Supplemental Materials

Supplemental Table 1: List of CON and AWB model parameters.

Model	Parameter	Value, if not varied	Units	Parameter Description
CON/AWB	I_S	0.0009	mg C g^{-1} soil h^{-1}	External SOC input rate
CON/AWB	I_D	0.0001	mg C g^{-1} soil h^{-1}	External DOC input rate
CON	$k_{S_{ref}}$	Dependent	mg C mg^{-1} C h^{-1}	SOC decay constant
CON	$k_{D_{ref}}$	Dependent	mg C mg^{-1} C h^{-1}	DOC decay constant
CON	$k_{M_{ref}}$	Dependent	mg C mg^{-1} C h^{-1}	MIC decay constant
CON	u_M	0.002	mg C g^{-1} DOC h^{-1}	DOC uptake rate of microbes
CON	Ea_S	Fitted by HMC	kJ mol^{-1}	SOC activation energy
CON	Ea_D	Fitted by HMC	kJ mol^{-1}	DOC activation energy
CON	Ea_M	Fitted by HMC	kJ mol^{-1}	MIC activation energy
CON	a_{DS}	Fitted by HMC		DOC to SOC transfer coefficient
CON	a_{SD}	Fitted by HMC		SOC to DOC transfer coefficient
CON	a_M	Fitted by HMC		MIC to SOC transfer coefficient
CON	a_{MS}	Fitted by HMC		Fraction of dead MIC transferred
AWB	K_{ref}	Dependent	mg C g^{-1} soil	SOC reference K_M
AWB	$K_{U_{ref}}$	Dependent	mg C g^{-1} soil	DOC uptake into MIC reference K_M
AWB	V_{ref}	Fitted by HMC	mg C mg^{-1} C h^{-1}	SOC reference *Vmax*
AWB	$V_{U_{ref}}$	Fitted by HMC	mg C mg^{-1} MIC h^{-1}	DOC uptake into MIC reference V_{max}
AWB	Ea_K	Fitted by HMC	kJ mol^{-1}	SOC K_M activation energy
AWB	Ea_{KU}	Fitted by HMC	kJ mol^{-1}	DOC uptake K_M activation energy
AWB	Ea_V	Fitted by HMC	kJ mol^{-1}	SOC V_{max} activation energy
AWB	Ea_{VU}	Fitted by HMC	kJ mol^{-1}	DOC uptake V_{max} activation energy
AWB	r_E	Dependent	mg C mg^{-1} MIC h^{-1}	Enzyme production rate
AWB	r_L	0.0005	mg C mg^{-1} C h^{-1}	Enzyme loss rate
AWB	r_M	Dependent	mg C mg^{-1} C h^{-1}	MIC death rate
AWB	$E_{C_{ref}}$	Fitted by HMC	mg C mg^{-1} C	Reference temperature C use efficiency (CUE)
AWB	m_t	Fitted by HMC	°C^{-1}	CUE temperature change slope

Supplemental Appendix 1

(a) CON ODE system equations

The conventional (CON) model consists of three C pools in SOC, DOC, and MIC. The mass transfer of C between these pools is represented as first-order linear decay processes. The CON model obeys the following dynamics:

$$\frac{dS}{dt} = I_S + a_{DS}k_D D + a_M a_{MS} k_M M - k_S S \tag{1}$$

$$\frac{dD}{dt} = I_D + a_{SD}k_S S + a_M(1 - a_{MS})k_M M - u_M D - k_D D \tag{2}$$

$$\frac{dM}{dt} = u_M D - k_M M \tag{3}$$

The decay constants k_I vary from their reference values k_{Iref} based on the Arrhenius equation of temperature dependence,

$$k_I = k_{I_{ref}} exp\left[-\frac{Ea_I}{R}\left(\frac{1}{T} - \frac{1}{T_{ref}}\right)\right] \tag{4}$$

where R is the ideal gas constant 8.314 J mol^{-1} K^{-1} and the reference temperature T_{ref} used was 283.15 K.

CO_2 soil flux is calculated from the CON model by summing the proportion of fluxes that do not enter soil C pools at each time step:

$$\text{CON flux } = k_S S(1 - a_{SD}) + k_D D(1 - a_{DS}) + k_M M(1 - a_M) \tag{5}$$

Response ratios are then calculated from the model output flux by dividing the flux calculated at a given time point by the pre-warming steady state flux.

(b) AWB ODE system equations

The Allison-Wallenstein-Bradford (AWB) model consists of four C pools in SOC, DOC, MIC, and ENZ (representing the extracellular enzyme C mass). In the AWB model, MIC accumulation and SOC decomposition follow a non-linear Michaelis-Menten function. Other processes, including ENZ production, ENZ loss, and MIC death still follow a first-order linear decay process. The AWB system equations are as follows:

$$\frac{dS}{dt} = I_S + a_{MS}r_M M - \frac{VES}{K + S} \tag{6}$$

$$\frac{dD}{dt} = I_D + (1 - a_{MS})r_M M + \frac{VES}{K + S} + r_L E - \frac{V_U MD}{K_U + D} \tag{7}$$

$$\frac{dM}{dt} = E_C \frac{V_U MD}{K_U + D} - r_M M - r_E M \tag{8}$$

Similar to the CON decay constants, the Michaelis-Menten function parameters K, K_U, V, and V_U vary from their reference values based on the Arrhenius equation. E_C, the AWB microbial C use efficiency parameter, depends linearly on temperature, following Li et al., 2014, and operates under the simplifying assumption that higher temperatures make C use slightly less efficient:

$$E_C = E_{C_{ref}} - m_t(T - T_{ref}) \tag{9}$$

The loss rate parameters r_I were not made to be temperature dependent.

AWB CO_2 flux is calculated as the proportion of the C transfer out of the DOC pool that is not partitioned into the MIC pool:

$$\text{AWB flux} = (1 - E_C)\frac{V_U M D}{K_U + D} \quad (10)$$

Supplemental Appendix 2

(a) Re-arranged CON steady state equations

The steady state solutions for the C pools in CON are as follows:

$$D_0 = \frac{a_{SD} I_S + I_D}{u_M + k_D + u_M a_M (a_{MS} - a_{MS} a_{SD} - 1) - a_{DS} k_D a_{SD}} \quad (11)$$

$$M_0 = \frac{u_M}{k_M} D \quad (12)$$

$$S_0 = \frac{I_S + D(a_{MS} k_D + u_M a_M a_{MS})}{k_S} \quad (13)$$

To set pre-warming steady state soil C densities to desired values, we re-arranged the steady state equations into the following forms to solve for the steady state values of parameters that depend on the soil C densities:

$$k_{Mref} = \frac{u_M D_0}{M_0} \quad (14)$$

$$k_{Dref} = \frac{-I_D - a_{SD} I_S + u_M D_0 - a_M D_0 u_M + a_M a_{MS} u_M D_0 - a_M a_{MS} a_{SD} u_M D_0}{(a_{DS} a_{SD} - 1) D_0} \quad (15)$$

$$k_{Sref} = \frac{I_S + D_0 \left(a_{DS} k_{Dref} + u_M a_M a_{MS} \right)}{S_0} \quad (16)$$

(b) Re-arranged AWB steady state equations

The steady state solutions for the C pools in AWB are as follows:

$$S_0 = \frac{-r_L K \left(I_S \left(r_M \left(1 + E_C (a_{MS} - 1)\right) + r_E (1 - E_C) \right) + E_C I_D a_{MS} r_M \right)}{I_S \left(r_M \left(r_L \left(1 + E_C (a_{MS} - 1)\right)\right) + r_E \left(r_L (1 - E_C) - E_C V\right) \right) + E_C I_D (a_{MS} r_M r_L - r_E V)} \quad (17)$$

$$M_0 = \frac{E_C (I_D + I_S)}{(1 - E_C)(r_M + r_E)} \quad (18)$$

$$D_0 = \frac{-K_U (r_M + r_E)}{r_M + r_E - E_C V_U} \quad (19)$$

$$E_0 = \frac{r_E M}{r_L} \quad (20)$$

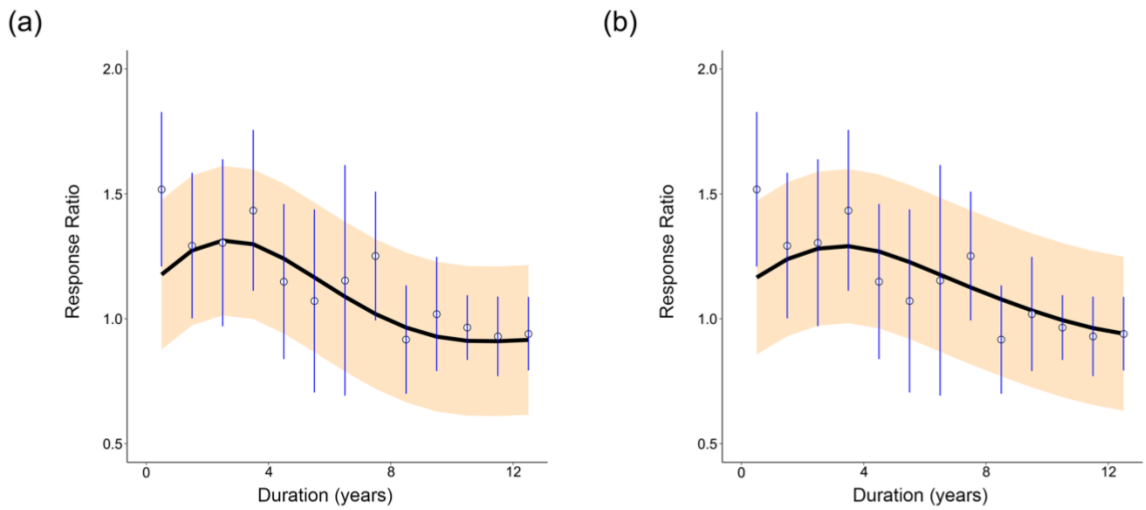
To set pre-warming steady state soil C densities to desired values, we re-arranged the steady state equations into the following forms:

$$r_{\mathrm{M}} = \frac{-E_{C_{ref}} (I_D + I_S) + M_0 r_E \left(1 - E_{C_{ref}}\right)}{M_0 \left(E_{C_{ref}} - 1 \right)} \quad (21)$$

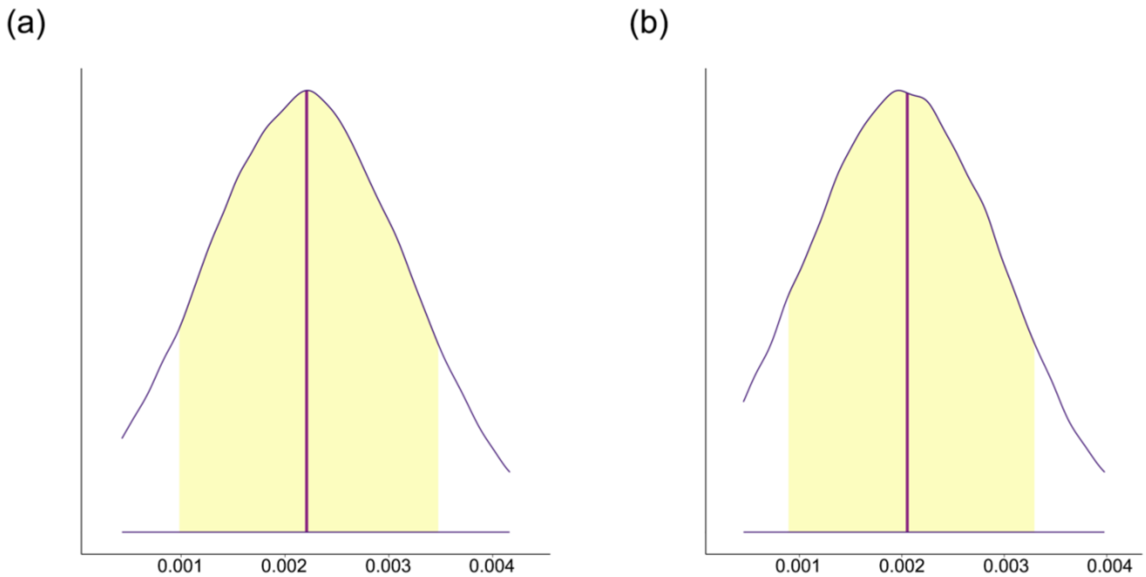
$$K_{U_{ref}} = \frac{-D_0 \left(r_M + r_E - E_{C_{ref}} V_{U_{ref}} \right)}{r_M + r_E} \quad (22)$$

$$K_{ref} = \frac{-S_0\left(-I_S r_E r_L + E_{C_{ref}} I_S r_E r_L - a_{MS} E_{C_{ref}} I_D r_L r_M - I_S r_L r_M + E_{C_{ref}} I_S r_L r_M - a_{MS} E_{C_{ref}} I_S r_L r_M + E_{C_{ref}} I_D r_E V_{ref} + E_{C_{ref}} I_S r_E V_{ref}\right)}{r_L\left(-I_S r_E + E_{C_{ref}} I_S r_E - a_{MS} E_{C_{ref}} I_D r_M - I_S r_M + E_{C_{ref}} I_S r_M - a_{MS} E_{C_{ref}} I_S r_M\right)} \tag{23}$$

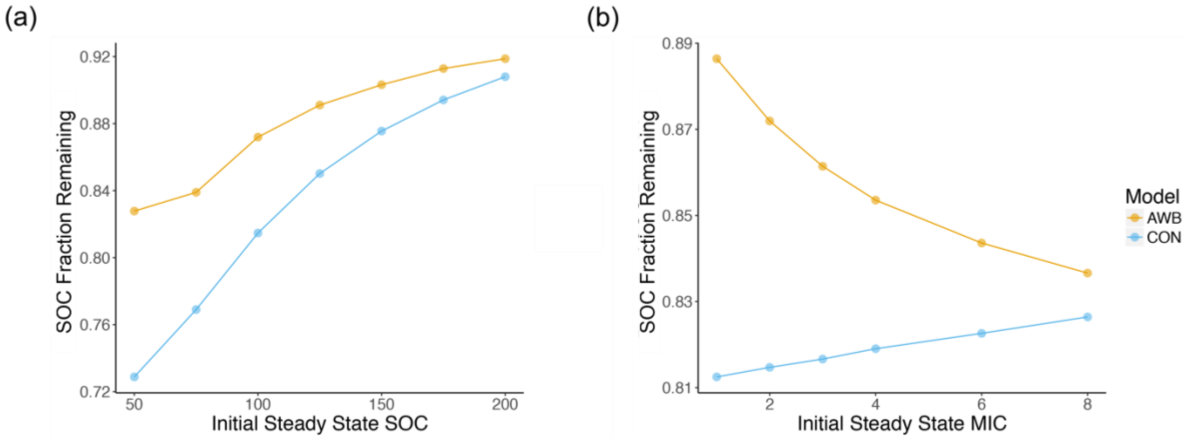
Supplemental Figure 1: Distribution of AWB fits to meta-analysis data (Romero-Olivares et al., 2017) with **(a)** MIC = 1 mg C g^{-1} soil; and **(b)** MIC = 8 mg C g^{-1} soil. Open circles show the meta-analysis data points. Blue vertical lines mark the 95% confidence interval for each data point calculated from the pooled standard deviation. The black line indicates the mean model fit. The orange shading marks the 95% posterior predictive interval for the fit. For **(a)** and **(b)**, the non-MIC pre-warming steady state soil C densities were set at SOC = 100 mg C g^{-1} soil, DOC = 0.2 mg C g^{-1} soil, and ENZ = 0.1 mg C g^{-1} soil.



Supplemental Figure 2: 95% credible areas for **(a)** AWB V_{Uref}; and **(b)** m_t parameters corresponding to pre-warming steady state SOC = 100 mg C g^{-1} soil, DOC = 0.2 mg C g^{-1} soil, MIC = 2 mg C g^{-1} soil, and ENZ = 0.1 mg C g^{-1} soil.



Supplemental Figure 3: Fraction of SOC remaining 12 years after warming perturbation in AWB and CON simulations. **(a)** Pre-warming steady state SOC varied from 50 to 200 mg C g^{-1} soil, with pre-warming MIC, DOC and ENZ held constant respectively at 2 mg C g^{-1} soil, 0.2 mg C g^{-1} soil, and 0.1 mg C g^{-1} soil; **(b)**, Pre-warming MIC varied from 1 to 8 mg C g^{-1} soil, with pre-warming SOC, DOC and ENZ held constant, respectively, at 100 mg C g^{-1} soil, 0.2 mg C g^{-1} soil, and 0.1 mg C g^{-1} soil.



Supplemental Table 2: AWB and CON prior distribution tables. Including σ, the residual error scale term, we fit 10 parameters in our AWB runs and 8 parameters in our CON runs. Normal, Gaussian priors were used for all fitted ODE model parameters. The notation we use for our normal distributions follows an N(mean, standard deviation) format. The Markov chain guess-scaling parameter, σ, was drawn from a more weakly informative half-Cauchy distribution per recommendations from literature (Gelman, 2006).

(a) CON priors

Parameter	Distribution	Parameter Description
Ea_S	N(50,25)	SOC activation energy
Ea_D	N(50,25)	DOC activation energy
Ea_M	N(50,25)	MIC activation energy
a_{DS}	N(0.3,0.15)	DOC to SOC transfer coefficient
a_{SD}	N(0.3,0.15)	SOC to DOC transfer coefficient
a_M	N(0.3,0.15)	MIC to SOC transfer coefficient
a_{MS}	N(0.5,0.25)	Fraction of dead MIC transferred
σ	Cauchy(0,1)	Residual Error Scale

(b) AWB priors

Parameter	Distribution	Parameter Description
V_{ref}	N(0.4,0.2)	SOC reference V_{max}
$V_{U_{ref}}$	N(0.01,0.005)	DOC reference V_{max}
Ea_V	N(50,25)	SOC V_{max} activation energy
Ea_{VU}	N(50,25)	DOC V_{max} activation energy
Ea_K	N(50,25)	SOC K_m activation energy
Ea_{KU}	N(50,25)	DOC K_m activation energy
$E_{C_{ref}}$	N(0.4,0.2)	Reference C use efficiency (CUE)
m_t	N(0.002,0.001)	CUE slope
a_{MS}	N(0.5,0.25)	Fraction of dead MBC transferred to SOC
σ	Cauchy(0,1)	Residual Error Scale

Supplemental Table 3: Posterior means calculated for parameters that were fit in HMC runs are displayed in the following tables.

(a) CON posterior distribution means for SOC-varied runs

Parameter	SOC = 50	SOC = 75	SOC = 100	SOC = 125	SOC = 150	SOC = 175	SOC = 200
Ea_S	77.5688	73.5806	66.6395	60.8239	56.6663	53.5904	51.3607
Ea_D	50.1994	50.327	50.2519	50.1629	50.1603	50.2286	50.1584
Ea_M	52.4829	52.4331	52.2358	52.0848	51.9255	51.9786	51.7802
a_{DS}	0.3246	0.3256	0.3271	0.3263	0.3262	0.3263	0.3271
a_{SD}	0.3337	0.3351	0.3363	0.3363	0.337	0.3364	0.337
a_M	0.3384	0.3397	0.337	0.3341	0.3333	0.3325	0.3314
a_{MS}	0.5048	0.5032	0.5006	0.5001	0.4962	0.4971	0.496
σ	0.1338	0.139	0.1552	0.1677	0.1765	0.183	0.1875

(b) AWB posterior distribution means for SOC-varied runs

Parameter	SOC = 50	SOC = 75	SOC = 100	SOC = 125	SOC = 150	SOC = 175	SOC = 200
V_{ref}	0.3792	0.4043	0.4103	0.4154	0.4174	0.4211	0.4225
$V_{U_{ref}}$	0.0104	0.0104	0.0104	0.0104	0.0104	0.0104	0.0104
Ea_V	74.5409	70.1111	65.0713	62.2252	60.5653	59.1068	58.3651
Ea_{VU}	50.2139	50.4011	50.6711	50.8293	50.9639	51.0867	51.0899
Ea_K	26.0240	30.3601	35.4526	38.3727	40.2573	41.3633	42.1677
Ea_{KU}	49.8581	49.5998	49.5098	49.2307	49.0796	48.9942	49.0002
$E_{C_{ref}}$	0.2055	0.2523	0.3381	0.4041	0.4538	0.4905	0.5126
m_t	0.0018	0.0021	0.0022	0.0022	0.0023	0.0023	0.0023
a_{MS}	0.494	0.4984	0.5013	0.5064	0.5129	0.521	0.526
σ	0.1521	0.1504	0.1601	0.1654	0.1698	0.1741	0.1781

(c) CON posterior distribution means for MIC-varied runs

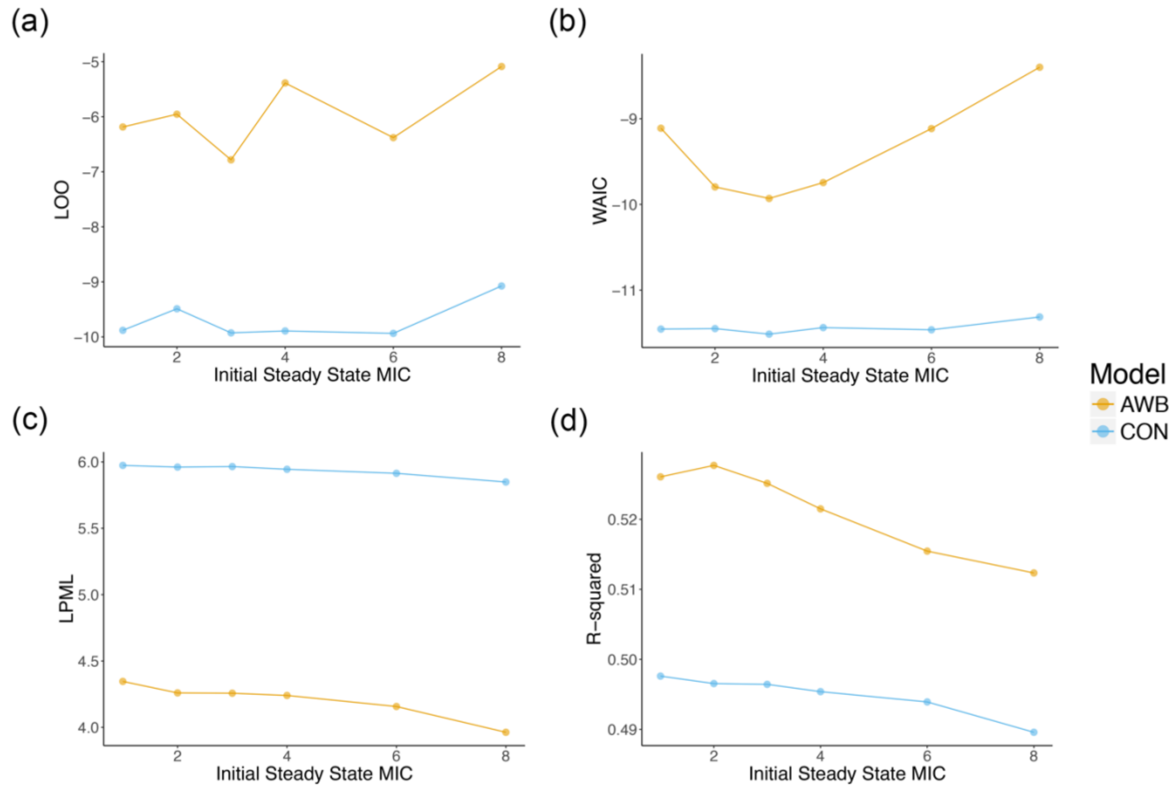
Parameter	MIC = 1	MIC = 2	MIC = 3	MIC = 4	MIC = 6	MIC = 8
Ea_S	67.3478	66.6395	66.0341	65.2664	64.1673	62.9712
Ea_D	50.0552	50.2519	50.2694	50.5307	50.7564	50.8176
Ea_M	50.8727	52.2358	53.0678	53.866	54.9957	55.6118
a_{DS}	0.3271	0.3271	0.3259	0.3265	0.3267	0.3259
a_{SD}	0.3328	0.3363	0.3402	0.3414	0.3455	0.3465
a_M	0.3347	0.337	0.3371	0.3367	0.3363	0.3348
a_{MS}	0.5044	0.5006	0.5	0.498	0.4957	0.494
σ	0.155	0.1552	0.1549	0.1556	0.1556	0.1563

(d) AWB posterior distribution means for MIC-varied runs

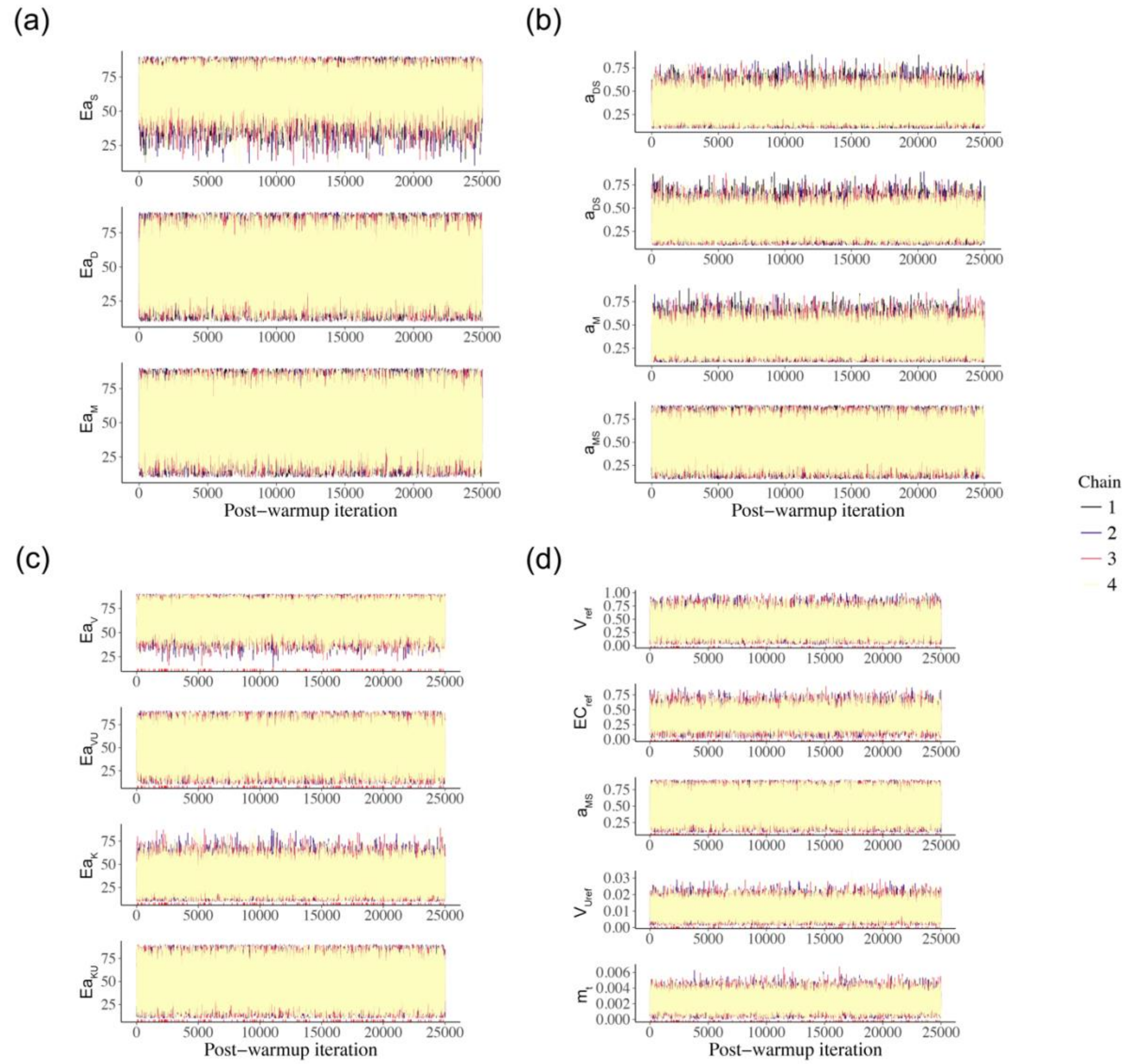
Parameter	MIC = 1	MIC = 2	MIC = 3	MIC = 4	MIC = 6	MIC = 8
V_{ref}	0.4026	0.4103	0.4149	0.4169	0.422	0.4241

$V_{U_{ref}}$	0.0104	0.0104	0.0104	0.0104	0.0104	0.0104
Ea_V	65.6762	65.0713	65.1575	65.314	65.6712	66.0533
Ea_{VU}	50.7039	50.6711	50.6658	50.7089	50.5448	50.4471
Ea_K	34.9008	35.4526	35.4906	35.3379	35.1293	34.871
Ea_{KU}	49.4109	49.5098	49.5087	49.4814	49.4972	49.6048
$E_{C_{ref}}$	0.2595	0.3381	0.395	0.4358	0.4959	0.5342
m_t	0.0022	0.0022	0.0022	0.0022	0.0023	0.0023
a_{MS}	0.4867	0.5013	0.5166	0.5348	0.5619	0.581
σ	0.1596	0.1601	0.1604	0.1609	0.1629	0.1643

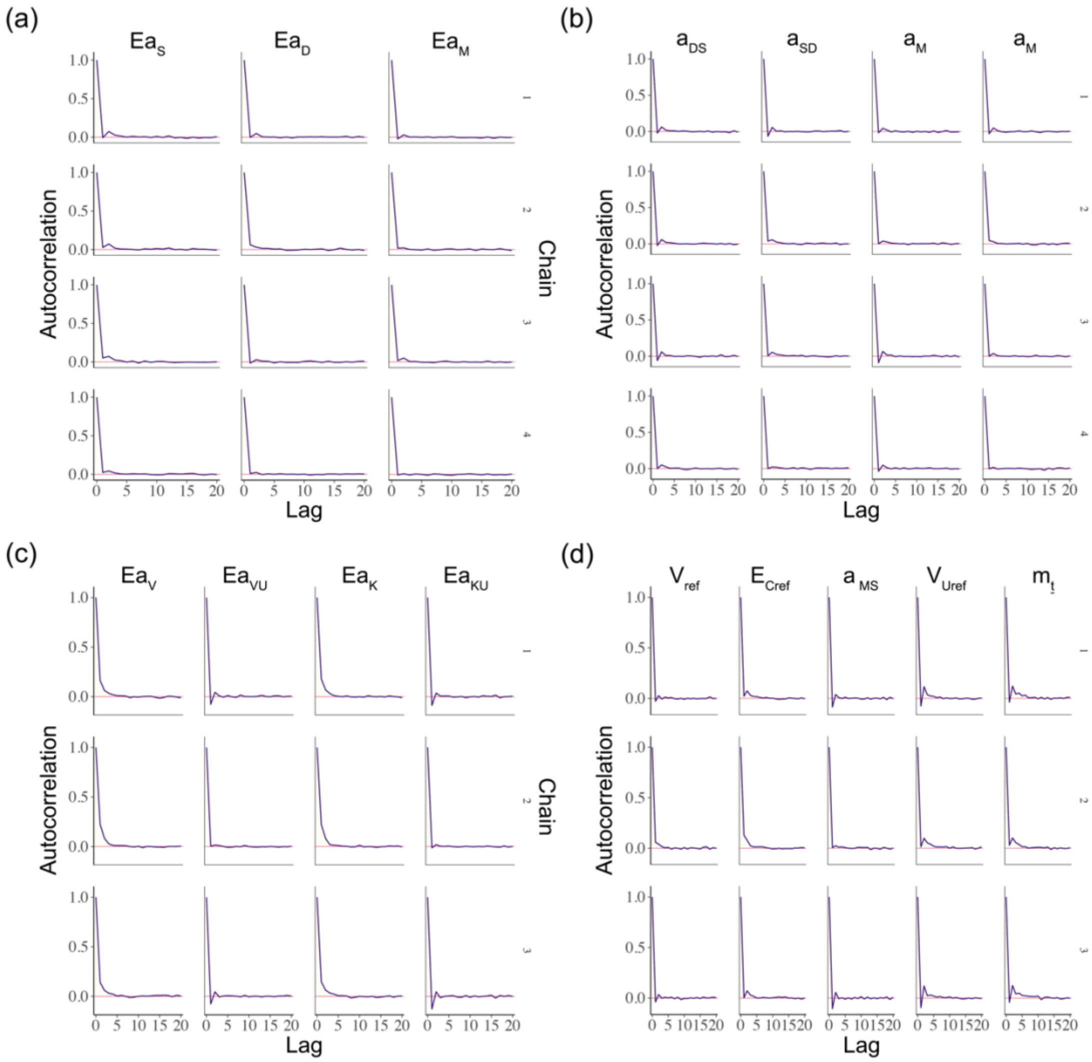
Supplemental Figure 4: Change in fit metrics for AWB and CON as pre-warming steady state MIC is varied from 1 to 8 mg C g^{-1} soil. **(a)** LOO; **(b)** WAIC; **(c)** LPML; **(d)** R^2



Supplemental Figure 5: Trace plots for AWB and CON parameters indicate that the Markov chains were well-mixed with appropriate burn-in. Example trace plots depicted in which pre-warming SOC = 100 mg C g^{-1} soil, MIC = 2 mg C g^{-1} soil, DOC = 0.2 mg C g^{-1} soil, and (for AWB) ENZ = 0.1 mg C g^{-1} soil. **(a)** CON E_a parameters; **(b)** CON partition fraction parameters; **(c)** AWB E_a parameters; **(d)** AWB parameters V_{ref}, EC_{ref}, a_{MS}, V_{Uref}, and m_t.

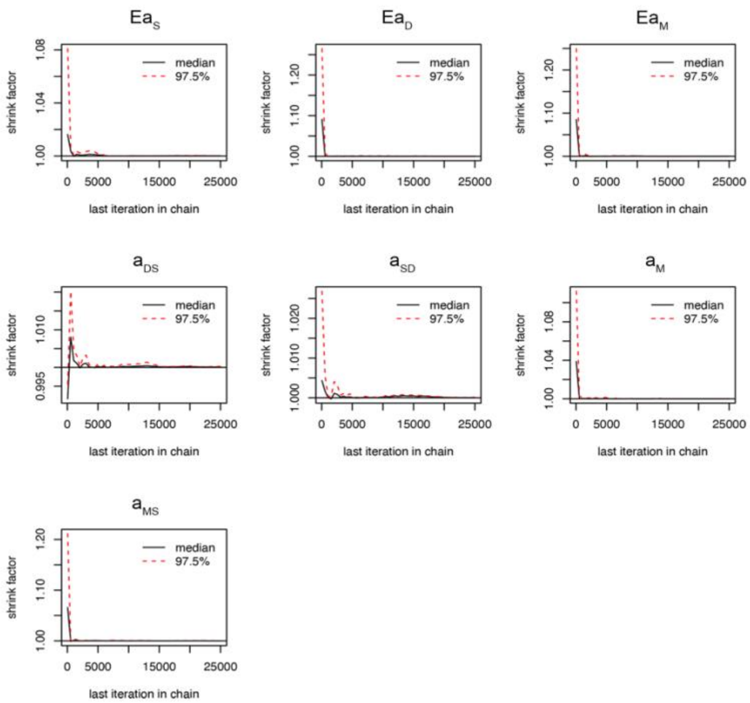


Supplemental Figure 6: Autocorrelation plots for pre-warming SOC = 100 mg C g^{-1} soil, MIC = 2 mg C g^{-1} soil, DOC = 0.2 mg C g^{-1} soil, and (for AWB) ENZ = 0.1 mg C g^{-1} soil indicate effective sample collection. For all fitted AWB and CON parameters, autocorrelation, or the dependence between values of the same parameter accepted by Markov chains, tends to drop as lag, the distance between MCMC iterations increases. Low autocorrelation indicates more independence between samples and more efficient collection of effective samples for inference. **(a)** CON E_a parameters; **(b)** CON partition fraction parameters; **(c)** AWB E_a parameters; **(d)** AWB parameters V_{ref}, E_{Cref}, a_{MS}, VU_{ref}, and m_t.

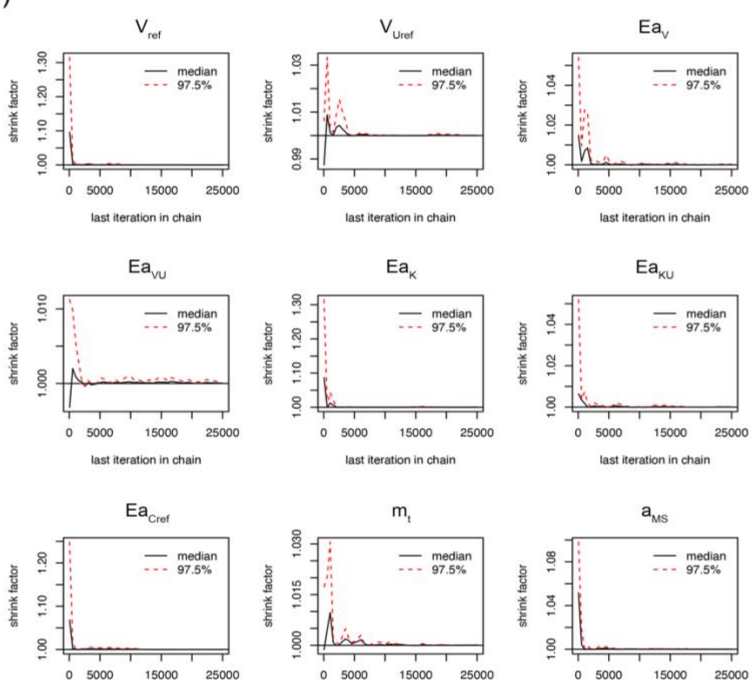


Supplemental Figure 7: $\hat{R}$ is a Bayesian diagnostic measure that estimates the degree of convergence between multiple Markov chains. An $\hat{R}$ value that approaches 1 as the number of Markov chain iterations increase is ideal. Plots of $\hat{R}$ values for **(a)** CON; and **(b)** AWB parameters corresponding to simulations using pre-warming SOC = 100 mg C g-1 soil, MIC = 2 mg C g-1 soil, DOC = 0.2 mg C g-1 soil, and (for AWB) ENZ = 0.1 mg C g-1 soil.

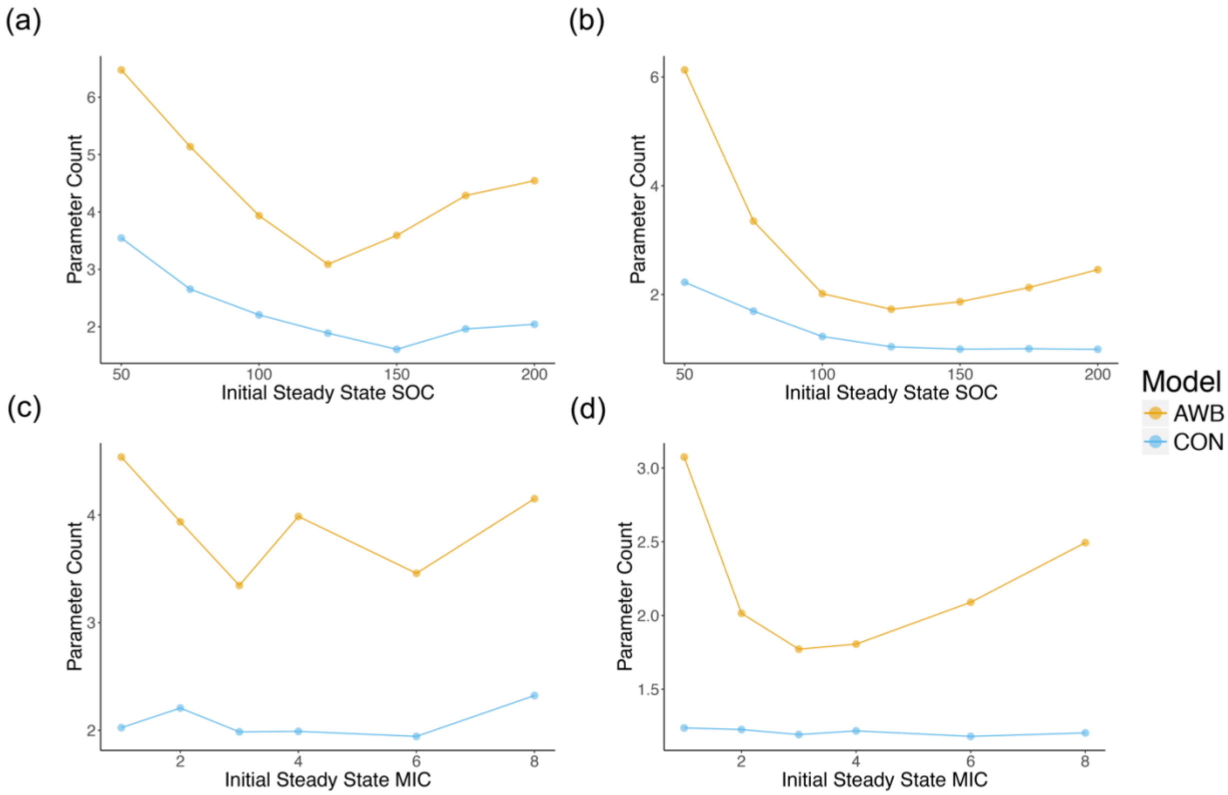
(a)



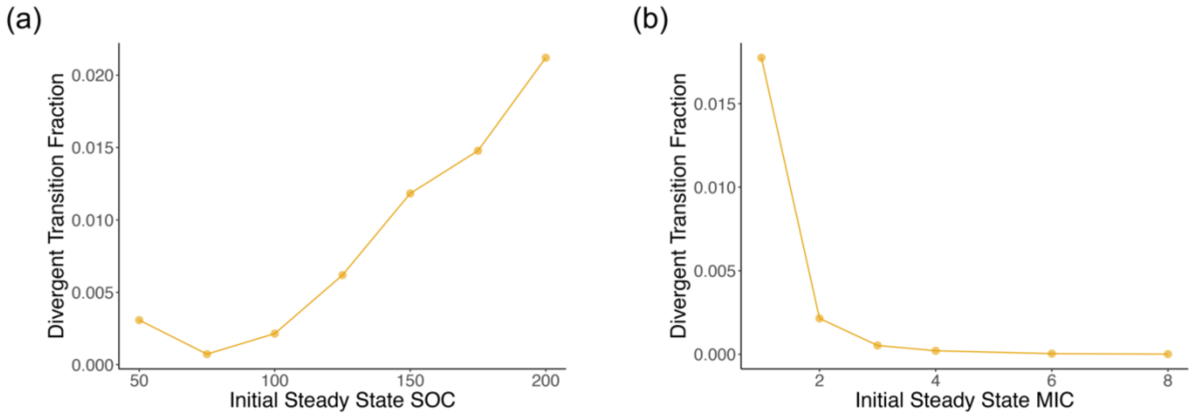
(b)



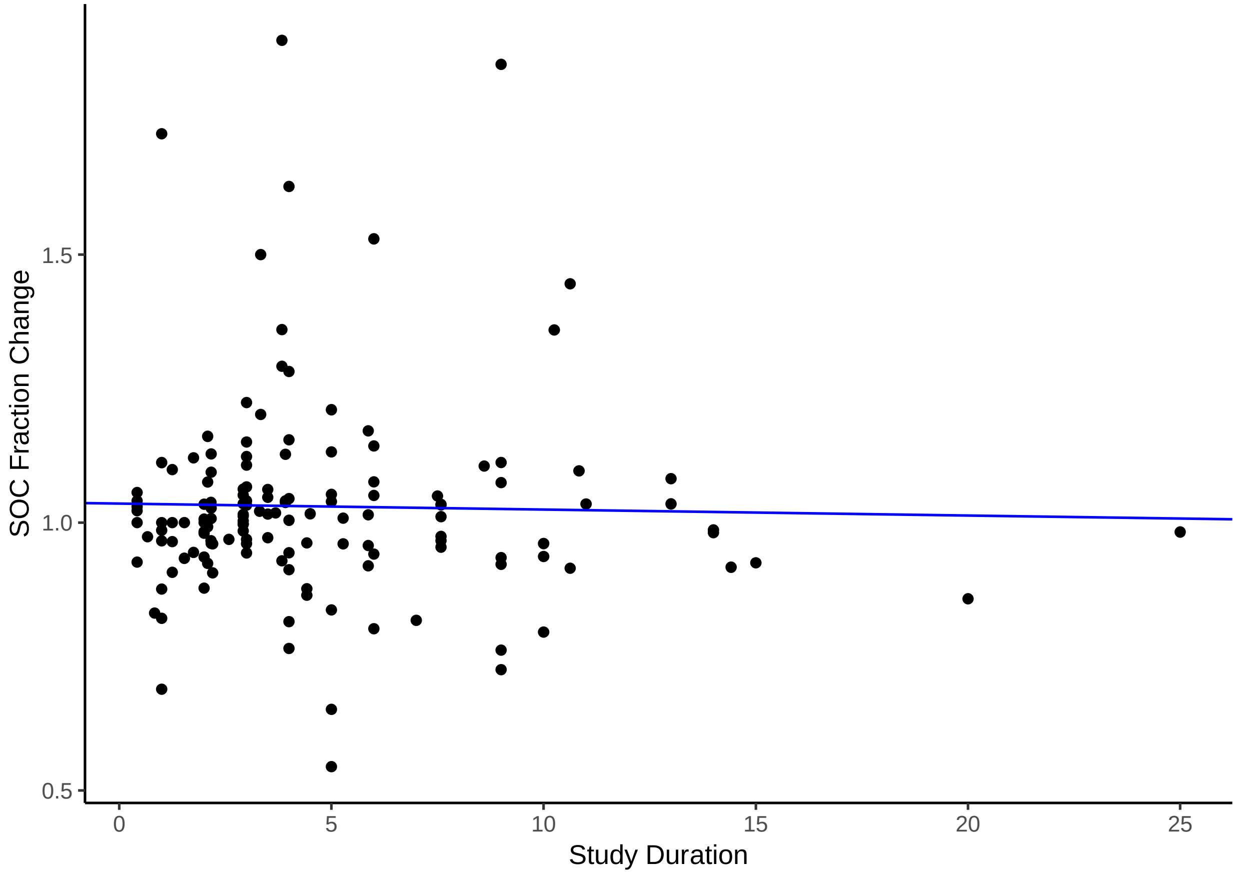
Supplemental Figure 8: Plots of effective parameter counts for CON and AWB in SOC-varied and MIC-varied HMC runs. Decreasing SOC in AWB and CON runs increased effective parameter count and over-fitting punishment in the LOO and WAIC calculations. Effective parameter counts computed as part of **(a)** LOO for SOC-varied runs; **(b)** WAIC for SOC-varied runs; **(c)** LOO for MIC-varied runs; and **(d**) WAIC for MIC-varied runs.



Supplemental Figure 9: Ratio of divergent transitions to total posterior samples collected in AWB runs. Decreasing the MIC-to-SOC ratio in AWB runs corresponded to an increase in the number of divergent transitions. Divergent transition frequencies in **(a)** varied SOC runs; and **(b**) in varied MIC runs.



Supplemental Figure 10: Fraction change of SOC stocks from 143 field warming studies versus study duration (van Gestel et al., 2018). A statistical analysis not accounting for sample size of each study found that the effect of duration on fraction change was insignificant (p = 0.7822). Fraction change ranged from 0.544 to 1.9. Mean fraction change was 1.03, not accounting for sample sizes.



Supplemental Table 4: N_{eff} / N (effective posterior sample ratio) for model parameters. Ratios of effective posterior samples to total posterior samples for each parameter fit to in the AWB and CON runs.

(a) CON SOC-varied runs

Parameter	SOC = 50	SOC = 75	SOC = 100	SOC = 125	SOC = 150	SOC = 175	SOC = 200
Ea_S	0.7708	0.7856	0.7921	0.8446	0.7685	0.8423	0.8359
Ea_D	1.0000	1.0000	0.8920	1.0000	0.9356	1.0000	1.0000
Ea_M	1.0000	1.0000	0.9421	1.0000	0.9136	1.0000	1.0000
a_{DS}	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
a_{SD}	1.0000	1.0000	1.0000	1.0000	0.8226	1.0000	1.0000
a_M	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
a_{MS}	1.0000	1.0000	0.9226	1.0000	0.9177	1.0000	1.0000
σ	0.6182	0.6641	0.6166	0.6432	0.6149	0.6821	0.6344

(b) AWB SOC-varied runs

Parameter	SOC = 50	SOC = 75	SOC = 100	SOC = 125	SOC = 150	SOC = 175	SOC = 200
V_{ref}	0.3642	1.0000	1.0000	0.7557	0.7705	0.8939	0.8038
$V_{U_{ref}}$	0.5190	1.0000	1.0000	0.5679	0.4973	1.0000	0.5206
Ea_V	0.0689	0.7158	0.6108	0.4743	0.4496	0.4597	0.4079
Ea_{VU}	0.7576	1.0000	1.0000	0.8374	0.8901	1.0000	0.8460
Ea_K	0.1994	0.7085	0.6148	0.4702	0.4545	0.4701	0.4287
Ea_{KU}	0.8343	1.0000	1.0000	0.8575	0.8936	1.0000	0.8390
$E_{C_{ref}}$	0.3305	0.6603	0.7058	0.5220	0.3752	0.3572	0.2342
m_t	0.6162	1.0000	1.0000	0.5341	0.5880	1.0000	0.5904
a_{MS}	0.8619	1.0000	1.0000	0.8114	0.9012	1.0000	0.8356
σ	0.4510	0.6577	0.6396	0.5424	0.5430	0.5882	0.5105

(c) CON MIC-varied runs

Parameter	MIC = 1	MIC = 2	MIC = 3	MIC = 4	MIC = 6	MIC = 8
Ea_S	1.0000	0.7921	1.0000	1.0000	0.8201	0.7856
Ea_D	1.0000	0.8920	1.0000	1.0000	0.8935	1.0000
Ea_M	1.0000	0.9421	1.0000	1.0000	0.9057	0.9182
a_{DS}	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
a_{SD}	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
a_M	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
a_{MS}	1.0000	0.9226	1.0000	1.0000	0.9255	1.0000
σ	0.6548	0.6166	0.6901	0.6759	0.6148	0.6258

(d) AWB MIC-varied runs

Parameter	MIC = 1	MIC = 2	MIC = 3	MIC = 4	MIC = 6	MIC = 8
V_{ref}	0.6324	1.0000	1.0000	0.8350	0.9067	0.8005
$V_{U_{ref}}$	0.7500	1.0000	1.0000	1.0000	1.0000	0.6008
Ea_V	0.4216	0.6108	0.5942	0.5510	0.6135	0.5555

Ea_{VU}	0.7500	1.0000	1.0000	1.0000	1.0000	0.8975
Ea_{K}	0.4283	0.6148	0.5959	0.5651	0.6101	0.5557
Ea_{KU}	0.7500	1.0000	1.0000	1.0000	1.0000	0.8842
E_{ef}	0.4559	0.7058	0.6996	0.5715	0.5393	0.4942
m_{t}	0.4294	1.0000	1.0000	0.6233	0.6211	0.6247
a_{MS}	0.7500	1.0000	1.0000	1.0000	1.0000	0.7735
σ	0.4438	0.6396	0.6555	0.5900	0.5916	0.5097