatments, i.e. a baseline and a perturbed ecosystem, thus it seems like the more useful application of this method would be as an analog to GWP (not AGWP): calculate CBS of both a directly-perturbed ecosystem and an unperturbed ecosystem (or relatively unperturbed, i.e. not* *logged or afforested or whatever the treatment is, but still subject to CO2 fertilization, changes in climate-driven mortality, etc), recognizing that, in a globally changed world, neither will likely be at steady state, and calculate a relative CBS as the ratio of the two absolute CBS.*

Again, we reconsidered this point and believe this manuscript is not the right venue to challenge the IRFs of Joos et al. (2013), so we removed this text from the manuscript. Instead, we take the same simple approach of replacing the infinity time scale in Joos' IRF and replaced it by a 1 million year timescale as in Millar et al. (2017). This removes the mathematical problem of finding a limit to our integrals and has no practical consequence for policy relevant timescales. We do appreciate the suggestion of the Reviewer about computing a relative metric to compare CBS for two cases, e.g. a perturbed and unperturbed system. We did something very similar in the forestry examples in the previous version of the manuscript, but instead of computing a ratio we computed a difference between the two CBS values. In the new version of the manuscript, we added now a computation of the ratio of CBS between two systems, and added a discussion about how one could use this ratio for problems similar as in the use of GWPs.

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# The Climate Benefit of Carbon Sequestration

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**Abstract.** Ecosystems play a fundamental role in climate change mitigation by ~~taking up~~ photosynthetically fixing carbon from the atmosphere and storing it for a period of time in organic matter. Although climate impacts of carbon emissions by sources can be quantified by global warming potentials, ~~it is not necessarily clear what are~~ the appropriate formal metrics to assess climate benefits of carbon removals by sinks are unclear. We introduce here the Climate Benefit of Sequestration (CBS), a metric that quantifies the radiative effect of ~~taking up~~ fixing carbon dioxide from the atmosphere and retaining it for a period of time in an ecosystem before releasing it back ~~to the atmosphere. To~~ as the result of respiratory processes. In order to quantify CBS, we ~~also propose~~ present a formal definition of carbon sequestration (CS) as the integral of an amount of carbon ~~taken up~~ removed from the atmosphere stored over the time horizon it remains ~~in~~ within an ecosystem. Both metrics incorporate the separate effects of i) inputs (amount of atmospheric carbon removal), and ii) transit time (time of carbon retention) ~~in~~ on carbon sinks, which can vary largely for different ecosystems or ~~management types. In three separate examples, we show how to compute and apply these metrics to compare different carbon management practices in forestry and soils. We believe these metrics~~ forms of management. These metrics can be useful ~~in resolving current controversies about the management of ecosystems for climate change mitigation~~for comparing the climate impacts of carbon removals by different sinks over specific time horizons, to assess the climate impacts of ecosystem management, and to obtain direct quantifications of climate impacts as the net effect of carbon emissions by sources versus removals by sinks.

## 1 Introduction

Terrestrial ecosystems exchange carbon with the atmosphere at globally significant quantities, thereby influencing Earth's climate and potentially mitigating warming caused by increasing concentrations of $CO_2$ in the atmosphere. Carbon ~~taken up~~ fixed during the process of photosynthesis remains stored in the terrestrial biosphere over a range of timescales, from days to millennia; timescales ~~comparable with those of~~ of relevance for affecting the concentration of greenhouse gases in the atmosphere (Archer et al., 2009; IPCC, 2014; Joos et al., 2013). During the time carbon is stored in the terrestrial biosphere, it is removed from the radiative forcing effect that occurs in the atmosphere; thus, it is of scientific and policy relevance to understand the timescale of carbon storage in ecosystems; i.e., for how long newly fixed carbon is retained in an ecosystem before it is released back to the atmosphere.

Timescales of element cycling and storage are unambiguously characterized by the concepts of *system age* and *transit time* (Bolin and Rodhe, 1973; Rodhe, 2000; Rasmussen et al., 2016; Sierra et al., 2017; Lu et al., 2018). In a system of multiple interconnected compartments, system age characterizes the time that the mass of an element observed in the system has remained there since its entry. Transit time characterizes the time that it takes element masses to traverse the entire system, from the time of entry until they are released back to the external environment (Sierra et al., 2017). Both metrics are excellent system-level diagnostics of the dynamics and timescales of ecosystem processes~~; and since they~~. Because system age and transit time both can be reported as mass- or probability distributions, they provide ~~information~~ different information about an ecosystem over a wide range in the time domain.

System age and transit time are closely related to the complexity of the ecosystem and its process rates, which are affected by the environment (Luo et al., 2017; Rasmussen et al., 2016; Sierra et al., 2017; Lu et al., 2018). ~~In ecosystems, mean transit times of carbon seem to be much lower than mean system ages (Sierra et al., 2018b)~~Mean system ages are consistently greater than mean transit time (Lu et al., 2018; Sierra et al., 2018b), suggesting that once a mass of carbon enters an ecosystem, a large proportion gets quickly released back to the atmosphere, but a small proportion ~~can remain~~ remains for very long times. Furthermore, differences in transit times across ecosystems suggest that not all carbon sequestered in the terrestrial biosphere spends the same amount of time stored; e.g., one unit of ~~phtosynthesized~~ photosynthesized carbon is returned back to the atmosphere faster in a tropical than in a boreal forest (Lu et al., 2018). Therefore, not all carbon ~~taken up~~ drawn down from the atmosphere should be treated equally for the purpose of quantifying the climate mitigation potential of sequestering carbon in ecosystems ~~.~~ as it is currently recommended in accounting methodologies (IPCC, 2006).

~~Transit time distributions in ecosystems can inform us about the time newly sequestered carbon will be removed from radiative effects. This is in contrast to global~~ Global warming potentials (GWPs~~), which~~ , see definition in section 2) quantify the radiative effects of greenhouse gases emitted to the atmosphere (Fig. 1), but do not consider the avoided radiative effect of storing carbon in ecosystems (Neubauer and Megonigal, 2015). GWPs are computed using the age distribution of $CO_2$ and other greenhouse gases in the atmosphere (Rodhe, 1990; Joos et al., 2013), but do not consider age or transit times of carbon in ecosystems ~~for~~ in the case of sequestration. Transit time distributions, in particular, can better inform us about the time newly sequestered carbon will be removed from radiative effects in the atmosphere.

For more comprehensive accounting of the contribution of carbon sequestration to climate change mitigation, it is necessary to quantify the avoided warming effects of sequestered carbon in ecosystems over the timescale the carbon is stored. The GWP metric is inappropriate to quantify avoided warming potential as a result of sequestration. A metric that can capture this avoided warming effect could have applications for 1) comparing different carbon sequestration ~~proposals considering differences in~~ activities considering the time carbon is stored in ~~different proposed options~~ecosystems, and 2) ~~better accounting~~ providing better accounting methods for the effect of removals by sinks in climate policy. Currently, the Intergovernmental Panel on Climate Change (IPCC) recommends countries and project developers to report only emissions by sources and removals by sinks of greenhouse gases (GHGs), treating all removals equally in terms of their fate (IPCC, 2006). ~~In other words, all carbon taken up from the atmosphere is treated equally despite current evidence of the contrary from transit time estimations.~~

~~It has been previously recognized that GWPs have problems when applied~~ Problems with applying GWPs to compute climate benefits of sequestering carbon in ecosystems are well documented (Moura Costa and Wilson, 2000; Fearnside et al., 2000; Brandão et al., 2013; Neubauer and Megonigal, 2015)~~, and several~~. Several approaches have been proposed to deal with the issue of timescales (Brandão et al., 2013)~~. While most of these approaches~~, many of which deal with time as some form of delay in emissions. However, to our knowledge, ~~none of them explicitly account~~ no solution proposed thus far explicitly accounts for the time carbon is sequestered in ecosystems, ~~since~~ from the time of ~~carbon fixation during photosynthesis~~ photosynthetic carbon fixation until it is returned back to the atmosphere by ~~respiratory processes of autotrophs and heterotrophs~~autotrophic and heterotrophic respiration.

~~The~~ Therefore, the main objective of this manuscript is to introduce a metric to assess the climate benefits of carbon sequestration ~~in ecosystems~~ while accounting for the time carbon is stored ~~, and to provide examples on how to use this framework for different problems of land use and ecosystem carbon management in the context of climate change mitigation~~in ecosystems. We first present the theoretical framework for the development of the metric, ~~and then provide three different~~ then provide simple examples for its ~~use in ecosystem management decisions~~ computation and discuss potential applications for ecosystem management and for climate change mitigation.

## 2 Theoretical framework

### 2.1 Absolute Global Warming Potential AWGP

The direction of carbon flow, into or out of ecosystems, is of fundamental importance to understand and quantify their contribution to climate change mitigation. The absolute global warming potential (AGWP) of carbon dioxide quantifies the radiative effects of a unit of $CO_2$ emitted to the atmosphere during its life time; in the direction land $\rightarrow$ atmosphere. It is expressed as (Lashof and Ahuja, 1990; Rodhe, 1990)

$$\mathrm{AGWP}(T, t_0) = \int_{t_0}^{t_0+T} k_{\mathrm{CO_2}} \, M_a(t) \, \mathrm{d}t \tag{1}$$

where $k_{\mathrm{CO_2}}$ is the radiative efficiency or greenhouse effect of one unit of ~~carbon dioxide~~ $CO_2$ (in mole or mass) in the atmosphere, and $M_a(t)$ is the amount of gas remaining in the atmosphere after some time $t$ (Rodhe, 1990; Joos et al., 2013). The AGWP quantifies the amount of warming produced by $CO_2$ while it stays in the atmosphere since the time the gas is emitted at time $t_0$ over a time horizon $T$. The function $M_a(t)$ quantifies the fate of the emitted carbon in the atmosphere and can be written in general form as

$$M_a(t) = h_a(t - t_0) M_a(t_0) + \int_{t_0}^{t} h_a(t - \tau) Q(\tau) \, \mathrm{d}\tau, \tag{2}$$

where $h_a(t-t_0)$ is the impulse response ~~or Green's~~ function of atmospheric $CO_2$ released into the atmosphere; $M_a(t_0)$ is the content of atmospheric $CO_2$ at time $t_0$, and $Q(\tau)$ is the perturbation of new incoming carbon to the atmosphere between $t_0$ and $t$.

For a pulse, or instantaneous emission of ~~carbon dioxide $M_a(t_0)=E_0$,~~ $CO_2$, $M_a(t_0)=E_0$, and

$$M_a(t) = h_a(t-t_0)E_0, \tag{3}$$

assuming no additional carbon ~~is entering~~ enters the atmosphere after the pulse. ~~In case~~ If the pulse is equivalent to 1 kg or mole of $CO_2$, then $E_0 = 1$ and $M_a(t) = h_a(t-t_0)$. For a pulse emission of any arbitrary size,

$$\mathrm{AGWP}(T, E_0, t_0) = k_{\mathrm{CO_2}}\, E_0 \int\limits_{t_0}^{t_0+T} h_a(t-t_0)\,\mathrm{d}t. \tag{4}$$

The AGWP can be computed for any other greenhouse gas using their respective radiative efficiencies and fate in the atmosphere (~~Green's~~ impulse response function). To compare different gases, the Global Warming Potential (GWP) is defined as the AGWP of a particular gas divided by the AGWP of ~~carbon dioxide (Shine et al., 1990; Lashof and Ahuja, 1990). Since our~~ $CO_2$ (Shine et al., 1990; Lashof and Ahuja, 1990). Our interest in this manuscript is on carbon fixation and respiration in the form $CO_2$, ~~we concentrate exclusively in~~ therefore we primarily concentrate here on AGWP.

The impulse response function $h_a(t-t_0)$ plays a central role within the AGWP framework. The function encodes information about the fate of a gas once it enters the atmosphere and determines for how long the gas will remain. Therefore, it can be interpreted as a density distribution for the the transit time of a gas, since the time of emission until it is removed by natural sinks (e.g. $CO_2$) or by chemical reactions (e.g. $CH_4$).

The function typically is assumed to be static, i.e. the time at which the gas enters the atmosphere is not relevant, only the time it remains there $(t-t_0)$. However, this function can be time-dependent, expressing different shapes depending on the time the gas enters the atmosphere, i.e. $h_a(t_0, t-t_0)$. For example, when natural sinks saturate, faster accumulation of $CO_2$ and longer transit times of carbon in the atmosphere would be observed (Metzler et al., 2018). In this situation, the specific time of an emission would lead to different response functions in the atmosphere. Because current research on impulse response functions primarily considers the static time-independent case (see Millar et al., 2017, for an exception), we will consider only the static case for the remainder of this manuscript.

### 2.2 The radiative efficiency of $CO_2$ and its impulse response function

The radiative efficiency of $CO_2$ is a function of the concentration of this gas and the concentration of other gases in the atmosphere with overlapping absorption bands (Lashof and Ahuja, 1990; Shine et al., 1990). Therefore, $k_{\mathrm{CO_2}}$ changes as the concentration of GHGs change in the atmosphere. For most applications however, the radiative efficiency of $CO_2$ has been assumed constant in the limit of a small perturbation at a specific background concentration (Lashof and Ahuja, 1990; Shine et al., 1990; Jo

Here, we use a constant value of $k_{\mathrm{CO_2}} = 6.48 \times 10^{-12}$ W m$^{-2}$ MgC$^{-1}$ based on results reported by Joos et al. (2013) for an atmospheric background of 389 ppm ($\sim$ present day). This radiative efficiency represents the change in radiative forcing caused by a change of 1 Mg of carbon in the atmosphere in the form of $CO_2$ in units of rate of energy transfer (Watt) per square meter of surface.

Joos et al. (2013) have also derived impulse response functions (IRFs) of $CO_2$ in the atmosphere using coupled carbon-climate models that include multiple feedbacks among Earth system processes. One function was obtained by emitting a pulse of 100 GtC to a 'pre-industrial' atmosphere with a background concentration of 280 ppm (PI100 function from here on), and another function was obtained by emitting 100 GtC to a 'present day' atmosphere with a background of 389 ppm (PD100 from here on). The functions they report are averages from the numerical output of multiple models fitted to a sum of exponential functions that include an intercept term. This intercept implies that a proportion of the added $CO_2$ never leaves from the atmosphere-ocean-terrestrial system to long-term geological reservoirs. Following Millar et al. (2017), we added a timescale of 1 million years that corresponds to the intercept term in the IRFs. The addition of this timescale has no effect on the results presented here, which are focused on much shorter timescales, but they avoid the mathematical problem that the integrals of the original functions go to infinity with time (Lashof and Ahuja, 1990; Millar et al., 2017).

### 2.3 Carbon sequestration CS, and the climate benefit of carbon sequestration CBS

GWPs are useful to quantify the climate impacts of increasing or reducing emissions of GHGs to the atmosphere. However, it is also necessary to quantify the climate benefits of carbon flows in the opposite direction, atmosphere $\rightarrow$ land. Furthermore, it is also important to quantify not only how much and how fast carbon enters ecosystems, but also for how long the carbon stays (Körner, 2017).

Carbon taken up from the atmosphere through the process of photosynthesis is stored in multiple ecosystem reservoirs for a particular amount of time. Carbon sequestration can be defined as the process of capture and long-term storage of $CO_2$ (Sedjo and Sohngen, 2012). We define here carbon sequestration CS over a time horizon $T$ as

$$\mathrm{CS}(T, S_0, t_0) := \int_{t_0}^{t_0+T} M_s(t - t_0)\,\mathrm{d}t, \tag{5}$$

where $M_s(t - t_0)$ represents the fate of ~~carbon in~~ a certain amount of carbon $S_0$ taken up by the sequestering system ~~, and $S_0$ is the amount of fixed carbon~~at a time $t_0$. Notice that this definition of carbon sequestration is very similar to that of AGWP for an emission, with the exception that the radiative efficiency term is omitted.

To obtain the fate of sequestered carbon over time, we represent carbon cycling and storage in ecosystems using the theory of compartmental dynamical systems (Luo et al., 2017; Sierra et al., 2018a). In their most general form, we can write carbon cycle models as

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \dot{x}(t) = u(x,t) + \mathbf{B}(x,t)\,x, \tag{6}$$

where $x(t) \in \mathbb{R}^n$ is a vector of $n$ ecosystem carbon pools, $u(x,t) \in \mathbb{R}^n$ is a time-dependent vector-valued function of carbon inputs to the system, and $\mathbf{B}(x,t) \in \mathbb{R}^{n \times n}$ is a time-dependent compartmental matrix. The latter two terms can depend on the vector of states, in which case the compartmental system is considered nonlinear. In case the input vector and the compartmental matrix have fixed coefficients (no time-dependencies), the system is considered autonomous, and non-autonomous otherwise (Sierra et al., 2018a). ~~Models expressed as an autonomous linear system~~ This distinction of models with respect to linearity and time-dependencies (autonomy) is fundamental to distinguish important properties of models. For instance, models expressed as autonomous linear systems have a steady-state solution given by $x^* = -\mathbf{B}^{-1}u$, where $x^*$ is a vector of steady-state contents for all ecosystem pools. Non-autonomous models have no steady-state solution.

The fate of the fixed carbon for the general non-autonomous case can be obtained as

$$M_s(t-t_0) = \|\mathbf{\Phi}(t,t_0)\beta(t_0)S_0\|, \tag{7}$$

where $\beta(t_0)S_0 = u(t_0)$, and $\beta(t_0)$ is an $n$-dimension vector representing the partitioning of the total sequestered carbon among $n$ ecosystem carbon pools (Ceballos-Núñez et al., 2020). The $n \times n$ matrix $\mathbf{\Phi}(t,t_0)$ is the state transition operator, which represents the dynamics of how carbon moves in a system of multiple interconnected compartments (see details in appendix). Throughout this document, we use the symbol $\|\ \|$ to denote the 1-norm of a vector, i.e. the sum of the absolute values of all elements in a vector.

Because ecosystems and most reservoirs are open systems, ~~carbon~~ the sequestered carbon $S_0$ returns back to the atmosphere (mostly as ecosystem respiration ~~Re~~$r(t)$) according to

$$r(t) = -\mathbf{1}^\intercal\,\mathbf{B}(t)\mathbf{\Phi}(t,t_0)\beta(t_0)S_0, \tag{8}$$

where $\mathbf{1}^\intercal$ is the transpose of the $n$-dimensional vector containing only 1s.

The link between the time it takes sequestered carbon $S_0$ to appear in the output flux $r(t)$ is established by the concept of transit time (Metzler et al., 2018). In particular, we define the forward transit time (FTT) as the age that fixed carbon will have at the time it is released back to the atmosphere, or, how long a mass fixed now will stay in the system. The backward transit time (BTT) is defined as the age of the carbon in the output flux since the time it was fixed, or, how long the mas leaving the system now had stayed. This implies that

$$r(t) = p_{\mathrm{BTT}}(t-t_0,t) = p_{\mathrm{FTT}}(t-t_0,t_0), \tag{9}$$

where $p_{\mathrm{BTT}}(t-t_0,t)$ is the backward transit time distribution of carbon leaving the system at time $t$ with an age $t-t_0$, while $p_{\mathrm{FTT}}(t-t_0,t_0)$ is the forward transit time distribution of carbon entering the system at time $t_0$ and leaving with an age $t-t_0$. For systems in equilibrium, both quantities are equal (Metzler et al., 2018). ~~Semi-explicit formulas for these~~ For systems not in equilibrium, semi-explicit formulas for their distributions are given in the appendix.

For the atmosphere, carbon sequestration is a form of 'negative emission', and we can represent its fate in the atmosphere as

$$M'_a(t) = -h_a(t-t_0)S_0 + \int_{t_0}^{t} h_a(t-\tau)r(\tau)\,\mathrm{d}\tau, \tag{10}$$

where the prime symbol represents a perturbed atmosphere as an effect of sequestration. The first term in the rhs represents the response of the atmosphere to an instantaneous sequestration $S_0$ at $t_0$, and the second term represents the perturbation in the atmosphere of the carbon returning back from the terrestrial biosphere. Notice that the integral in this equation can be written as a convolution $(h_a \star r)(t)$ between the impulse response function of atmospheric $CO_2$ and the respired carbon returning from ecosystems to the atmosphere.

We define now the climate benefit of sequestration for a pulse of $CO_2$ into an ecosystem as

$$\begin{aligned} \mathrm{CBS}(T, S_0, t_0) &:= \int_{t_0}^{t_0+T} k_{\mathrm{CO}_2} M_a'(t)\,\mathrm{d}t, \\ &= -k_{\mathrm{CO}_2} \int_{t_0}^{t_0+T} \left(h_a(t-t_0)S_0 - (h_a \star r)(t)\right)\mathrm{d}t. \end{aligned} \tag{11}$$

This metric integrates over a time horizon $T$ the radiative effect avoided by sequestration of ~~carbon in an ecosystemduring the time it is stored , and takes into account the subsequent return of the gas to the atmosphere~~an amount of carbon $S_0$ taken up at time $t_0$ by an ecosystem. It captures the timescale at which the carbon is stored and gradually returns back to the atmosphere. It can also be interpreted as the atmospheric response to carbon sequestration in the form of a negative emission of $CO_2$ during a time horizon of interest. It relies on knowledge of the atmospheric response to perturbations in the form of an impulse response function, and the transit time of carbon in an ecosystem.

### 2.4 ~~Carbon sequestration~~ Ecosystems in ~~linear systems at~~ equilibrium: the linear, steady-state case

The computation of CS and CBS is simplified for systems in equilibrium. For linear systems at steady-state, the time at which the carbon enters the ecosystem is irrelevant (Kloeden and Rasmussen, 2011; Rasmussen et al., 2016); one only needs to know for how long the carbon has been in the system to predict how much of it remains. Mathematically, this implies

$$\mathbf{\Phi}(t, t_0) = e^{a \cdot \mathbf{B}} \quad \text{for all} \quad t_0 \le t \quad \text{and} \quad a = t - t_0. \tag{12}$$

Therefore, for linear systems at steady state, we have the special cases

$$M_s(a) = \|e^{a \cdot \mathbf{B}}\, u\|, \tag{13}$$

and

$$M_{s1}(a) = \left\| e^{a \cdot \mathbf{B}} \frac{u}{\|u\|} \right\|, \tag{14}$$

where $M_{s1}$ represents the fate of one unit of fixed carbon, which can also be interpreted as the proportion of carbon remaining after the time of fixation~~$(a = t - t_0)$~~.

The amount of released carbon returning to the atmosphere is therefore

$$r(a) = -\mathbf{1}^{\intercal}\, \mathbf{B}\, e^{a \cdot \mathbf{B}}\, u, \tag{15}$$

which for one unit of fixed carbon is equal to the transit time density distribution $f(\tau)$ of a linear system (Metzler and Sierra, 2018, see also appendix)

$$r_1(a) = -\mathbf{1}^{\intercal}\, \mathbf{B}\, e^{a \cdot \mathbf{B}} \frac{u}{\|u\|}. \tag{16}$$

where $r_1(a) = f(\tau)$, with mean (expected value) transit time given by

$$\mathbb{E}(\tau) = -1^{\intercal}\,\mathbf{B}^{-1}\,\frac{u}{\|u\|} = \frac{\|x^*\|}{\|u\|}. \tag{17}$$

We can now derive the steady-state expression of CS as

$$\mathrm{CS}(T) = \int_0^T \|e^{a\cdot\mathbf{B}}\,u\|\,\mathrm{d}a. \tag{18}$$

Furthermore, it is possible to find a closed-form expression for this integral

$$\mathrm{CS}(T) = \|\mathbf{B}^{-1}\left(e^{T\cdot\mathbf{B}} - \mathbf{I}\right)u\|, \tag{19}$$

where $\mathbf{I} \in \mathbb{R}^{n\times n}$ is the identity matrix. Similarly, for one unit of carbon entering a steady-state system at any time, we define $\mathrm{CS}_1$ as

$$\mathrm{CS}_1(T) = \int_0^T \left\|e^{a\cdot\mathbf{B}}\,\frac{u}{\|u\|}\right\|\,\mathrm{d}a, \tag{20}$$

which by integration gives

$$\mathrm{CS}_1(T) = \left\|\mathbf{B}^{-1}\left(e^{T\cdot\mathbf{B}} - \mathbf{I}\right)\frac{u}{\|u\|}\right\|. \tag{21}$$

These steady-state expressions can be very useful to compare different systems or changes to a particular system if the steady-state assumption is justified. Furthermore, it can be shown that in the long term, as the time horizon $T$ goes to infinity ($\infty$), the term $(e^{T\cdot\mathbf{B}} - \mathbf{I})$ converges to $-\mathbf{I}$, and therefore equation (19) converges to the expression

$$\lim_{T\to\infty} \mathrm{CS}(T) = \|x^*\|, \tag{22}$$

which means that the total amount of carbon at steady-state is equal to the long-term carbon sequestration of an instantaneous amount of fixed carbon at an arbitrary time.

Similarly, for one unit of carbon entering a system at steady-state, the long-term $\mathrm{CS}_1$ from equation (21) can be obtained simply as

$$\lim_{T\to\infty} \mathrm{CS}_1(T) = \mathbb{E}(\tau), \tag{23}$$

by using the definition of mean transit time of equation (17). This means that long-term sequestration of one unit of $CO_2$ converges to the mean transit time of carbon in an ecosystem.

## 2.5 ~~From instantaneous to~~ Dynamic ecosystems out of equilibrium: the continuous ~~fluxes~~sequestration and emissions case

In addition of considering isolated pulses of emissions $E_0$ or sequestrations $S_0$, we can also consider permanently ongoing emissions $e : t \mapsto E(t)$ and sequestration $s : t \mapsto S(t)$, respectively. Hence,

$$\mathrm{CS}(T, s, t_0) := \int_{t_0}^{t_0+T} M_s(t)\,\mathrm{d}t, \tag{24}$$

where

$$M_s(t) = \int_{t_0}^{t} \|\boldsymbol{\Phi}(t,\tau)\,\beta(\tau)\,s(\tau)\|\,\mathrm{d}\tau. \tag{25}$$

Here $s(\tau)$ is a scalar flux of sequestration at time $\tau$. This leads to

$$r(t) = -\mathbf{1}^{\mathsf{T}}\,\mathbf{B}(t)\int_{t_0}^{t} \boldsymbol{\Phi}(t,\tau)\,\beta(\tau)\,s(\tau)\,\mathrm{d}\tau. \tag{26}$$

The fate of sequestered carbon, for the atmosphere in the form of a balance between simultaneous sequestration and return of carbon, can now be obtained as

$$\begin{aligned} M_a'(t) &= -\int_{t_0}^{t} h_a(t-\tau)\,s(\tau)\,\mathrm{d}\tau + \int_{t_0}^{t} h_a(t-\tau) r(\tau)\,\mathrm{d}\tau \\ &= -\int_{t_0}^{t} h_a(t-\tau)\,[s(\tau) - r(\tau)]\,\mathrm{d}\tau \\ &= -(h_a \star (s-r))(t). \end{aligned} \tag{27}$$

We can now define the climate benefit of sequestration for a dynamic ecosystem with continuous sequestration and respiration as

$$\begin{aligned} \mathrm{CBS}(T, s, t_0) &:= \int_{t_0}^{t_0+T} k_{\mathrm{CO_2}} M_a'(t)\,\mathrm{d}t, \\ &= -k_{\mathrm{CO_2}} \int_{t_0}^{t_0+T} (h_a \star (s-r))(t)\,\mathrm{d}t. \end{aligned} \tag{28}$$

This expression of CBS accounts for the dynamic behavior of inputs and outputs of carbon in ecosystems, and can be used to represent time-dependencies resulting from environmental changes, disturbances, or produced by emission scenarios or scheduled management activities. This time-dependent CBS is computed for a time horizon $T$ starting at any initial time $t_0$. In other words, it can be used to analyze specific time windows of interest, accounting for the fate of all carbon sequestered during specific time intervals.

### 2.6 ~~The radiative efficiency $k_{CO_2}$ of carbon dioxide and its impulse response function $h_a(t)$~~

~~The radiative efficiency of carbon dioxide is a function of the concentration of this gas and the concentration of other gases in the atmosphere with overlapping absorption bands (Lashof and Ahuja, 1990; Shine et al., 1990). Therefore, $k_{CO_2}$ changes as the concentration of GHGs change in the atmosphere. Time-dependent radiative efficiencies $k_g(t)$ of a gas $g$ can be considered in the calculation of CBS, which would imply a numerical integration of the time-dependent integrals presented above. For most applications however, the radiative efficiency of $CO_2$ has been assumed constant in the limit of a small perturbation at a specific background concentration (Lashof and Ahuja, 1990; Shine et al., 1990; Joos et al., 2013; Myhre et al., 2013).~~

~~Here, we use a constant value of $k_{CO_2} = 6.48 \times 10^{-3}$ W m$^{-2}$ PgC$^{-1}$ based on results reported by Joos et al. (2013) for an atmospheric background of 389 ppm ($\sim$ present day). This radiative efficiency represents the change in radiative forcing caused by a change of 1 Pg of carbon in the atmosphere in the form of carbon dioxide in units of rate of energy transfer (Watt) per square meter of surface.~~

~~Joos et al. (2013) have also derived impulse response functions of carbon dioxide in the atmosphere using coupled carbon-climate models (Figure **??**). One function was derived for a pre-industrial atmosphere with 280 ppm and another for a present day atmosphere with 389 ppm. The functions they report are averages for multiple models fitted to a sum of exponential functions that include an intercept term. This intercept implies that a proportion of the added carbon dioxide never leaves from the atmosphere-ocean-terrestrial system to long-term geological reservoirs. An alternative function was proposed by Lashof and Ahuja (1990) that omits the intercept term (Figure **??**). We evaluate these different functions for the purpose of this manuscript.~~

~~Impulse response function of carbon dioxide in the atmosphere ($h_a(t)$) as predicted by different authors. Joos PD100 represents the impulse response experiment reported in Joos et al. (2013) for a present day (PD) atmosphere with a 100 PgC emission pulse. Joos PI100 represents an impulse response experiment from the same study using a pulse of 100 PgC in a pre-industrial (PI) atmosphere. In addition, we include the impulse response curve reported in Lashof and Ahuja (1990).~~

## 3 ~~Example 1: the climate benefit of carbon sequestration in the pre-industrial biosphere~~

~~We will show now~~

## 3 Example 1: CS and CBS for linear systems in equilibrium

### 3.1 The fate of a pulse of inputs through the system

A simple ecosystem carbon model, the Terrestrial Ecosystem Model (TECO), will now demonstrate an application of the theory ~~using a simple global carbon model~~ to compute CS and CBS assuming a ~~terrestrial biosphere in~~ linear system at steady-state ~~. The model was initially developed by Emanuel et al. (1981) and contains five~~ (i.e., in equilibrium). The TECO model is described by Weng and Luo (2011) with parameter values obtained through data assimilation using observations from the Duke forest in North Carolina, USA. It contains eight main compartments: ~~non-woody tree parts~~ foliage $x_1$, woody ~~tree parts~~ biomass $x_2$, ~~ground vegetation~~ fine roots $x_3$, ~~detritus/decomposers~~ metabolic litter $x_4$, ~~and active soil carbon~~ structural litter $x_5$, fast soil organic matter (SOM) $x_6$, slow SOM $x_7$, and passive SOM $x_8$ (Figure 2). In addition to its simplicity and tractability, there are two advantages of using this model over others: 1) it provides reasonable ~~values of carbon stocks and fluxes for a pre-industrial biosphere~~predictions of net ecosystem carbon fluxes and biometric pool data (Weng and Luo, 2011), 2) ~~its impulse response function and distributions of system age and transit time have been studied previously (Emanuel et al., 1981; Thompson and Randerson, 199~~ is commonly used to express complex ecosystem-level concepts such as the matrix generalization of carbon cycle models, their traceability, and transient behavior (e.g. Luo and Weng, 2011; Luo et al., 2012; Xia et al., 2013; Luo et al., 2017; Sierra, 2019).

The model ~~, expressed as a linear autonomous compartmental system, is given by~~

$$\dot{x} = u + \mathbf{B}\,x,$$

$$= \begin{pmatrix} 77 \\ 0 \\ 36 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -77/37 & 0 & 0 & 0 & 0 \\ 31/37 & -31/452 & 0 & 0 & 0 \\ 0 & 0 & -36/69 & 0 & 0 \\ 21/37 & 15/452 & 12/69 & -48/81 & 0 \\ 0 & 2/452 & 6/69 & 3/81 & -11/1121 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix},$$

~~where mass of~~ is commonly expressed as

$$\frac{d\mathbf{X}(t)}{dt} = \mathbf{b}U(t) + \xi(t)\mathbf{A}\mathbf{C}\mathbf{X}(t), \tag{29}$$

where $\mathbf{X}$ is a vector of ecosystem carbon pools, $\mathbf{C}$ is a diagonal matrix with cycling rates for each pool, $\mathbf{A}$ is a matrix of transfer coefficients among pools, and $\mathbf{b}$ is a vector of allocation coefficients to plant parts. The function $U(t)$ determines the carbon ~~is in units of PgC, and fluxes in units of PgC yr$^{-1}$. Total carbon~~ inputs to the ~~terrestrial biosphere (gross primary production GPP)are 113 PgC yr$^{-1}$~~system as net primary production (NPP), and $\xi(t)$ is a time-dependent function that modifies ecosystem cycling rates according to changes in the environment.

~~Applying equations and , it is possible to observe the fate of the total incoming carbon entering~~ For this steady state example, we assume constant inputs ($U(t) = U$) and constant rates ($\xi(t) = 1$). Furthermore, defining $\mathbf{B} := \mathbf{A}\mathbf{C}$, and $u := \mathbf{b}U$, we can write this model as a linear, autonomous compartmental system of the form

$$\dot{x} = u + \mathbf{B}\,x, \tag{30}$$

with values for $\mathbf{B}$ and $u$ as in Luo et al. (2012) and provided in the appendix.

The fate of a pulse of carbon input entering the ecosystem at an arbitrary time (time-independent) may be observed by applying equations (13) and (14) (Figure 3). Carbon enters the ~~terrestrial biosphere through the non-woody vegetation (leaves) and ground vegetation~~ ecosystem through foliage, wood, and fine root pools. A large proportion of ~~the carbon that enters at any given time is quickly respired or~~ this carbon is quickly transferred from these pools to the ~~woody tree parts, detritus~~

~~and soil. After some decades, most of the remaining carbon is transferred to the active soil carbon compartment where it is eventually respired by microorganisms and~~ fine and metabolic litter pools. Subsequently, the carbon moves to the SOM pools with important respiration losses during these transfers. Most carbon is returned back to the atmosphere with a mean transit time of ~~15.1~~ 35.8 yr for the whole system. Half of the sequestered carbon is returned back to the atmosphere in ~~2.3~~ 14.1 yr, and 95% in ~~74.5~~ 134.5 yr.

~~Carbon sequestration~~Ecosystem-level CS, i.e., the area under the curve of the amount of remaining carbon over time, ~~shows an increasing and asymptotic behavior~~ increases towards an asymptote as the time horizon of integration increases (Figure ~~??~~4a). Here, CS is reported in units of ~~PgC~~ MgC ha$^{-1}$ yr, because this is the amount of carbon retained in organic matter over a fixed time horizon. For relevant time horizons of 50, 100, 500, and 1000 yr, CS was ~~1012.19, 1300.83, 1704.49, and 1711.22 PgC~~ 145.85, 200.70, 236.52, and 237.69 MgC ha$^{-1}$ yr, respectively. In the long-term (i.e., as the time horizon goes to infinity), ~~carbon sequestration~~ CS converges to the steady-state carbon stock predicted by the model of ~~1711.3 PgC~~237.86 MgC ha$^{-1}$.

A similar computation can be made for one unit of fixed carbon ($CS_1$, unitless). In this case $CS_1$ was ~~8.96, 11.51, 15.08, and 15.14~~ 21.96, 30.21, 35.61, and 35.79 yr for time horizons of 50, 100, 500, and 1000 yr, respectively. In the long-term, $CS_1$ converges to the mean transit time of carbon, ~~15.14~~ 35.81 yr (Figure ~~??~~4b).

Due to sequestration at $t_0$, the CBS shows a rapid negative increase in radiative forcing, which decreases as the time horizon increases due to the return of carbon to the atmosphere as an effect of respiration (Figure ~~??~~4c). The shape of the curve however, depends strongly on the ~~impulse response function~~ IRF for atmospheric $CO_2$. CBS is larger over the long-term (~~> 300~~ > 200 yr) for the ~~PI100~~ present day (PD100) curve proposed by Joos et al. (2013) ~~due to its intercept. This means that emitted carbon that never leaves the atmosphere-ocean-terrestrial system can be retained in the biosphere with a fixed benefit over an infinite time horizon. This is in contrast to the impulse response function proposed by Lashof and Ahuja (1990) in which emitted carbon eventually leaves the atmosphere-ocean-terrestrial system and returns to a geological reservoir. The CBS in this case returns to zero in the long-term, which means that the climate benefit of sequestration is temporary and does not last forever. We believe this latter case is more realistic, and therefore~~ than for the pre-industrial curve (PI100). Impulse response functions depend strongly on the magnitude and timing of the pulse (Joos et al., 2013; Millar et al., 2017). Therefore, estimates of climate impacts of emissions (AGWP, Figure 5) and climate benefits to sequestration (CBS, Figure 4c,d) depend strongly on the choice of the IRF. For the purpose of this manuscript, we will use the ~~impulse response function of Lashof and Ahuja (1990)~~ present day curve (PD100) from here on.

~~The AGWP computed with both impulse response functions predicts very large impacts of carbon emissions compared to sequestration (Figure ??d) . Over a 100 year time horizon, the AGWP is above 45~~ Because AGWP and CBS are based on similar concepts and share similar units, it becomes possible to directly compare one another (Figure 6) and obtain an estimate of the climate impact of emissions versus sequestration. This can be done either as the ratio of the absolute value of CBS to AGWP, i.e. | CBS | /AGWP (unitless); or as the net radiative balance CBS+AGWP (W m$^{-2}$ yr~~; and over a 1000 year time horizon, AGWP is above 170 W m$^{-2}$ yr. In contrast, CBS values are never larger (in absolute value)than 4 W m$^{-2}$ yr, suggesting that the effects of emitting~~). It is possible to compute these relations using the CBS for one unit of sequestered carbon, which provides a direct estimate of the impact of one unit of sequestration versus one unit of emission; or corresponding to the amount of NPP sequestered in one year (6.6 MgC ha$^{-1}$ yr$^{-1}$ for Duke forest).

In our example, the emission of 1 ~~PgC to the atmosphere are much more dramatic than the avoided warming produce by the sequestration of 113 PgC in~~ MgC to the atmosphere has a predominant warming effect that cannot be compensated by the sequestration of 1 MgC at the Duke forest (Figure 6). However, the ~~pre-industrial biosphere.~~ sequestration of the equivalent of NPP in one year can have a significant climate benefit compared to the emission of 1MgC, depending on the time horizon of analysis. When one integrates in time horizons lower than 200 years, CBS outweighs AGWP in this example. However, because the lifetime of an emission of $CO_2$ is much longer in the atmosphere than the transit time of carbon through a forest ecosystem, AGWP outweighs CBS on longer timescales.

## ~~4 Carbon management to maximize the climate benefit of carbon sequestration~~

The time of integration in the computation of GWP has been a heavily debated topic in the past, and this is related to the topic of 'permanence' of sequestration in carbon accounting and climate policy (Moura Costa and Wilson, 2000; Noble et al., 2000; Sedjo and Sohn One problem in these previous debates is that the timescale of carbon in ecosystems was not considered explicitly while the timescale of carbon in the atmosphere was. With the approach proposed here, both are explicitly taken into account, and can better inform management and policy debates about sequestration of carbon in natural and man-made sinks.

### 3.1 Carbon management to maximize the climate benefit of carbon sequestration

In the context of climate change mitigation, management of ecosystems may be oriented to increase carbon sequestration and its climate benefit. In the recent past, scientists and policy makers have advocated increasing the amount of inputs to ecosystems as an effective form of carbon management (e.g. Silver et al., 2000; Grace, 2004; Lal, 2004; Chabbi et al., 2017; Minasny et al., 2017). Although increases in carbon inputs can increase the amount of stored carbon in an ecosystem with related climate benefits, it does not necessarily increase the amount of time the sequestered carbon will stay in the system. Therefore, strategies that focus on increasing carbon inputs alone, do not take full advantage of the potential of ecosystems to mitigate climate change.

We can conceptualize any management activity that increases or reduces carbon inputs to an ecosystem by a factor $\gamma$, so the new inputs are given by the product $\gamma u$. For example, if we increase carbon inputs to an ecosystem by 10%, $\gamma = 1.1$. Increasing (decreasing) carbon inputs increase (decrease) carbon storage at steady state by an equal proportion since

$$
\begin{aligned}
-\mathbf{B}^{-1}(\gamma u) &= \gamma(-\mathbf{B}^{-1}u), \\
&= \gamma x^{*}.
\end{aligned}
$$

$$\begin{aligned} -\mathbf{B}^{-1}(\gamma u) &= \gamma(-\mathbf{B}^{-1}u), \\ &= \gamma x^*. \end{aligned} \tag{31}$$

However, the time carbon requires to travel through the ecosystem is still the same since the transit time does not change, as we can see from the mean transit time expression

$$-\mathbf{1}^{\mathsf{T}}\mathbf{B}^{-1}\frac{\gamma u}{\|\gamma u\|} = \mathbb{E}(\tau). \tag{32}$$

Both the transit time distribution (eq. B4) and the mean transit time (eq. 17) only take into account the proportional distribution of the carbon inputs to the different pools $(u/\|u\|)$, but not the total amount of inputs. Therefore, a unit of carbon that enters an ecosystem stays there for the same amount of time independent of how much carbon is entering the system. Although these results only apply to linear systems at steady-state, they provide some intuition about what might be the case in systems out of equilibrium, such as while in transition from one steady state to another following a land use or management change.

Carbon management can also be oriented to modify process rates in ecosystems as encoded in the matrix $\mathbf{B}$. A proportional decrease (increase) in process rates by a factor $\xi$ not only increases (decreases) carbon storage as

$$\begin{aligned} -(\xi\mathbf{B})^{-1}u &= \tfrac{1}{\xi}(-\mathbf{B}^{-1}u), \\ &= \tfrac{x^*}{\xi}, \end{aligned}$$

$$\begin{aligned} -(\xi\mathbf{B})^{-1}u &= \frac{1}{\xi}(-\mathbf{B}^{-1}u), \\ &= \frac{x^*}{\xi}, \end{aligned} \tag{33}$$

it also increases (decreases) the mean transit time as

$$-\mathbf{1}^{\mathsf{T}}(\xi\mathbf{B})^{-1}\frac{u}{\|u\|} = \frac{\mathbb{E}(\tau)}{\xi}. \tag{34}$$

Based on these results, it is now clear that carbon management to increase carbon inputs alone can only increase CS, but not $\mathrm{CS}_1$; i.e. the new carbon inputs have a sequestration benefit only through increase of carbon storage, but not through a longer transit time in ecosystems. Management to decrease process rates on the contrary, can increase both CS and $\mathrm{CS}_1$ because the new carbon entering the system stays there for longer.

We can see these effects of carbon management on CS by running simulations using the ~~model of Example 1~~ same steady-state model (Figure 7). ~~In this example~~Now, we modified carbon inputs and process rates by either increasing them by 10 and 50% ($\gamma$, $\xi$ = 1.1, 1.5), or decreasing them by 10 and 50% ($\gamma$, $\xi$ = 0.9, 0.5). The simulations showed that increasing or decreasing carbon inputs increase or decrease CS for any time horizon (Figure 7a), but it does not modify the behavior of one unit of sequestered carbon ($CS_1$) (Figure 7b). On the contrary, decreasing or increasing process rates increase or decrease both CS (Figure 7c) and $CS_1$ (Figure 7d).

~~Management of~~ The resultant effects of changes in management of inputs or process rates ~~have subsequent effects on CBS . If the amount of sequestered carbon is larger than a reference system, the climate benefit is larger and can last for a longer time horizon~~ on CBS can differ substantially. Increases or decreases of carbon inputs have similar proportional effects on CBS, but differences in processes rates are not equally proportional. While an increase in inputs by 50% would increase CBS by 50%, a decrease in process rates by 50% would have an increase in CBS by more than 100% for time horizons longer than 300 years (Figure 8). Similarly, ~~if process rates decrease with respect to a reference system, the CBS is larger and last for a longer time horizon. Therefore, a combination of management of carbon inputs and process rates can have large benefits for climate change mitigation~~while a decrease in inputs by 50% would reduce CBS by 50%, an increase in process rates by 50% would decrease CBS by only ∼40%.

These results ~~imply that carbon management can be planned to maximize the climate benefits of carbon sequestration by both increasing carbon inputs and increasing transit time~~show that management of transit time, e.g. by decreasing process rates, may lead to stronger climate benefits than managing carbon inputs alone. Furthermore, one could think about optimization scenarios in which both inputs and transit times are managed to achieve larger climate benefits given certain constraints. The concept of CBS is thus a useful mathematical framework to formally pose such an optimization problem. ~~Examples on how to increase transit time in ecosystems are discussed in subsequent examples.~~

## 4 ~~CS and CBS during the industrial period~~

~~During the industrial period, starting in calendar year 1850, concentrations of carbon dioxide in the atmosphere increased steadily with a number of consequences on Earth system processes. Models participating in the Coupled Model Intercomparison Project Phase 6 (CMIP6) (Eyring et al., 2016), predict an increase in carbon uptake by the terrestrial biosphere, i.e. gross primary production, from a multi-model average of 133.6 PgC yr$^{-1}$ in 1850 to 160.8 PgC yr$^{-1}$ in 2014 (Figure **??**a). To demonstrate the application of our framework for the out of steady-state case, we use here this time series of average GPP to drive the model of Emanuel et al. (1981) during the industrial period.~~

~~Simulations of time-dependent CS and CBS for the industrial period using the model of Emanuel et al. (1981) driven by the average GPP of models from the CMIP6 archive (*esm-hist* experiment). a) Average GPP and predicted respiration fluxes of the input and their difference. b) Amount of carbon remaining of the inputs for every year calculated using equation . The areas under the curve are values of CS for time horizons of 50 yr starting in 1850, 1900, and 1950, computed using equation . c) Continuous values of CS (areas under the curve of panel b for time horizons from 1 to 150 yr. The dash line represents CS for the equilibrium case with constant inputs of 113 PgC yr$^{-1}$. d) Continuous values of CBS for time horizons from 1 to 150 yr computed using equation .~~

~~Carbon incorporated into the terrestrial biosphere returns back to the atmosphere with an average transit time of 15.1 yr; therefore a large proportion of carbon incorporated during the industrial period is respired back quickly while some carbon is stored for a longer time (Fig. ??a). Using equation , we can compute the integrated fate of carbon incorporated every year $M_s(t)$, from 1850 to any other year until 2014 (Fig. ??b), which is the total amount of all remaining carbon since the beginning of the industrial period in $t_0 = 1850$. This amount of carbon can be integrated further for different time horizons $T$, using equation to obtain time-dependent values of CS (Fig. ??c); i.e. for different lengths of the time horizon $T$.~~

~~Starting in the year 1850 and for time horizons of 50, 100, and 150 yr, the predicted values of CS are 40,732, 110,811, and 196,569 PgC yr, respectively (Fig. ??b). As the time horizon increases , CS increases due to the continuous accumulation of extra carbon in the terrestrial biosphere (Fig. ??c). This upward trend is not surprising because even if the system were at~~

## 4 Example 2: CS and CBS for dynamic systems out of equilibrium

### 4.1 Pulses entering at different times and experiencing different environments

The steady-state ~~, the continuous accumulation of annual fluxes would lead to a linear trend with time horizon (dashed line in Fig. ??c).~~

~~As the time horizon increases and more carbon is retained, the climate benefit of sequestration increases (negative increase)(Fig. ??d). This prediction of CBS for the industrial period represents the total cumulative amount of avoided warming prevented by carbon sequestered in the terrestrial biosphere. It can be computed for different time horizons and initial times $t_0$, and could be useful to compare with values of AGWP for GHG emissions or with different forms of biospheric management.~~

## 5 ~~Example 2: Carbon management in forests~~

~~Land use and land use change are important drivers of changes in the terrestrial carbon cycle, with forest-related activities considered as a major player in the global carbon balance. Over the years, there has been considerable debate about how to quantify the role of forestry in mitigating climate change (e.g. Harmon et al., 1990; Fearnside, 1995; Tipper and de Jong, 1998; Winjum et~~ ~~One contended issue is how to quantify the contribution of forest products with relatively long lifetimes that can help to lockup carbon. How to account for time and permanence of sequestered carbon has been another contended issue (Fearnside et al., 2000; Moura Co~~ ~~well as quantifying the climate benefits of bioenergy as a substitute of fossil-fuel derived energy (Schulze et al., 2019). The definition of carbon sequestration we introduced here, and the CBS concept we developed, can help to better address these issues.~~

~~Commercial forest management is commonly advocated as an activity that can play a major role in climate change mitigation as opposed to preserving old-growth forests. The rationale is that young managed forests have generally a larger rate of biomass growth than old-growth forests, and in addition, wood products can retain carbon for a considerable amount of time (Schulze et al., 2019). Arguments against commercial forestry highlight that total carbon stocks are lower in younger than in old-growth forests, which results in a~~ examples above are useful to gain some intuition about potential long-term ~~net~~

~~loss of carbon to the atmosphere. Also, inefficiencies in the wood production chain result in significant carbon losses due to waste with only minor proportions of harvested wood ending up in long-duration products (Harmon et al., 1990)~~patterns in CS and CBS, but for real-world applications it is necessary to consider systems out of equilibrium and driven by specific time-dependent signals. We will consider now the case of an ecosystem driven by increases in atmospheric $CO_2$ concentrations that lead to higher photosynthetic uptake, and increasing temperatures that lead to faster cycling rates. We will thus consider a non-autonomous version of the TECO model that follows the general form

$$\dot{x}(t) = \gamma(t)u + \xi(t)\mathbf{B} \cdot x(t), \tag{35}$$

where the time-dependent function $\gamma(t)$ incorporates the effects of temperature and atmospheric $CO_2$ on primary production, and the function $\xi(t)$ incorporates the effects of temperature on respiration rates. Specific shapes for these functions were taken from Rasmussen et al. (2016), and are described in detail in the appendix. When applied to the CASA model in Rasmussen et al. (2016), these functions predicted an increase in primary production and an increase in process rates, which resulted in a decrease in transit times over a simulation of 600 years.

~~In this example, we will not advocate any of the fore-mentioned points of view, but will show that the CS and CBS concepts can be an important tool to address this debate. For the sake of simplicity, let's consider the model of Emanuel et al. (1981) again, and introduce a flow of carbon from the woody tree parts pool $x_2$ to a set of five new pools, namely: long- $x_6$, mid- $x_7$, and short-duration $x_8$ forest products, as well as a bioenergy $x_9$ and a waste pool $x_{10}$ (Figure 2). Carbon in each of these pools have a distinct cycling rate according to Table **??**. This choice of values is somewhat arbitrary, and they may change according to tree species, silvicultural practices, and climatic factors. Other transfers among these and other compartments (e.g. landfill disposal) are also ignored in this simple example.~~

~~We will consider three different cases of carbon management with different silvicultural practices and efficiencies in terms of the amount of carbon that ends up in long-duration forest products. In scenario S1, 60% of the harvested wood is transferred to long-duration products and only 5% ends up in waste, and 5% is used in bioenergy. Carbon in these two last pools is released quickly to the atmosphere. In scenario S2, only 30% of the harvested wood is transferred to long-duration products and much larger proportions, 30~~ We used the same simulation setup here starting from an empty system ($x(0) = 0$), and ~~10%, end up in bioenergy and waste, respectively (Table **??**). In scenarios S1 and S2, silvicultural practices are such that they result in an increase in the flow from the non-woody tree parts to the detritus pool by 30% in comparison to the original model~~obtained similar results in terms of primary production and transit times as in Rasmussen et al. (2016). We used these simulation results to compute CS and CBS for carbon entering the ecosystem at different times during the simulation window. In particular, we considered the case of the amount of carbon sequestered at years 100 and 300 after the start of the simulation; i.e. ~~an additional flux of 11 PgC yr$^{-1}$ among these pools. In scenario S3, we considered improved silvicultural practices that decrease the transfer of non-woody tree parts to detritus by 10 PgC yr$^{-1}$, and instead, this amount is transferred to the woody-tree parts. The distribution of the harvested wood in S3 is similar as in S2 (Table **??**), which implies that S3 is ascenario of improved silvicultural practices but low efficiency in the transformation of harvested wood.~~ we considered the cases $t_0 = 100$

and $t_0 = 300$ (Figure 9a) and computed the fate of this carbon ($M_s(t,t_0,u_0)$), its carbon sequestration (CS($T,u_0,t_0$)) and the climate benefit of sequestration (CBS($T,u_0,t_0$)) for different time horizons $T$.

~~One main consequence of transferring a large amount of carbon from wood biomass to forest products is that the steady-state ecosystem carbon stock is lower by 588.5 PgC in the S1 and S2 scenarios, and by 652.0 PgC in the S3 scenario, compared to the no management case. For S1 and S2, this is 34% less carbon compared to the original amount of carbon in the no management case, and 38% less carbon in S3. However, if we include the amount of carbon stored in the forest products, the steady-state carbon stock is 187.4 PgC higher in the high efficiency transfer scenario S1, 11% higher with respect to the original model. In the low efficiency transfer scenario S2, the steady-state carbon stock is lower by 131 PgC (8% less) with respect to the no management case. In S3, improved silvicultural practices compensate losses due to low efficiencies in transformations to long-duration products, resulting in 39.5 PgC more (23%) than in the no management case.~~

~~Due to carbon transfers to wood products, the amount of carbon that~~ Although more carbon enters the ecosystem ~~in any given year decreases faster in all three scenarios during the first decades in comparison with the no management case (Figure ??a,b). In subsequent decades, due to transfers to wood products, carbon is lost to the atmosphere more slowly with differences among the scenarios according to transfer efficiencies and silvicultural management. In the long term (> 200 yr), all scenarios converge, which implies that in all cases the carbon that enters in a particular year is eventually returned back to the atmosphere (Figure ??a,b).~~

~~The CS concept helps to disentangle the contrasting effects of fast losses in the initial years after carbon enters versus the slow loss in later decades because it integrates the mass loss curves (Figure ??~~at simulation year 300 than at year 100 due to the $CO_2$ fertilization effect, it is lost much faster because of higher temperatures that result in faster transit times for simulation times above 300 years (Figure 9a). The slower transit times experienced by the carbon that enters at year 100 due to lower temperature results then in much higher values of CS for time horizons $T > 100$ yr (Figure 9c). ~~Management practices that can retain carbon for a longer time result in higher amounts of carbon sequestration in the long-term. If we subtract the CS computed for the original model with no management from the CS computed for the three scenarios, we see that scenarios S1 and S3 result in an increase in carbon sequestration~~ Similarly for CBS, where differences are evident much earlier, for time horizons $T > 50$ yr (Figure 9d).

This simple example highlights the importance of time-dependent transit times in determining CS and CBS. If changes in climate lead to faster carbon processing rates, we would thus expect carbon to transit faster through the ecosystem, returning faster to the atmosphere, and therefore with lower values for carbon sequestration and its climate benefit.

### 4.1 Continuous inputs into a changing environment

In the previous example, we considered the case of two single pulses entering the ecosystem at different times under changing environmental conditions during a simulation. A consolidated view can be obtained by taking all single pulses and integrate them continuously in time to compute CS and CBS using equations (24) and (28), respectively. In this case, CS increases monotonically, and CBS decreases monotonically with time horizon (Figure 10, continuous black lines), ~~while the low efficiency transfer scenario S2 results in a decrease (Figure ??d). Low efficiencies in carbon transfers to long-duration products can be~~

~~compensated by silvicultural practices that reduce the amount of detritus and increase transfers to wood as shown by the CS of the S3 scenario (Figure **??**d).~~ which is somewhat obvious because as the ecosystem accumulates carbon, more of it is retained in the ecosystem and is isolated from atmospheric radiative effects. However, ~~if the CS were computed for short time horizons (< 30 yr), the CS difference would be negative in all cases due to the fast losses of carbon in the initial decades.~~

~~These differences in the fate of carbon and CS lead to similar qualitative differences in terms of CBS (Figure **??**e, f). In all cases, CBS is large after the instantaneous uptake and differences among scenarios spread as the time horizon increases depending on how carbonis transferred among the different reservoirs and is returned back to the atmosphere. In the long-term, all scenarios converge to zero, which implies that~~ this simulation only considers carbon that enters the ecosystem from the beginning of the simulation until the end of the time horizon, from $t_0$ to $T$. An important aspect to consider is the ~~benefits of sequestering a given amount of carbon at a particular time decline over large time horizons.~~ role of carbon already present in the ecosystem at $t_0$.

~~Comparison of the fate of carbon~~ $\cancel{M_s(t)}$~~, carbon sequestration CS, and the climate benefit of carbon sequestration CBS of an instantaneous uptake of 113 PgC among different management scenarios for forest products. Plots on the left (a, c, d) show the values obtained for the no management case and the three scenarios, while panels on the right (b, d, f) show the difference between each scenario and the no-management case. S1 has a high efficiency of transfer to long-duration products and S2 a low efficiency of transfers. S3 includes low efficiencies in transfers as in S2, but increases allocation to wood due to silvicultural management. Colors in all plots follow the same legend as in a.~~

~~These results suggest that forest management for climate change mitigation may be a viable alternative if it leads to a considerable increase in transit time; i.e., if a relatively large proportion of harvested wood is transferred to long-duration products with small losses to waste, and silvicultural practices are adopted that reduce the amount of detritus and increase allocation of C to wood.~~

~~As mentioned before, another contended issue related to forestry projects for climate change mitigation is how to quantify potential benefits due to fuel substitution by bioenergy (Werner et al., 2010; Schulze et al., 2019). The CBS can be computed for individual pools if the curve of mass of remaining carbon for the pool is known (Figure **??**). Since the CBS represents the quantity of avoided warming in units of W m~~$^{-2}$ ~~yr, it can be easily compared to the AGWP of any amount of fossil fuel emitted to the atmosphere that corresponds to the substitution. The fact that the CBS and the AGWP share the same units makes such comparisons and computations straightforward, which can be used in integrated assessments that also consider~~ We will consider now the case of continuous sequestration and release of carbon with differences in the ~~quantity of fossil fuels required to manage and harvest forests.~~ initial conditions in the simulation, which can vary according to land use changes. For example, when changing land use from agriculture to forest, or from natural forest to plantation, there are carbon legacies that have an influence on future carbon trajectories (Harmon et al., 1990; Janisch and Harmon, 2002; Sierra et al., 2012). These carbon legacies are usually dead biomass and detritus, which cause ecosystems to lose carbon via decomposition before photosynthesis from new biomass compensates for the losses. In these initial stages of recovery, ecosystems are usually net carbon sources, but they still may store more carbon than an ecosystem developing from bare ground.

~~For the three scenarios considered in this example, the CBS is larger for the bioenergy pool in the S3 scenario because more carbon is allocated to wood and more of the harvested wood is transferred to the bioenergy pool in comparison to the S1 and S2 case (Figure **??**). In the long-term, the CBS for the bioenergy pool can be simply computed as $k_{\mathrm{CO}_2} x_9^*$ following equation applied to individual pools. Therefore, quantifying the long-term contribution of bioenergy as a climate benefit can be an easy calculation if the steady-state assumption can be justified.~~

~~Fate of carbon in the bioenergy pool for the three scenarios (a), and the corresponding climate benefit of sequestration CBS for this pool (b).~~

~~The role of forestecosystems in global climate however, goes beyond the effects related to the carbon cycle. Forests also influence climate by effects on albedo, partitioning of latent and sensible heat fluxes, and effects on land surfaces roughness (Bonan, 2008). These effects can be quantified as radiative effects in energy units (e.g. W m$^{-2}$), therefore the CBS provides information in units more comparable to those used to assess the overall effect of forests on climate.~~

## 5 ~~Example 3: climate benefit of carbon sequestration in soils~~

~~In addition to carbon management in forests and wood products, soils are commonly considered as a promising alternative to sequester carbon and mitigate climate change (Lal, 2004; Minasny et al., 2017). Carbon in soils can be stabilized by a variety of physicochemical and biological mechanisms that can considerably prolong the time carbon stays in the soil system. Mechanisms for carbon stabilization and destabilization in soils interact in multiple forms (Sollins et al., 1996; von Lützow et al., 2006; Du which results in a large heterogeneity of process rates and therefore in the transit time of carbon (Sierra et al., 2018b). Mean transit times of soil carbon can vary from a few years to centuries as predicted by different global-scale soil carbon models (Luo et al., 2017; Sierra et al., 2018b; Lu et al., 2018). This is because once organic matter enters the soil in the form of plant detritus , it gets quickly consumed by microorganisms, most of it gets quickly respired and emitted to the atmosphere in the form of $CO_2$, but a small proportion can be transformed to different chemical forms and can also get sorbed into mineral surfaces where it can be retained for centuries to millennia (Trumbore, 2009; Rasmussen et al., 2018). Understanding the fate of carbon in soils, particularly in the long-term, is of fundamental importance to better understand the climate benefits of carbon sequestration in terrestrial ecosystems.~~

~~The model of Emanuel et al. (1981) that we have considered so far, contains a simple representation of soil carbon in a homogeneous active pool. To better explore the fate of carbon in a heterogeneous soil and how this representation affects the CBS, we will now replace the active pool of this model with the structure of the well-known Century model (Parton et al., 1987), which contains an active, aslow, and a passive pool, with sequential transfers of carbon among them (Figure 2). Century is the basis of many Earth system models, and although it has been criticized for not including detailed processes, it still has been useful in many different studies to predict long-term soil carbon storage (Blankinship et al., 2018). The implementation of the model included here follows the description of the original model in Parton et al. (1987), with default parameters not modified by temperature, moisture, or soil texture.~~

~~The inclusion of a heterogeneous soil structure results in a more rapid release of carbon in the first decades after carbon enters the system in comparison to~~ The CS and CBS concepts can be very useful to compare contrasting trajectories of ecosystem development and assess their role in terms of carbon sequestration alone and their climate impact. For this purpose, we performed an additional simulation in which at the starting time there is no living biomass, but the detritus pools and the ~~original model (Figure ??a). However, after the first 70 years more carbon stays in the system and it is lost at a much slower rate in comparison to the original model. The reason for this behavior is that the active pool in Century cycles much faster than~~ SOM pools are 1.5 and 1.0 times as large as in the ~~aggregated active pool of the original model. The slow and passive pools on the contrary, cycle carbon at much slower rates and therefore carbon is retained in the system for a much longer time.~~ equilibrium case, respectively ($x(0) = 98.7$ MgC ha$^{-1}$). In this simulation, the ecosystem losses a significant amount of carbon in the early stages of development and respiration is much larger than primary production ($r(t) > u(t)$) (Fig. 10a, dashed magenta line). Because soils are already close to an equilibrium value, the ecosystem has already a large amount of carbon stored, therefore in the computation of the fate of carbon $M_s(t, t_0)$ there is already a larger amount of carbon to consider, which causes CS to be larger for the land-use-change case than for the bare ground case (Figure 10c). On the contrary, because there are more emissions from the ecosystem in early development stages, CBS is lower for the land-use-change case than for the bare ground case (Figure 10d).

~~The long-term CS of an annual input of 113 PgC in the model with soil-pool structure according to Century is much larger than that of the original model and all other forest management scenarios considered in the previous example (Figure ??b). However, at short-time scales (< 150 yr) the CS of the extended model is lower than in all other cases, except for S2. This means that if we compare carbon sequestration in wood products versus soil carbon, the climate benefit would depend strongly on the time horizon of integration, and one can obtain contrasting answers depending on this integration time. More generally, the integration time in the computation of CS must capture as much as possible the range of timescales of carbon cycling in a system. If the integration time is too short and does not capture long-term slow processes, it would underestimate the long-term CS.~~ These contrasting results between CS and CBS for the continuous case with contrasting initial conditions, can be very useful to address debates and controversies about the role of land use change and baselines in carbon accounting. The results show that carbon sequestration can still be high in ecosystems where emission fluxes are large, but climate impacts can differ significantly. By using two different metrics, these two different aspects of carbon sequestration can be discussed separately.

~~Similarly, the CBS in the system with soil structure according to Century is much larger than the CBS in the other management cases considered before and can last for much longer (Figure ??c) . Since the CBS in these examples is computed for the same amount of carbon uptake (113 PgC), we can see the importance of long transit times for climate change mitigation. The mean transit time of the original model of Emanuel et al. (1981) is 15.1 yr, and for the forest management scenarios S1, S2, and S3, it is 16.8, 14.0, and 15.5 yr, respectively. For the expanded model with soil-pool structure according to Century, the mean transit time is 49.5 yr. We can thus see that the large value of CBS for this last case is due to a longer transit time of carbon in the ecosystem.~~

~~Soils are indeed a promising reservoir to store carbon in ecosystems, and the climate benefits of sequestering carbon in soils may be larger than the climate benefits of other forms of ecosystem management. However, our steady-state assumption implies~~

~~that changes in management in soils must be sustained for very long times for them to be relevant. It becomes then a practical challenge to promote sustained carbon sequestration in soils over centuries (Amundson and Biardeau, 2018; Schlesinger and Amundson, 20~~

## 5 Discussion

~~Debates~~ The metrics introduced here, carbon sequestration (CS) and the climate benefit of sequestration (CBS), integrate both the amount of carbon entering an ecosystem and the time it is stored there and thus avoiding radiative effects in the atmosphere. Disproportionate attention is given to quantifying sources and sinks of carbon in ongoing debates about the role of ecosystems in climate change mitigation~~have given disproportionate attention to quantifying sources and sinks of carbon, but~~ , with much less attention to the fate of carbon once it enters an ecosystem. The time carbon remains ~~stored~~ in an ecosystem~~is relevant~~ , encapsulated in the concept of transit time, is critical for climate change mitigation because during this time ~~the~~ carbon is removed from radiative effects in the atmosphere. ~~In this manuscript, we propose a metric that can integrate both the amount of carbon that enters the ecosystem and the time it is stored there while avoiding radiative effects: the climate benefit of sequestration (CBS ). This metric is strongly controlled by both the amount of carbon inputs to an ecosystem and the aggregated effect of all process rates at which this carbon is cycled before getting released back to the atmosphere. This aggregated effectcan be encapsulated in the concept of transit time, which determines long-term carbon sequestration CS.~~

The CS and CBS concepts unify atmospheric and ecosystem approaches to quantifying the greenhouse effect. The CBS concept builds on ~~the concept~~ that of absolute global warming potential (AGWP) of a greenhouse gas~~, with the main difference~~ . The main difference is that CBS quantifies avoided warming during the time carbon is stored in an ecosystem, while AGWP quantifies potential warming when the carbon enters the atmosphere. Both metrics rely on the quantification of the fate of carbon (or other GHGs for AGWP) once it enters the particular system. For atmospheric systems, a significant amount of work has been done in determining the fate of GHGs once they enter the atmosphere after emissions (e.g. Rodhe, 1990; O'Neill et al., 1994; Prather, 1996; Archer et al., 2009; Joos et al., 2013). For terrestrial ecosystems however, robust methods to quantify the fate of ~~sequestered carbon~~ carbon as it flows through terrestrial system components have been developed only recently (Rasmussen et al., 2016; Metzler and Sierra, 2018; Metzler et al., 2018).

Global warming potential (GWP), or the climate impact of an emission of a certain gas in relation to the impact of an emission of $CO_2$, is often used to assess climate impacts of actions, e.g., avoided deforestation, land use change, and even enhanced carbon sequestration. However, this metric has two limitations when applied to carbon sequestration and in comparison to the combined use of CBS and AGWP we advocate here: 1) it only quantifies the climate effects of emissions but not of sequestration, and treats all fixed carbon equally independent of its transit time in the ecosystem, 2) it is a relative measure with respect to the emission of $CO_2$. GWPs are commonly reported in units of $CO_2$-equivalents, which only address indirectly the effect of a gas in producing warming. In contrast, CBS quantifies the effects of avoided warming in units of W $m^{-2}$ over the period of time carbon is retained, facilitating comparisons to other effects of ecosystems on climate (Bonan, 2008).

Other concepts have been proposed in the past to account for the ~~time carbon is stored in ecosystems~~ temporary nature of carbon sequestration (see review by Brandão et al., 2013, and references therein), with special interest in accounting for ~~carbon credits . None of these concepts explicitly account for~~ credits in carbon markets. In fact, 'ton-year' accounting methods (Noble et al., 2000) resemble our definition of carbon sequestration; however, none of these previous concepts explicitly considers the time carbon is retained in the ecosystem~~, but rather use concepts related~~ . Instead, these approaches relate carbon sequestration to delay in ~~emissions (Fearnside et al., 2000) or equivalence of carbon storage in relation~~ fossil fuel emissions (Fearnside et al., 2000), or as the equivalence of the amount of carbon storage to AGWP (Moura Costa and Wilson, 2000). ~~One notable exception are the~~ The concepts of sustained global warming potential SGWP and sustained global cooling potential SGCP proposed by Neubauer and Megonigal (2015) ~~. Our~~ are notable exceptions. The CBS concept captures some of the ideas of the SGCP concept, but differs in some fundamental assumptions related to the interpretation of ~~Green's~~ the impulse response functions, the treatment of time-dependent fluxes and rates, and reporting. While SGCP reports values in reference to $CO_2$ as is commonly done for GWP, we report CBS for individual gases as it is done for AGWP. Appendix A elaborates on other aspects of the SGWP and SGCP concepts.

~~Compared to previously proposed metrics, the~~ The concept of CBS ~~can be very useful~~ improves our ability to address some of the existing debates about the role of ecosystems in mitigating climate change ~~.~~ and enhances our potential to provide decision-support. In combination with quantifications of AGWP, CBS provides the net climate effect of an ecosystem or some management. For example, ~~it~~ CBS can be used to better ~~account for~~ understand the climate impacts of storing carbon in long-term reservoirs ~~and~~ such as soils and wood products, and the climate benefits of increasing the transit time in these systems. ~~It can also~~ CBS can be used to better quantify the ~~role of bioenergy as~~ climate benefits of using biofuels as fossil fuel substitution by computing the CBS of the ~~bioenergy pool~~ whole bioenergy production system and adding the negative AGWP ~~caused by~~ attributed to the avoided emission. Similarly, it can be incorporated in assessments of sequestration in industrial systems with associated carbon capture and storage. ~~Furthermore, it can be combined with quantifications of AGWP that assess the climate impact of emissions to obtain assessments of the net climate effect of an ecosystem or some management.~~

~~The global warming potential (GWP) is commonly used to quantify the climate impact of an emission of a certain gas in relation to the impact of an emission of~~ ~~$CO_2$~~~~. Projects on avoided deforestation, land use change, and even enhanced carbon sequestration have relied on this metric to assess climate benefits or impacts. However, this metric has two limitations in comparison to the combined use of CBS and AGWP we advocate here: 1) it only quantifies the climate effects of emissions but not of sequestration, and treats all fixed carbon equally independent of its transit time in the ecosystem, 2) it is a relative measure with respect to the emission of~~ ~~$CO_2$~~~~. Therefore, GWPs are reported in units of~~ ~~$CO_2$~~~~-equivalents, which only address indirectly the effect of a gas in producing warming. The CBS on the contrary, quantifies the effects of avoided warming in units of~~ ~~$W\ m^{-2}$~~ ~~times the amount of time an amount of carbon is retained, which can be better compared to other effects of ecosystems on climate (Bonan, 2008).~~

~~Carbon management in ecosystems can be targeted to~~ Carbon management of ecosystems can maximize CS and~~CBS, which can be achieved~~ /or CBS by not only increasing carbon inputs, but also by increasing the transit time of carbon. There are many ways in which the transit time of carbon can be increased; for instance, by increasing transfers of carbon to slow cycling pools such as the case of increasing wood harvest allocation to long-duration products (Schulze et al., 2019), or addition of biochar to soils, or by reducing cycling rates of organic matter such as the case of soil flipping (Schiedung et al., 2019). Independently of the management activity, CS and CBS can be powerful metrics to quantify their climate benefits, make comparisons among them, and compare against baselines or no management ~~cases~~scenarios.

The examples we provided in this manuscript ~~are only for illustration purposes of the CBS concept, and by no means we advocate any of the management activities discussed here before they are studied more carefully. For more precise quantifications~~ illustrate the use and interpretation of CS and CBS metrics under the assumptions of linearity, steady-state, or time-dependences in carbon cycle dynamics with subsequent consequences for carbon sequestration and its climate benefits. The computation of the CBS ~~, more detailed and reliable models should be used. The model of Emanuel et al. (1981)~~ relies on a model, which can be as simple as a one-pool model or a state-of-the-science land surface model. The TECO model is an excellent tool to illustrate ~~carbon cycle related~~ ecosystem-level concepts because of its simplicity and tractability, but other models with more accurate parameterizations and including more processes should be considered for practical applications. ~~In any case, the computation of the CBS relies on a model, which can be as simple as a one-pool model or a state-of-the-science land surface model. For simplicity, we relied on the steady-state assumption in some of our examples, but the~~ The formulas and formal theory developed in Section 2 are general enough to deal with the non-steady-state case as well as with models with nonlinear interactions among state variables.

The concepts of CS and CBS present improvements to the current guidelines for carbon inventories that treat all carbon removals by sinks equally (IPCC, 2006) by explicitly considering the transit time of carbon in ecosystems. Therefore, these new concepts have potential for being incorporated in revised policies for carbon accounting in the context of international climate agreements and carbon markets. CS and CBS can aid in the economic valuation of carbon by adding economic incentives to sequestration activities that retain carbon in ecosystems for longer times. In addition, the concepts can help in dealing with the issue of permanence of carbon by explicitly quantifying climate benefits of sequestration that can be compared directly with the climate impacts of emissions on a similar time horizon.

Two potential limitations to apply the concepts of CS and CBS are that 1) they rely on a model that tracks the fate of the fixed carbon and 2) on ~~a Green's or~~ an impulse response function of $CO_2$ in the atmosphere. Reliable models may not be available for certain type of ecosystems or may include large uncertainties that propagate to CS and CBS estimates. Also, estimates of ~~Green's~~ impulse response functions for atmospheric $CO_2$ seem to have also ~~large~~ uncertainties, particularly ~~for long timescales (Archer et al., 2009; Lashof and Ahuja, 1990). The~~ related to the size of the emission pulse, the atmospheric background at which the pulse is applied, and the long-term behavior of the curve for timescales longer than 1000 years (Archer et al., 2009; Lashof and Ahuja, 1990; Joos et al., 2013; Millar et al., 2017). However, one advantage of the functions proposed by Joos et al. (2013) ~~, derived from complex simulations of~~ is that they are derived from coupled climate-carbon models ~~, produce unrealistic behaviors in the long-term due to the infinite storage of an emission in the atmosphere-ocean-terrestrial system. They are also derived from models out of equilibrium, violating the steady-state and linearity assumptions of the linear response theory. Advances~~ that include multiple feedbacks. Therefore, when computing CS and CBS for small perturbations of the carbon cycle, it is not necessary to explicitly compute carbon-climate feedbacks. Also, when comparing two different systems with a CBS ratio as in Figure (8) or a ratio CBS to AGWP (Figure 6), uncertainties in the IRFs would tend to cancel each other out. Nevertheless, advances in our understanding of the fate of emitted $CO_2$ ~~in~~ to the atmosphere will consequently derive in better estimates of the climate benefits of carbon sequestration.

## 6 Conclusions

Analyses of carbon sequestration for climate change mitigation purposes must consider both the amount of carbon inputs and the transit time of carbon. Both concepts are encapsulated in the unifying concepts of carbon sequestration ~~CS~~ (CS) and climate benefit of sequestration ~~CBS~~ (CBS) that we propose. Carbon management can be oriented to maximize CS and CBS, which can be achieved by managing both rates of carbon input and process rates in ecosystems. We believe the use of these metrics can help to better deal with current discussions about the role of ecosystems in mitigating climate change, ~~and will~~ provide better estimates of avoided or human-induced warming, and have potential to be included in accounting methods for climate policy.

*Code availability.* Code to reproduce all results is available at https://git.bgc-jena.mpg.de/csierra/cbs. Upon acceptance for publication, a copy of this repository will be archived in a permanent location with a respective digital object identifier.

## Appendix A: Comment on Neubauer and Megonigal (2015)

Neubauer and Megonigal (2015) proposed two metrics, the sustained global warming potential SGWP and the sustained global cooling potential SGCP, to overcome issues with GWP. However, there is an important misconception in their study that we would like to address here. In particular, these authors state "*...GWPs requires the implicit assumption that greenhouse gas emissions occur as a single pulse; this assumption is rarely justified in ecosystem studies*". The use of pulse emissions in computing AGWP, as shown in equation (3), is done with the purpose of obtaining a representation of the fate of a unit of emissions under the assumption that the system is in equilibrium. This is a mathematical property of linear time-invariant dynamical systems, by which an impulse response function can provide a full characterization of the dynamics of the system (Hespanha, 2009). In other words, the emission pulse is a mathematical method to obtain a description of the fate of incoming mass into the system, but it is not an assumption ~~required~~ imposed on the system.

To use impulse response functions, it is necessary to assume that a system is in equilibrium and that all rates remain constant for all times. It is this assumption that is problematic and difficult to impose on ecosystems, but not the pulse emission because it is simply a method. Therefore, we are of the opinion that the sustained-flux global warming potential metric proposed by these authors is unjustified on the argument that it removes the assumption of pulse emissions.

One interesting characteristic of the study of Neubauer and Megonigal (2015) is that it uses a model that couples an ecosystem compartment with the atmosphere, and their computation of SGWP and SGCP captures the interactions between these two reservoirs similarly as in the framework described here in section 2. The SGCP is very similar in spirit to the CBS. However, their approach differs from the approach we present here in that our mathematical framework is general enough to deal with ecosystem models of any level of complexity, not restricted to a one pool model and constant parameters and sequestration rates. Furthermore, we abstain from proposing a metric that is relative to $CO_2$. We are rather interested in an absolute metric that quantifies the effect of $CO_2$ sequestration on radiative forcing, and not in equivalents to sequestration or emissions of other gases.

## Appendix B: Fate and timescales of carbon in compartmental systems

Carbon cycling in the terrestrial biosphere is well characterized by a particular type of dynamical systems called *compartmental systems* (Anderson, 1983; Jacquez and Simon, 1993). These systems of differential equations generalize mass-balanced models and therefore generalize element and carbon cycling models in ecosystems (Rasmussen et al., 2016; Luo et al., 2017; Sierra et al., 2018a). In their most general form, we can write carbon cycle models as

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \dot{x}(t) = u(x,t) + \mathbf{B}(x,t)\,x, \tag{B1}$$

where $x(t) \in \mathbb{R}^n$ is a vector of ecosystem carbon pools, $u(x,t) \in \mathbb{R}^n$ is a time-dependent vector-valued function of carbon inputs to the system, and $\mathbf{B}(x,t) \in \mathbb{R}^{n \times n}$ is a time-dependent compartmental matrix. The latter two terms can depend on the vector of states, in which case the compartmental system is considered nonlinear. In case the input vector and the compartmental matrix have fixed coefficients (no time-dependencies), the system is considered autonomous, and non-autonomous otherwise (Sierra et al., 2018a). At steady-state, the autonomous linear system has the general solution $x^* = -\mathbf{B}^{-1}\,u$.

The probability density function (pdf) for system age of linear autonomous models at steady-state can be computed by the following expression (Metzler and Sierra, 2018)

$$f(a) = -1^{\mathsf{T}}\,\mathbf{B}\,e^{a \cdot \mathbf{B}}\,\frac{x^*}{\|x^*\|}, \quad a \geq 0, \tag{B2}$$

where $a$ is the random variable age, $1^{\mathsf{T}}$ is the transpose of the $n$-dimensional vector containing ones, $e^{a \cdot \mathbf{B}}$ is the matrix exponential computed for each value of $a$, and $\|x^*\|$ is the sum of the stocks of all pools at steady-state.

The mean, i.e. the expected value, of the age pdf can be computed by the expression

$$\mathbb{E}(a) = -1^{\mathsf{T}}\,\mathbf{B}^{-1}\,\frac{x^*}{\|x^*\|} = \frac{\|\mathbf{B}^{-1}\,x^*\|}{\|x^*\|}. \tag{B3}$$

The pdf of the transit time variable $\tau$ for linear autonomous systems in equilibrium is given by (Metzler and Sierra, 2018)

$$f(\tau) = -1^{\mathsf{T}}\,\mathbf{B}\,e^{\tau \cdot \mathbf{B}}\,\frac{u}{\|u\|}, \quad \tau \geq 0, \tag{B4}$$

and the mean transit time by

$$\mathbb{E}(\tau) = -1^{\mathsf{T}}\,\mathbf{B}^{-1}\,\frac{u}{\|u\|} = \frac{\|x^*\|}{\|u\|}. \tag{B5}$$

For the most general case of nonlinear non-autonomous systems, we follow the approach described in Metzler et al. (2018). For these systems, the age distribution of mass is given by

785 $$\begin{array}{l}\text{Mass in the system at}\\ \text{time } t \text{ with age } a\end{array} = \begin{cases}\mathbf{\Phi}(t, t-a)\, u(t-a), & a < t - t_0,\\ \mathbf{\Phi}(t, t_0)\, f^0(a - (t - t_0)), & a \geq t - t_0\end{cases}$$

where $\mathbf{\Phi}$ is a state-transition matrix, and $f^0$ is an initial age density distribution at initial time $t_0$. We obtain $\mathbf{\Phi}$ by taking advantage of an existing numerical solution $x(t)$, which we plug in the original system, obtaining a new compartmental matrix $\tilde{\mathbf{B}}(t) := \mathbf{B}(x(t), t)$ and a new input vector $\tilde{u} := u(x(t), t)$. Then, the new linear non-autonomous compartmental system

$$\dot{y}(t) = \tilde{\mathbf{B}}(t)\, y(t) + \tilde{u}(t), \quad t > t_0, \tag{B6}$$

790 has the unique solution $y(t) = x(t)$, which emerges from the fact that both systems are identical. The solution of the system is then given by

$$x(t) = \mathbf{\Phi}(t, t_0)\, x^0 + \int_{t_0}^{t} \mathbf{\Phi}(t, s)\, u(s)\, \mathrm{d}s, \tag{B7}$$

where $x^0 = \int_0^\infty f^0(a)\, \mathrm{d}a$ is the initial vector of carbon stocks. We obtain the state-transition matrix as the solution of the following matrix differential equation

795 $$\frac{\mathbf{\Phi}(t, t_0)}{\mathrm{d}t} = \mathbf{B}(t)\, \mathbf{\Phi}(t, t_0), \quad t > t_0, \tag{B8}$$

with initial condition

$$\mathbf{\Phi}(t_0, t_0) = \mathbf{I}, \tag{B9}$$

where $\mathbf{I} \in \mathbb{R}^{n \times n}$ is the identity matrix. For the special case in which the time-dependent metric can be expressed as a product between a time-dependent scalar factor $\xi(t)$ and a constant value matrix $\mathbf{B}$, i.e. $\mathbf{B}(t) = \xi(t)\mathbf{B}$, we obtain the state-transition
800 matrix as

$$\mathbf{\Phi}(t, t_0) = \exp\left( \int_{t_0}^{t} \xi(\tau)\mathrm{d}\tau \cdot \mathbf{B} \right). \tag{B10}$$

These formulas can be applied to any carbon cycle model represented as a compartmental system to obtain the fate of carbon once it enters the ecosystem as well as timescale metrics such as age and transit time distributions.

**Computation of the mass remaining in the system**

805 From equation (B7), we can see from the first term that the initial amount of carbon in the system $x^0$ changes over time according to the term $\mathbf{\Phi}(t, t_0)\, x^0$. Rasmussen et al. (2016) showed that under certain circumstances, equation (B7) is exponentially

stable as long as $\mathbf{B}$ is invertible, and the state transition operator acts as a term that exponentially 'decomposes' the initial amount of carbon. Furthermore, the state transition operator tracks the dynamics of the incoming carbon and how it is transferred among the different pools before it is respired. Therefore, this operator can be used to compute the fate of an amount of carbon sequestered at time $t_s$ as

$$M_s(t-t_s) = M_s(a) = \|\mathbf{\Phi}(t,t_s)\,u(t_s)\|, \quad a = t - t_s. \tag{B11}$$

Similarly, the fate of one unit of sequestered carbon at time $t_s$ can be computed as

$$M_{s1}(a) = \left\| \mathbf{\Phi}(t,t_s) \cdot \frac{u(t_s)}{\|u(t_s)\|} \right\|, \tag{B12}$$

where the subscript 1 denotes that the function predicts the fate of one unit of carbon.

## Appendix C: Detailed representation of the ~~modified models~~ TECO model and the transient simulations used in examples

~~For the scenario S1 with high efficiency in transfers to the long-duration products, the matrix of cycling rates~~ The terrestrial ecosystem model TECO described in Weng and Luo (2011) and Luo et al. (2012) has eight pools to simulate ecosystem-level carbon dynamics, with a parameterization for the Duke Forest, a temperate forest in North Carolina, USA. The annual amount of photosynthetically fixed carbon predicted by the model in this forest (GPP) is $U = 12.3$ Mg C ha$^{-1}$ yr$^{-1}$. The vector of carbon allocation is given by

$$\cancel{\mathbf{B}}\mathbf{b} = \begin{pmatrix} 0.14 \\ 0.26 \\ 0.14 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

which ~~results in a steady-state vector of pool sizes given by~~ shows that from all photosynthetically fixed carbon, 14% is allocated to foliage, 26% to woody biomass, and 14% to roots. Net primary production (NPP) in this case is the proportion of GPP that stays in the system, i.e. NPP = $U(0.14+0.26+0.14) = 6.64$ Mg C ha$^{-1}$ yr$^{-1}$.

$$x^* = \begin{pmatrix} 32.38 \\ 72.72 \\ 69.00 \\ 72.12 \\ 876.59 \\ 654.48 \\ 111.88 \\ 6.82 \\ 1.56 \\ 1.09 \end{pmatrix}, \quad \|x^*\| = 1898.62.$$

~~For the scenario S2 of low efficiency transfers to long-duration pools, the cycling rate matrix is given by~~ Each pool in the model cycles at annual rates given by the diagonal elements of the matrix

$$\mathbf{B}\mathbf{C} = \begin{pmatrix} 0.942 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.021 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.872 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 3.978 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.347 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 3.833 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.036 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.004 \end{pmatrix},$$

~~which results in a steady-state vector of pools sizes as~~ with a matrix of transfer coefficients as

$$x^* \mathbf{A} = , \|x^*\| = 1580.26 \begin{pmatrix} -1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & -1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.90 & 0.00 & 0.20 & -1.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.10 & 1.00 & 0.80 & 0.00 & -1.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.45 & 0.28 & -1.00 & 0.42 & 0.45 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.28 & 0.30 & -1.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.01 & -1.00 \end{pmatrix}.$$

~~For scenario S3 of low efficiency transfers and improved silviculture, the matrix of cycling rates is given by~~ Defining $\mathbf{B} := \mathbf{AC}$ and $u = \mathbf{b}U$, we obtained the steady-state solution as

$$x^* = -\mathbf{B}^{-1} \cdot u$$
$$= \begin{pmatrix} 1.83 \\ 149.52 \\ 1.97 \\ 0.48 \\ 13.69 \\ 0.81 \\ 61.29 \\ 8.27 \end{pmatrix}. \tag{C1}$$

For the simulation with initial conditions as in a land-use-change case, the initial conditions $x_0$ of the simulation where set as

$$x_0 = \begin{pmatrix} 0 & 0 & 0 & 1.5 & 1.5 & 1.0 & 1.0 & 1.0 \end{pmatrix}^{\mathsf{T}} \circ x^*, \tag{C2}$$

where the symbol $\circ$ represents entry-wise multiplication.

For the transient simulations, we derived time-dependent modifiers for inputs $\gamma(t)$ and for process rates $\xi(t)$ following the approach described in Rasmussen et al. (2016). Atmospheric $CO_2$ concentrations increase following a sigmoid curve given by

$$\cancel{\mathbf{B} =,} x_a(t) = 284 + 1715 \exp\left(\frac{0.0305t}{(1715 + \exp(0.0305t) - 1)}\right), \tag{C3}$$

~~which leads to the vector of steady-state contents~~ and surface air temperature increases with $CO_2$ concentrations according to

$$\cancel{x^* =, \|x^*\| = 1750.76.} T_s(t) = T_{s0} + \frac{\sigma}{\ln(2)} \ln(x_a(t)/285). \tag{C4}$$

The ~~modified version of the model in which the active soil pool is replaced by the pool structure of~~ combined effect of $CO_2$ concentrations and air surface temperature on primary production are then computed as

$$\gamma(t) = (1 + \beta(x_a(t), T_s(t)) \ln(x_a(t)/285)), \tag{C5}$$

with

$$\beta(x_a(t), T_s(t)) = \frac{3\rho x_a(t)\Gamma(T_s(t))}{(\rho x_a(t) - \Gamma(T_s(t)))(\rho x_a(t) + 2\Gamma(T_s(t)))}, \tag{C6}$$

where $\beta(x_a(t), T_s(t))$ is the ~~Century model has the following matrix of cycling rates~~

$$
\mathbf{B} = \begin{pmatrix}
-2.08108 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 \\
0.83784 & -0.06858 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 \\
0.00000 & 0.00000 & -0.52174 & 0.00000 & 0.00000 & 0.00000 & 0.00000 \\
0.56757 & 0.03319 & 0.17391 & -0.59259 & 0.00000 & 0.00000 & 0.00000 \\
0.00000 & 0.00442 & 0.08696 & 0.03704 & -0.07175 & 0.00160 & 0.00006 \\
0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.04219 & -0.00380 & 0.00000 \\
0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00029 & 0.00011 & -0.00013
\end{pmatrix},
$$

~~which results in a steady-state vector of pools sizes as~~ sensitivity of primary production with respect to atmospheric $CO_2$ and air surface temperature, and $\rho = 0.65$ is the ratio of intracellular $CO_2$ to $x_a(t)$. The response function with respect to temperature $\Gamma(T_s(t))$ is given by

$$\Gamma(T_s(t)) = 42.7 + 1.68(T_s(t) - 25) + 0.012(T_s(t) - 25)^2. \tag{C7}$$

The separate effect of air surface temperatures on process rates are computed with a power function of the form

$$x^* =, \|x^*\| = 5598.616. \xi(T_s(t)) = \xi_b^{0.1 T_s(t) - 1.5}, \tag{C8}$$

with $\xi_b = 2$.

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*Competing interests.* No competing interests to declare

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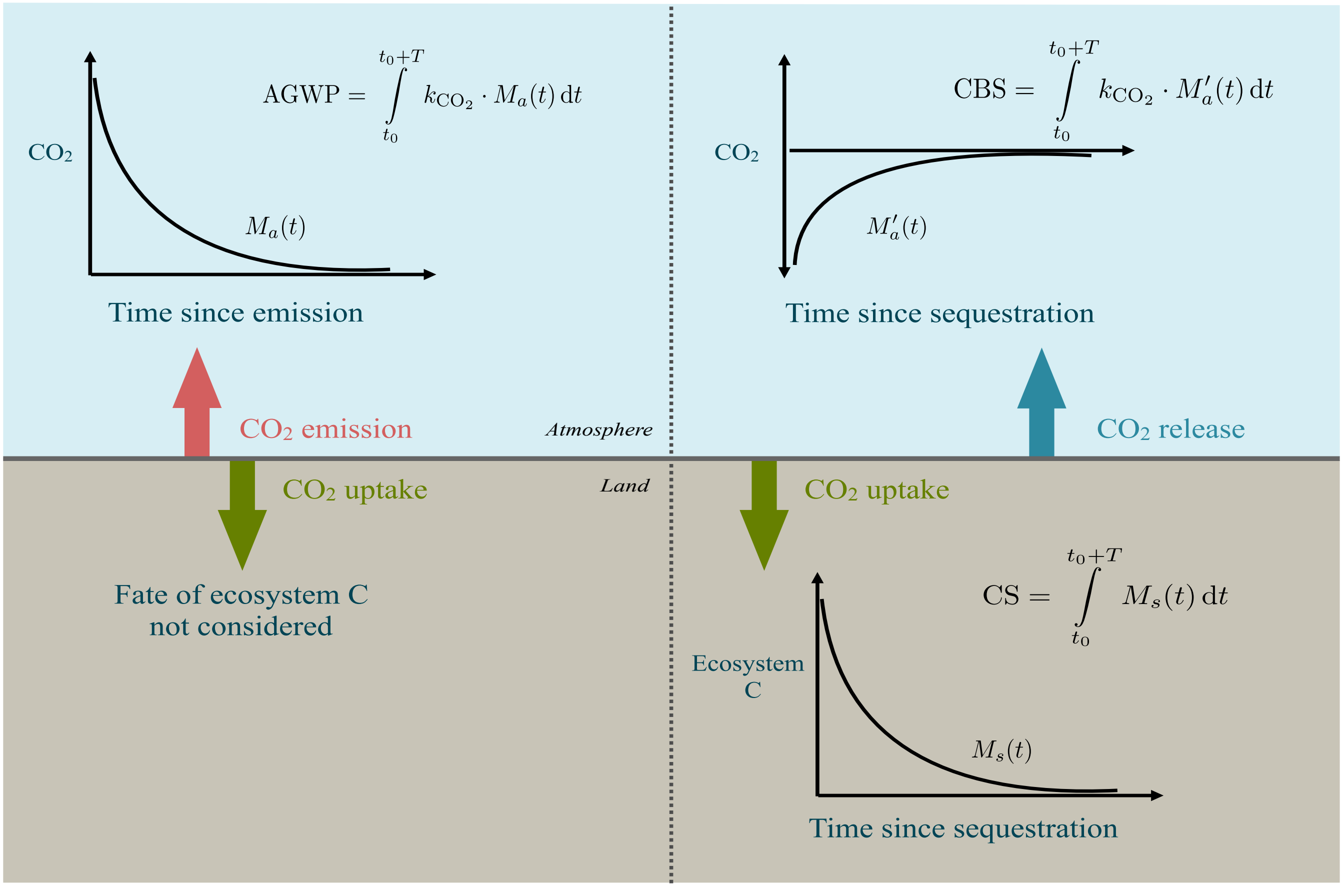


**Figure 1.** Contrast between current approach to quantification of climate effects of emissions and sequestration (left), and the proposed approach for sequestration (right). Plots and equations represent the concepts of absolute global warming potential (AGWP) of an emission of $CO_2$, carbon sequestration (CS), and climate benefits of sequestration (CBS). AGWP integrates over a time horizon $T$ the fate of an instant emission at time $t_0$ of a gas ($M_a(t)$) and multiplies by the radiative efficiency $k$ of the gas. A similar idea can be used to define CS as the integral of the fate $M_s(t)$ of an instant amount of carbon uptake $S_0$ over $T$. The CBS captures the atmospheric 'disturbance' caused by $CO_2$ uptake and subsequent release by respiration as the integral over $T$ of the fate of sequestered carbon $M'_a(t)$ multiplied by the radiative efficiency of $CO_2$.

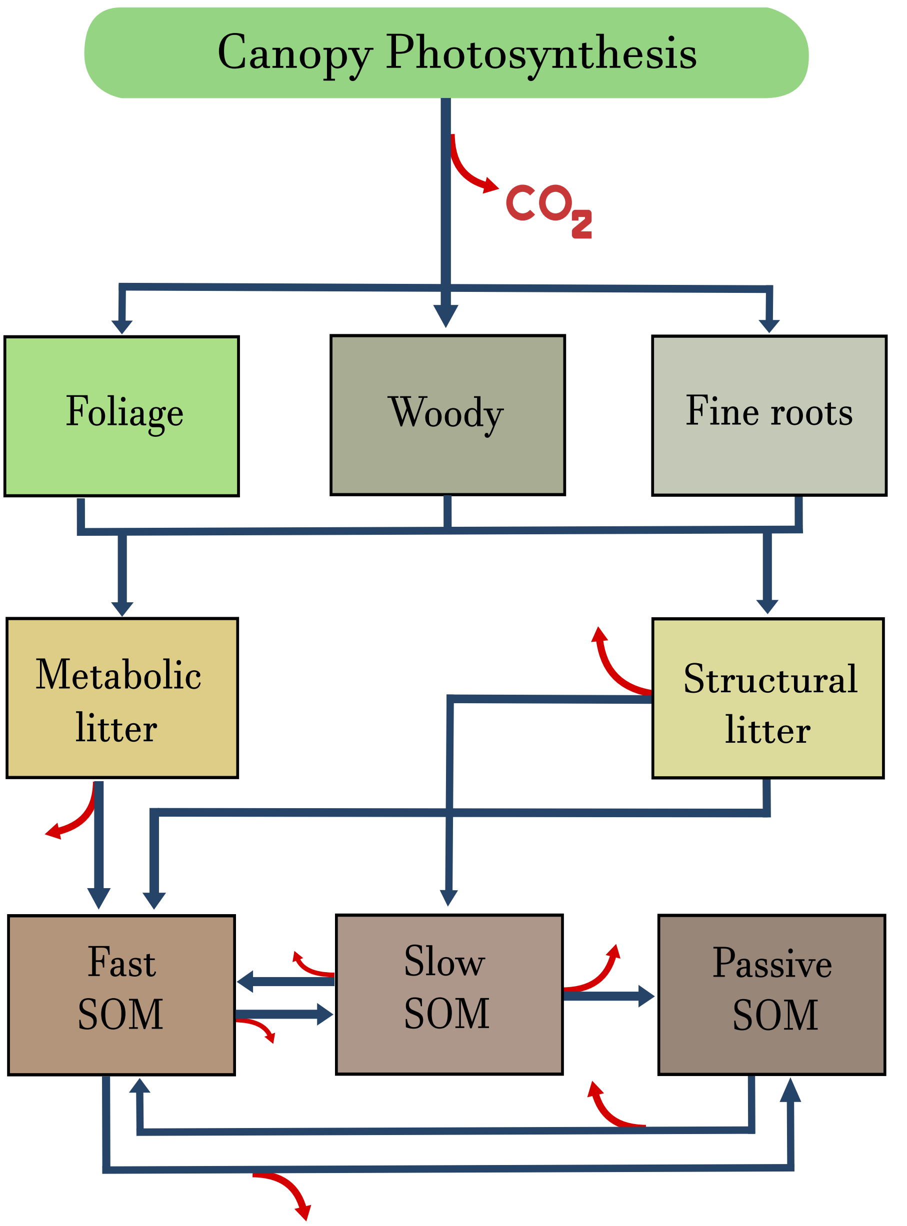


**Figure 2.** Graphical representation of the terrestrial ~~carbon cycle~~ ecosystem model ~~developed by Emanuel et al. (1981) with the modified structure used for the examples presented~~ TECO described in ~~this manuscript. The original model is presented in the center of the figure in black with respective values for stocks~~ Weng and Luo (2011) and ~~fluxes~~Luo et al. (2012). ~~The modified version used in Example 2 introduced~~ Carbon enters the ~~wood-product~~ ecosystem through canopy photosynthesis and is allocated to three biomass pools~~presented in blue on the left of the diagram~~: foliage, woody biomass and fine roots. ~~The modified version used in Example 3~~From these pools, ~~replaces~~ carbon is transferred to metabolic and structural litter pools, from where it can be respired as $CO_2$ or transferred to the ~~active~~ soil ~~pool by~~ organic matter (SOM) pools. Blue arrows represent transfers among compartments and red arrows release to the ~~pool structure used~~ atmosphere in the ~~Century model~~form of $CO_2$.

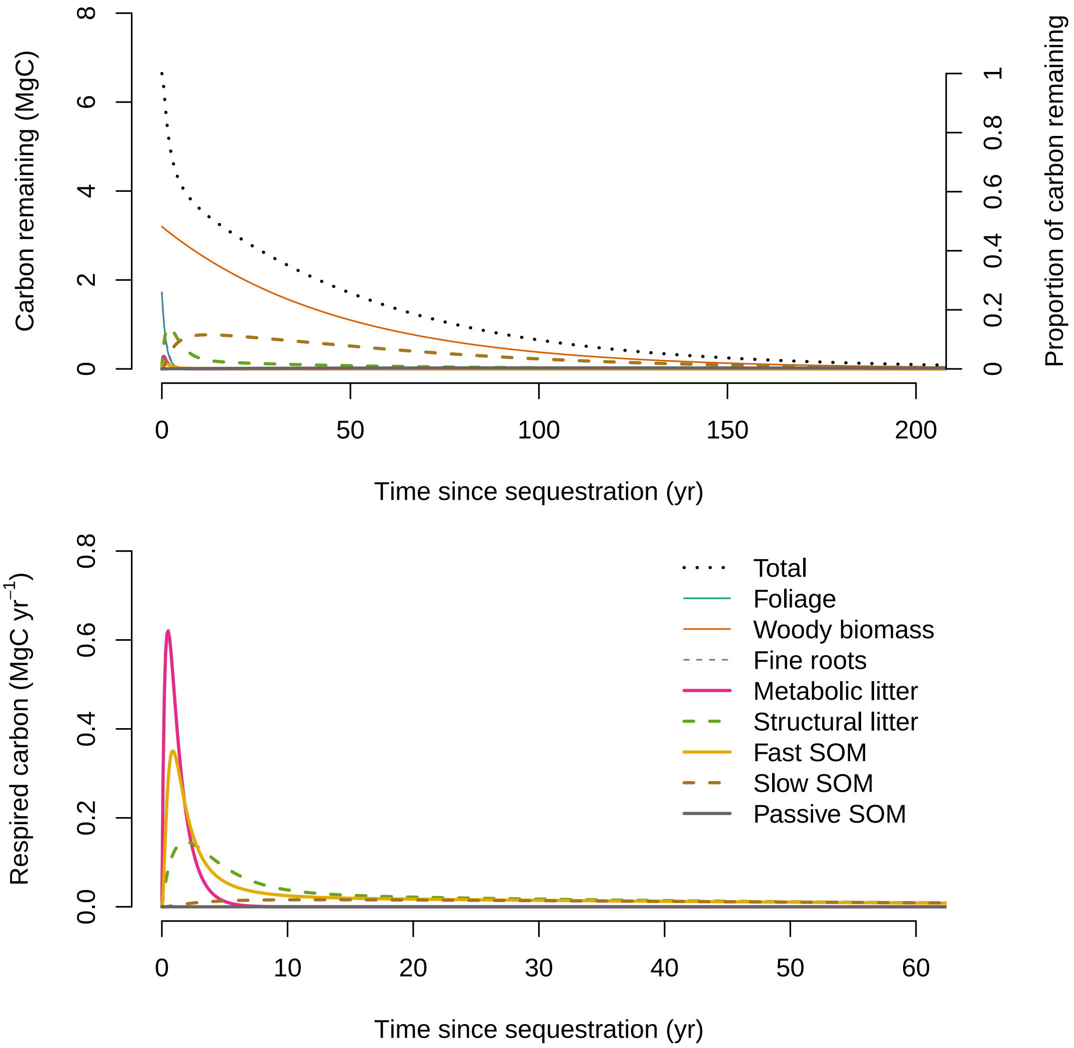


**Figure 3.** Fate of carbon ($M_s(t)$ left axis and $M_{s1}(t)$ right axis) entering the terrestrial biosphere according to the TECO model ~~of Emanuel et al. (1981)~~ parameterized for the Duke Forest and calculated using equation (13) for the upper panel, and respired carbon ($r(t)$) returning back to the atmosphere calculated using equations (15).

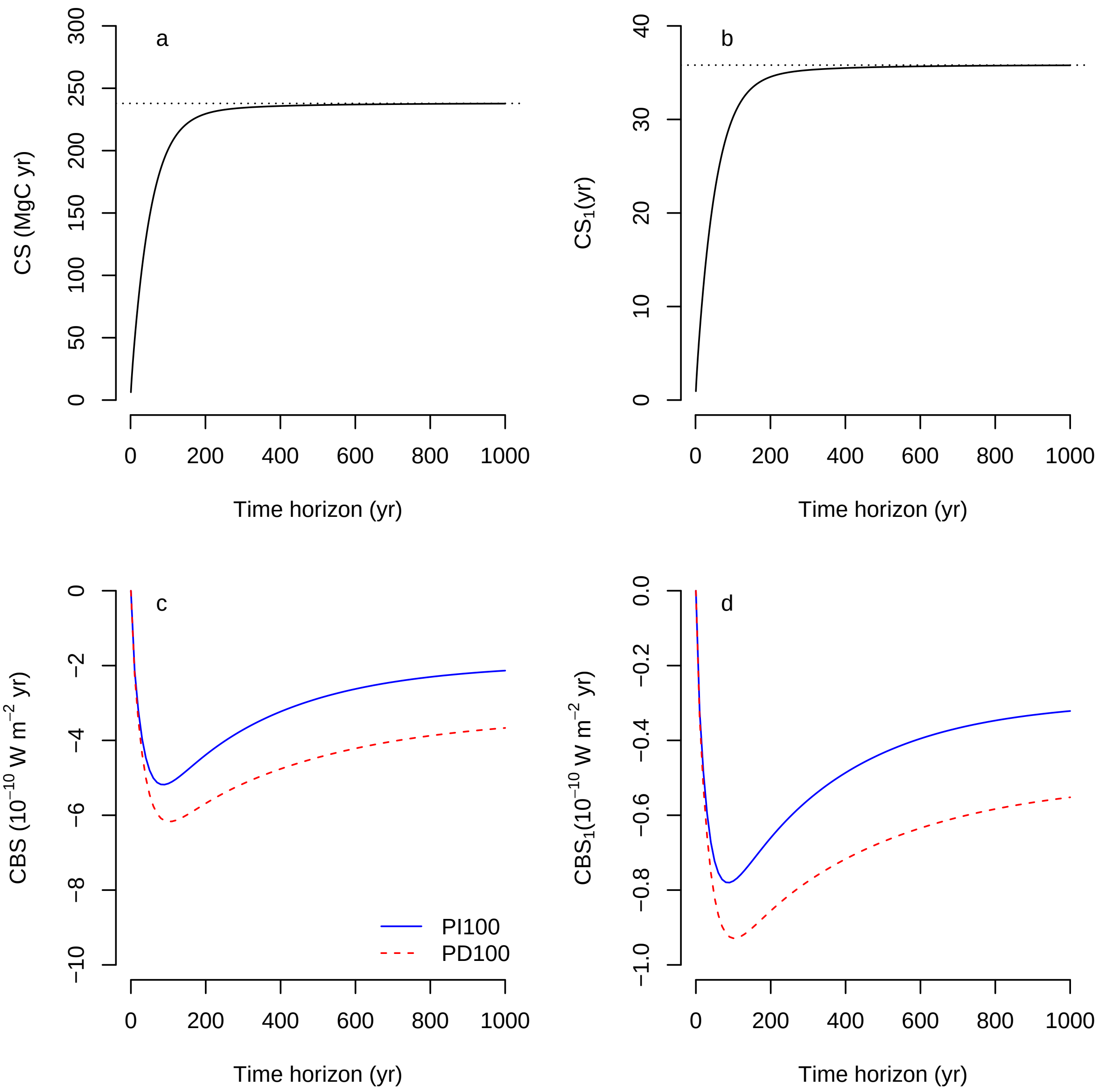


**Figure 4.** Carbon sequestration (CS) ~~,~~ and climate benefit of sequestration (CBS) ~~, and absolute global warming potential (AGWP)~~ for instantaneous carbon uptake ~~or emissions~~at any given time. a) CS due to the uptake of ~~113 PgC~~6.64 MgC ha$^{-1}$, b) CS due to the uptake of one unit of carbon ($CS_1$). c) CBS due to the uptake of ~~113 PgC~~ 6.64 MgC ha$^{-1}$ for two different impulse response functions (~~IRF~~PI100 and PD100). d) ~~AGWP~~ CBS due to the ~~emission~~ uptake of ~~1 PgCO$_2$-C to the atmosphere~~ one unit of carbon ($CBS_1$) for two different ~~IRFs, the pre-industrial experiment from Joos et al. (2013) and the~~ impulse response function~~reported in Lashof and Ahuja (1990)~~. ~~Dashed~~ Dotted lines in panels a and b represent ~~the~~ steady-state carbon storage and mean transit time, respectively.

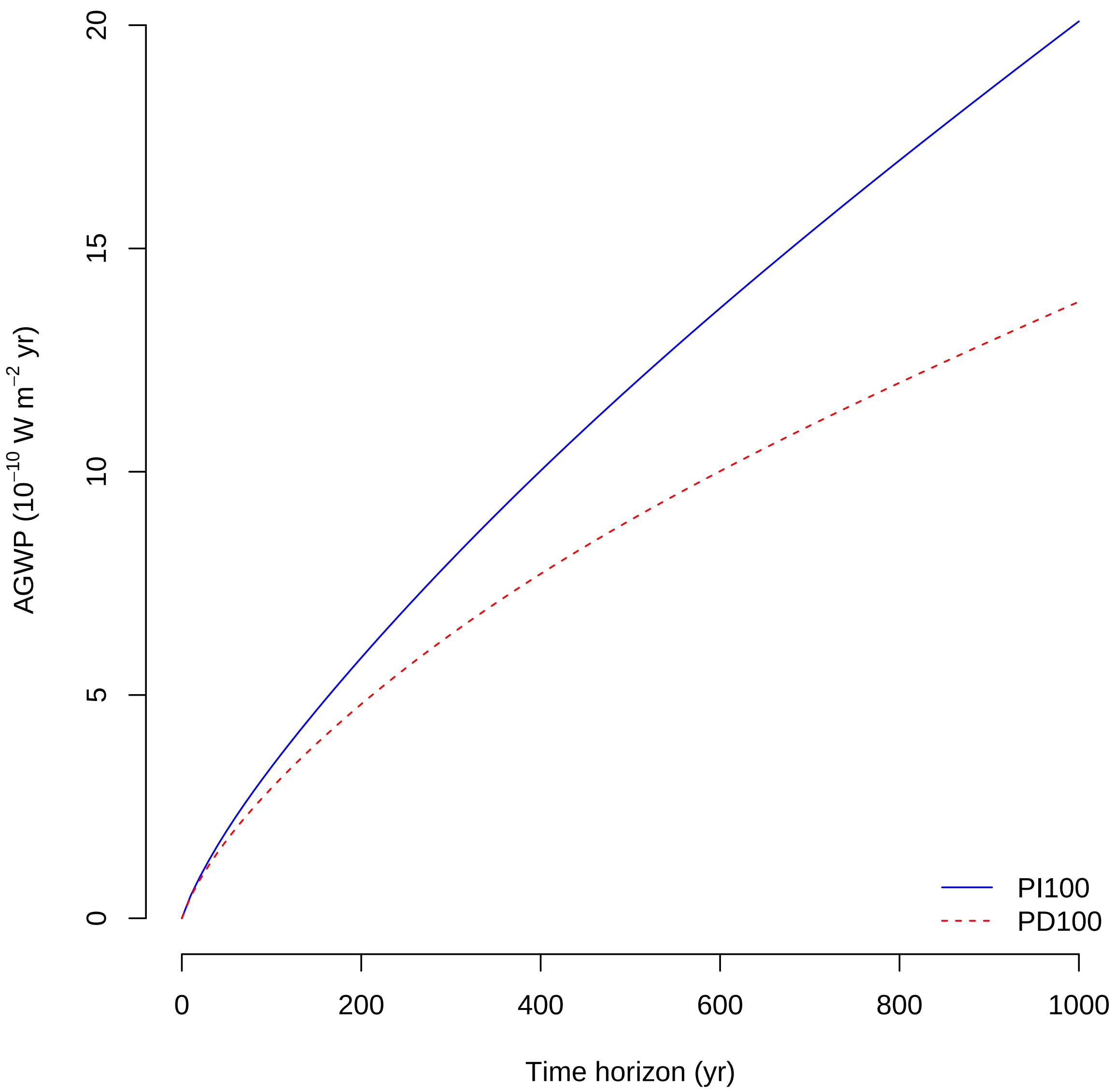


**Figure 5.** Absolute global warming potential (AGWP) due to the emission of 1 Mg$CO_2$-C to the atmosphere for the two different IRFs (PI100 and PD100) reported by Joos et al. (2013).

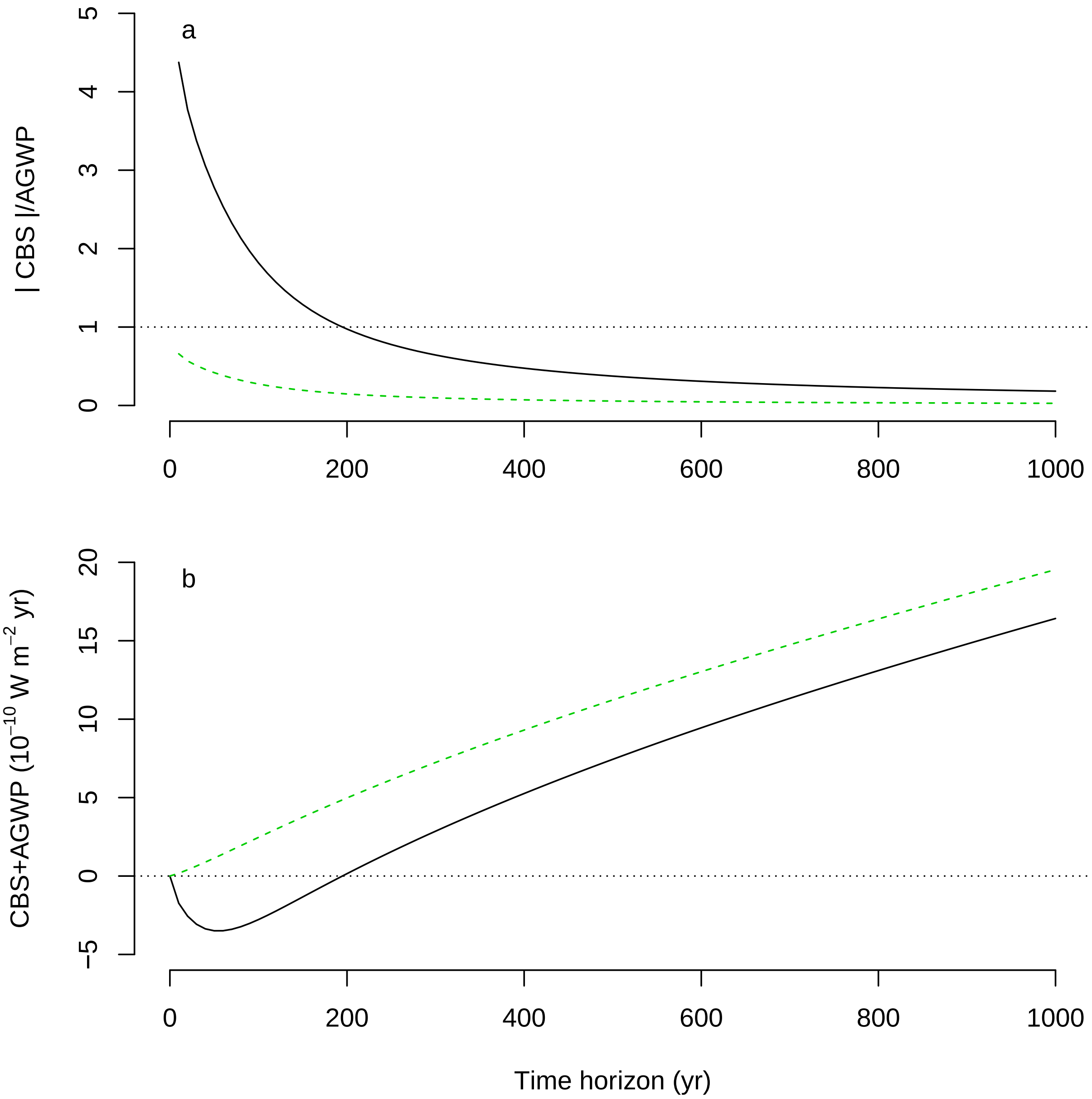


**Figure 6.** Relations between CBS and AGWP for the IRF PD100 as a function of time horizon $T$. a) Ratio between the absolute value of CBS and AGWP, based on a total sequestration 6.6 MgC (back line, NPP equivalent for one hectare and one year at Duke forest), versus a sequestration of 1 MgC (dashed green line). b) Radiative balance (net difference) between CBS and AGWP for the sequestration of 6.6. MgC (black line), and 1 MgC (dashed green line).

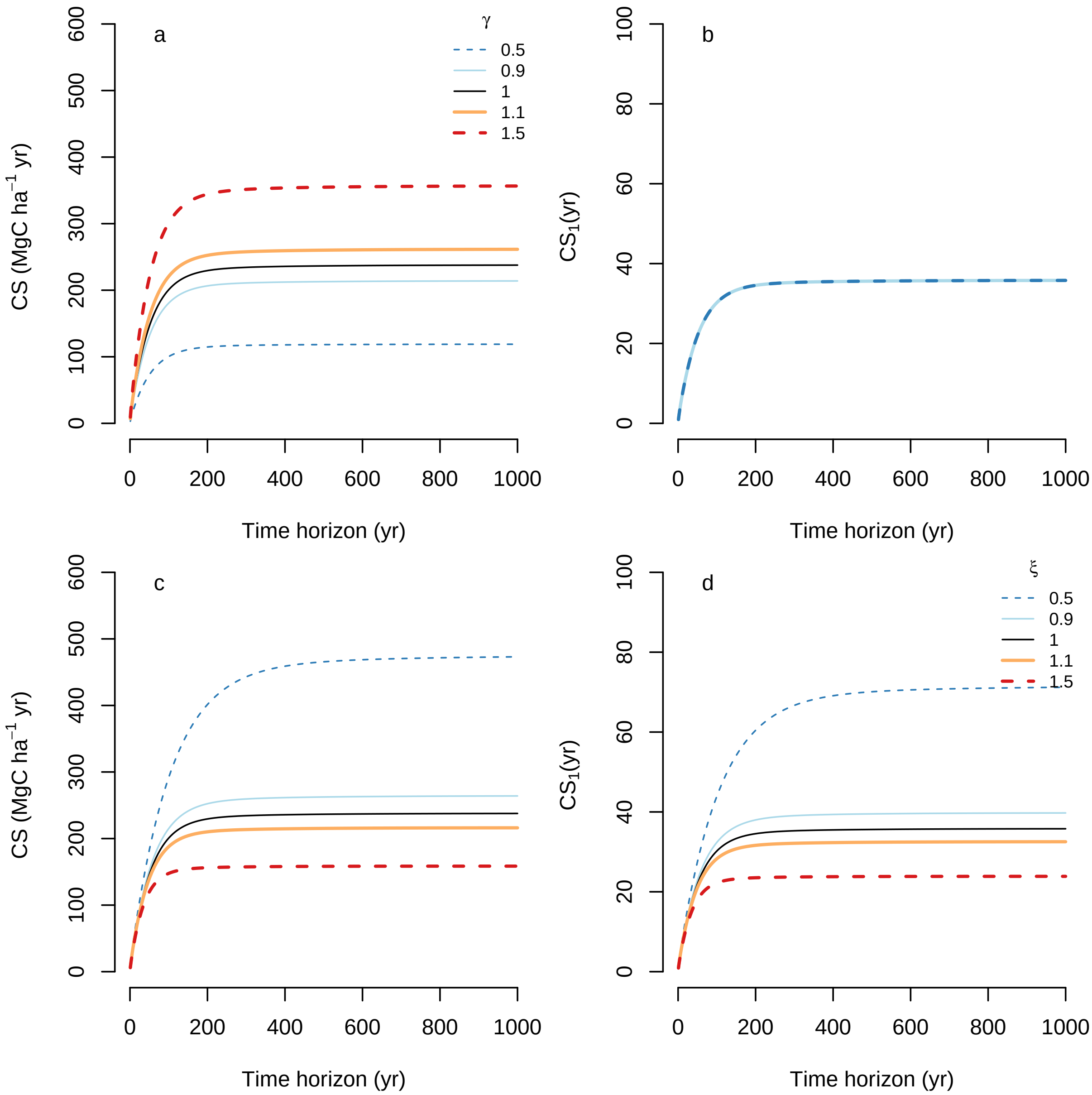


**Figure 7.** Different carbon management strategies and their effect on the CS and $\mathrm{CS}_1$. Management to increase or decrease carbon inputs in the vector $u$ by specific proportions $\gamma$ are shown in panels a and b. Management to increase or decrease process rates in the matrix $\mathbf{B}$ by a proportion $\xi$ are shown in panels c and d. Since $\mathrm{CS}_1$ quantifies carbon sequestration of one unit of carbon, management of the amount of carbon inputs does not modify $\mathrm{CS}_1$ in panel b, and all lines overlap.

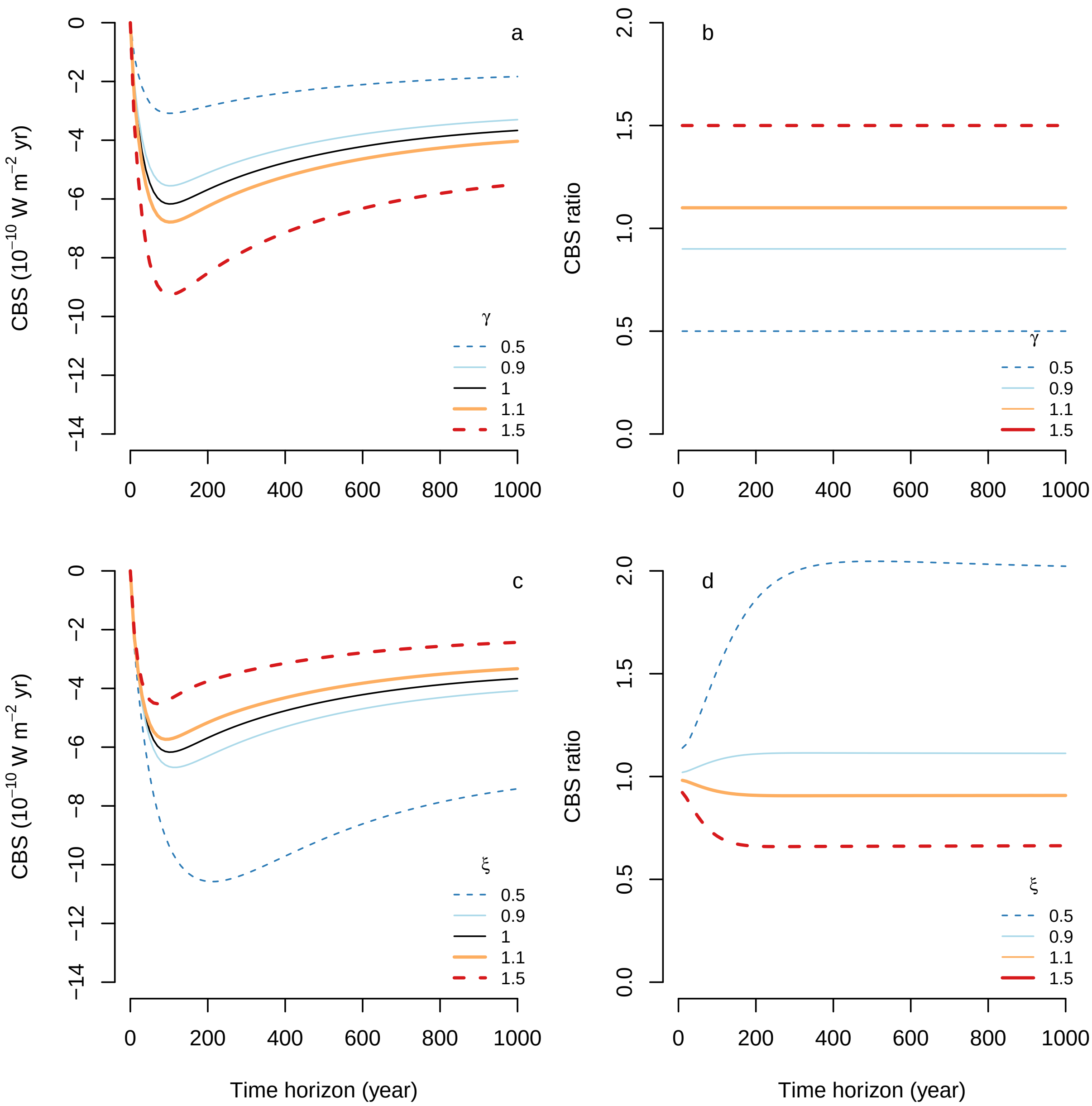


**Figure 8.** Effects of different management strategies on CBS. ~~The upper panel shows the effect~~ a) Effect of increasing or decreasing carbon inputs by a proportion $\gamma$ on CBS, ~~while~~ b) same effect of $\gamma$ expressed as a ratio with respect to the ~~lower panel shows the~~ reference case of $\gamma = 1$. c) effects of decreasing or increasing process rates in the matrix $\mathbf{B}$ by a proportion $\xi$ on CBS, d) same effect of $\xi$ expressed as a ratio with respect to the reference case $\xi = 1$.

~~Additional pools included in the model to represent the fate of harvested wood from the tree woody pool. $B_i$ represents the cycling rate of each pool in units of $yr^{-1}$. Harvest distributions represent the proportion of the harvested wood that is transferred to each pool. $B_i$ Harvest distribution, S1 Harvest distribution, S2 and S3 Long duration $x_6$ 0.02 0.60 0.30 Mid duration $x_7$ 0.04 0.20 0.20 Short duration $x_8$ 0.32 0.10 0.10 Bioenergy $x_9$ 0.70 0.05 0.30 Waste $x_{10}$ 1.00 0.05 0.10~~

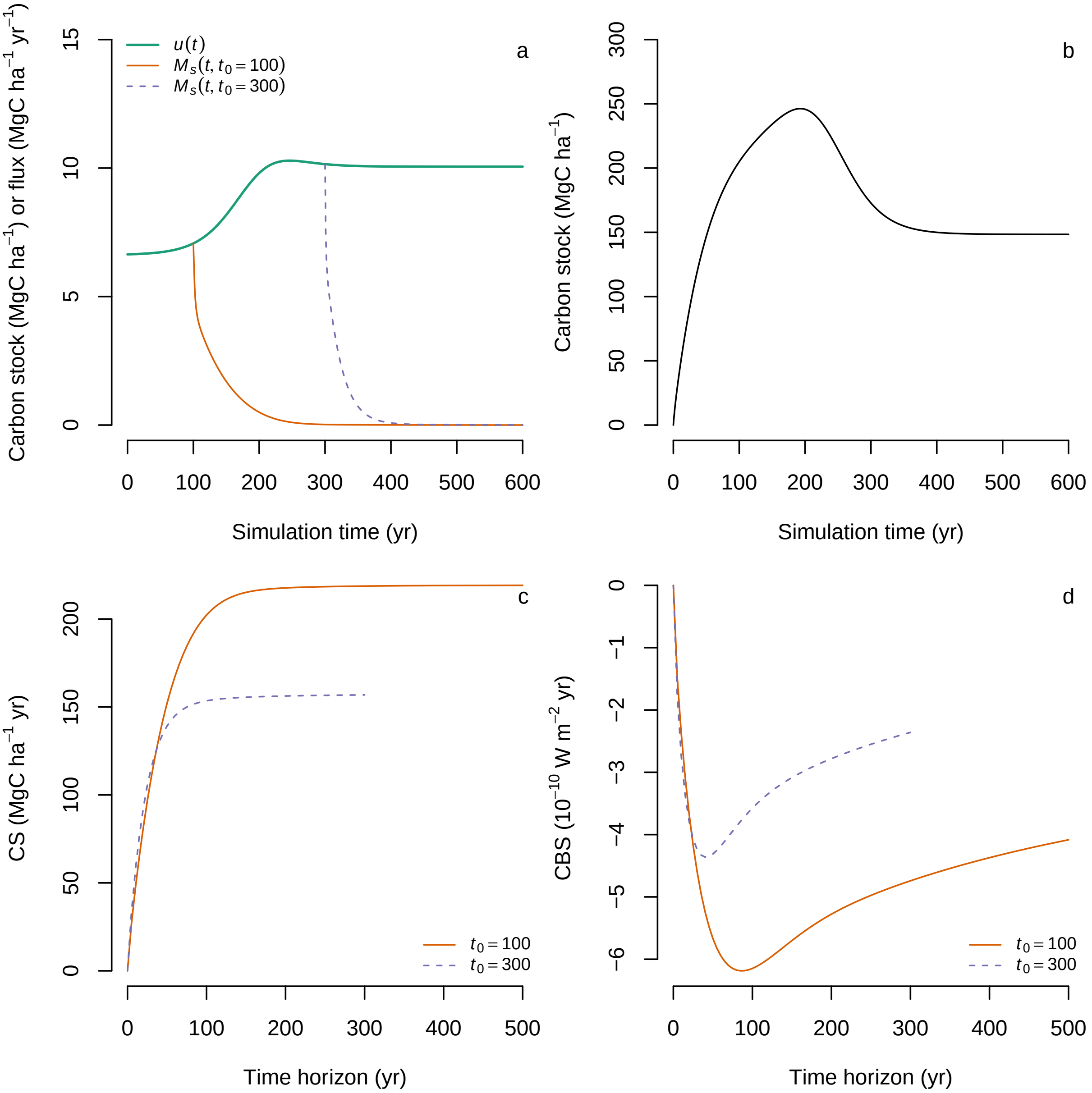


**Figure 9.** Prediction of CS and CBS for a non-steady-state case with time-dependent inputs $u(t)$ controlled by $CO_2$ fertilization and temperature, and process rates controlled by temperature modified by a time-dependent factor $\xi(t)$. a) Predicted time-dependent inputs $u(t)$, and the fate of carbon entering the ecosystem at simulation year 100 ($M_s(t, t_0 = 100)$) and simulation year 300 ($M_s(t, t_0 = 300)$). b) Predicted carbon accumulation in the ecosystem ($\|x(t)\|$) for the entire simulation period. c) Carbon sequestration for the amount of inputs entering at simulation years 100 and 300 calculated for different time horizons $T$. d) Climate benefit of sequestration for carbon entering the ecosystem at simulation years 100 and 300 integrated for different time horizons $T$.

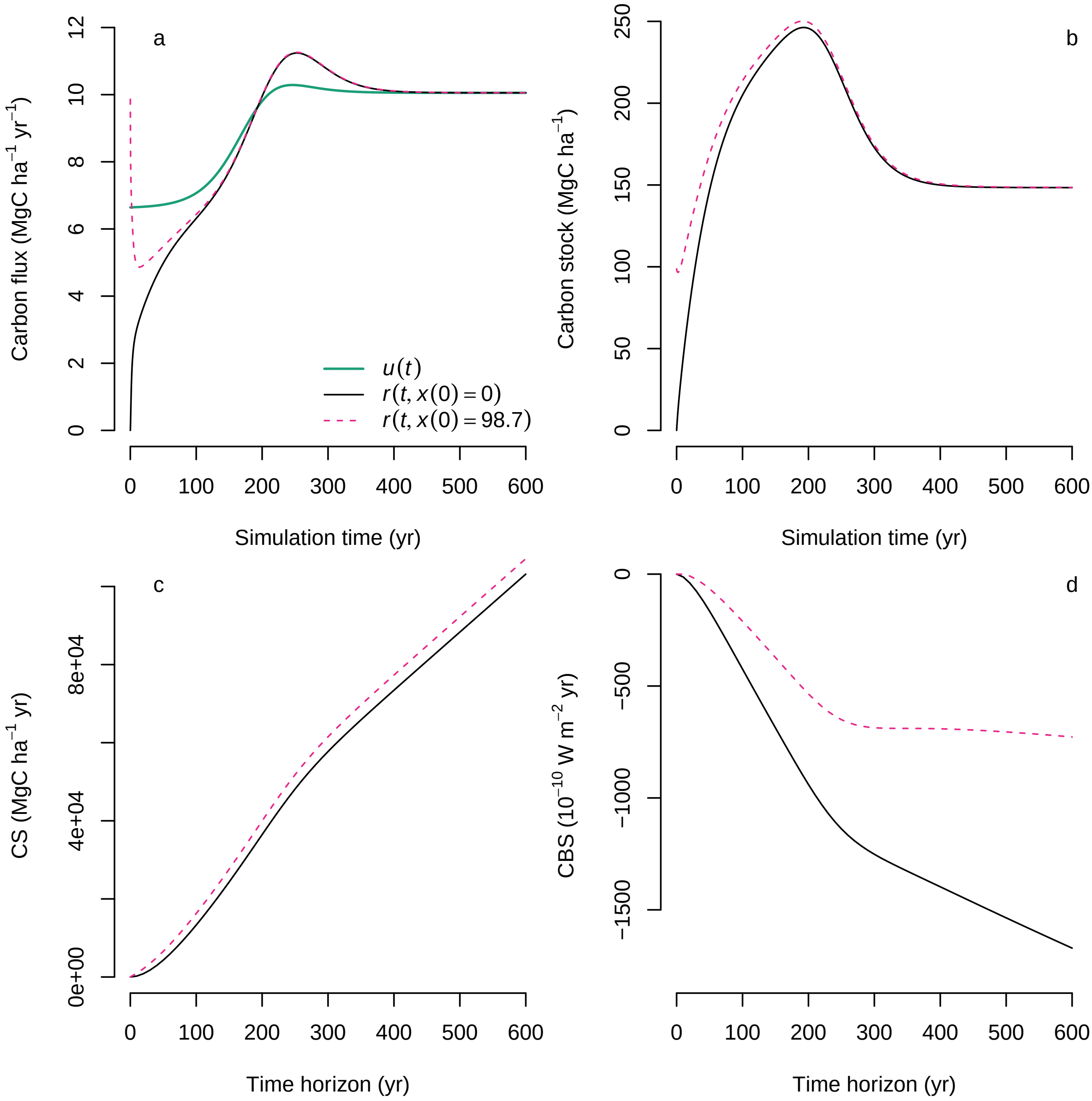


**Figure 10.** ~~Effect~~ Computation of ~~replacing the active soil pool~~ CS and CBS for continuous inputs and release of ~~the Emanuel et al. (1981) model~~ carbon in simulations with different initial conditions $x_0$: in one simulation the ~~pool structure of the Century model described~~ ecosystem develops from empty pools ($x(0) = 0$, i.e. bare ground, black lines), and in ~~Parton et al. (1987)~~the second simulation the ecosystem develops from existing litter and SOM pools, but empty biomass pools ($x(0) = 98.7$ MgC ha$^{-1}$, dashed magenta color lines). a) ~~Differences in~~ Inputs $u(t)$ and release fluxes $r(t)$ along the ~~proportion of carbon remaining in~~ simulation time. b) Carbon stocks predicted by the model ~~with soil structure according to Century versus~~ along the ~~original model, b~~simulation time. c) ~~carbon~~ Carbon sequestration CS for ~~all scenarios, and e~~a sequence of time horizons. d) ~~climate~~ Climate benefit of sequestration CBS for ~~all scenarios~~a sequence of time horizons.