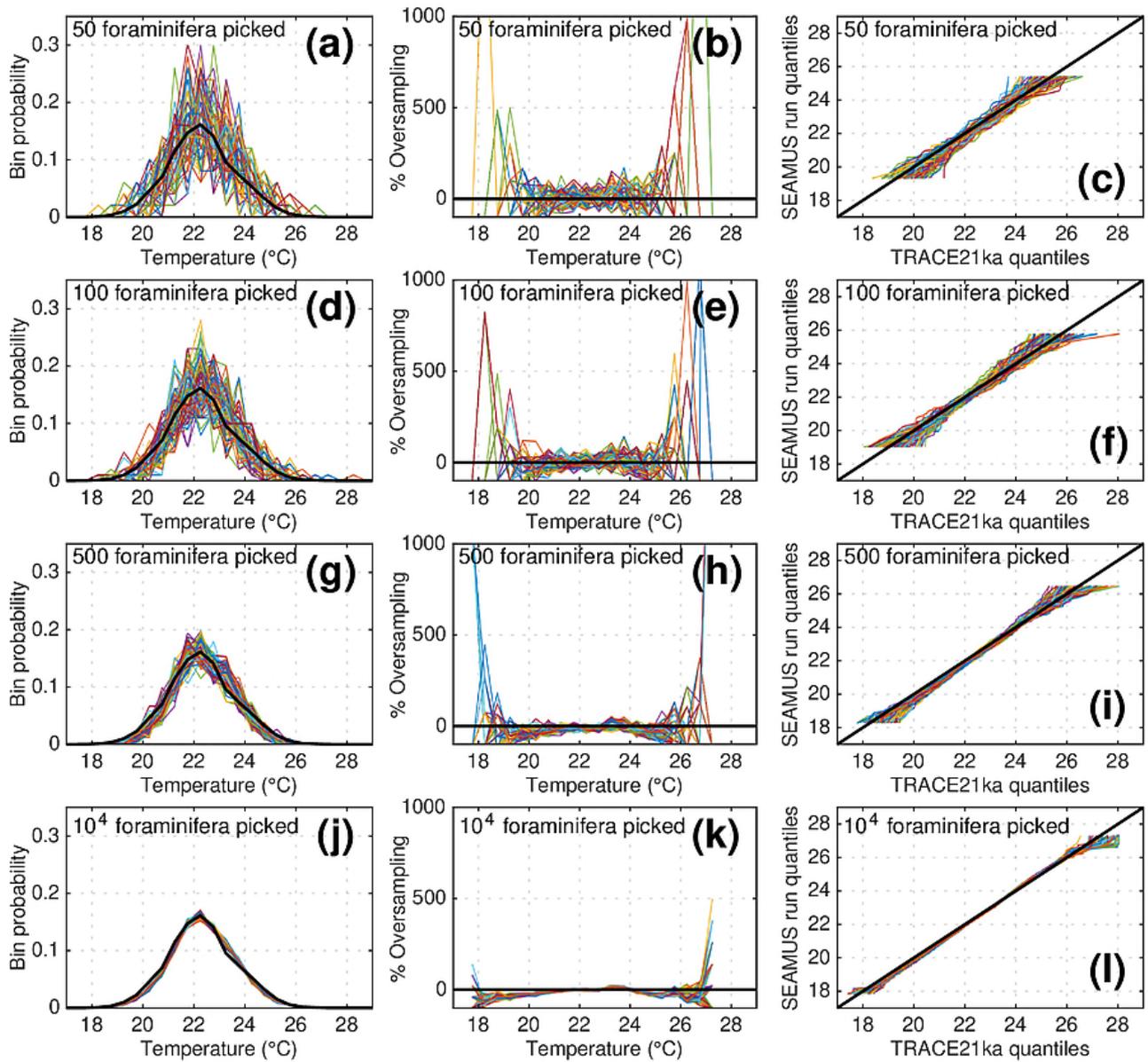


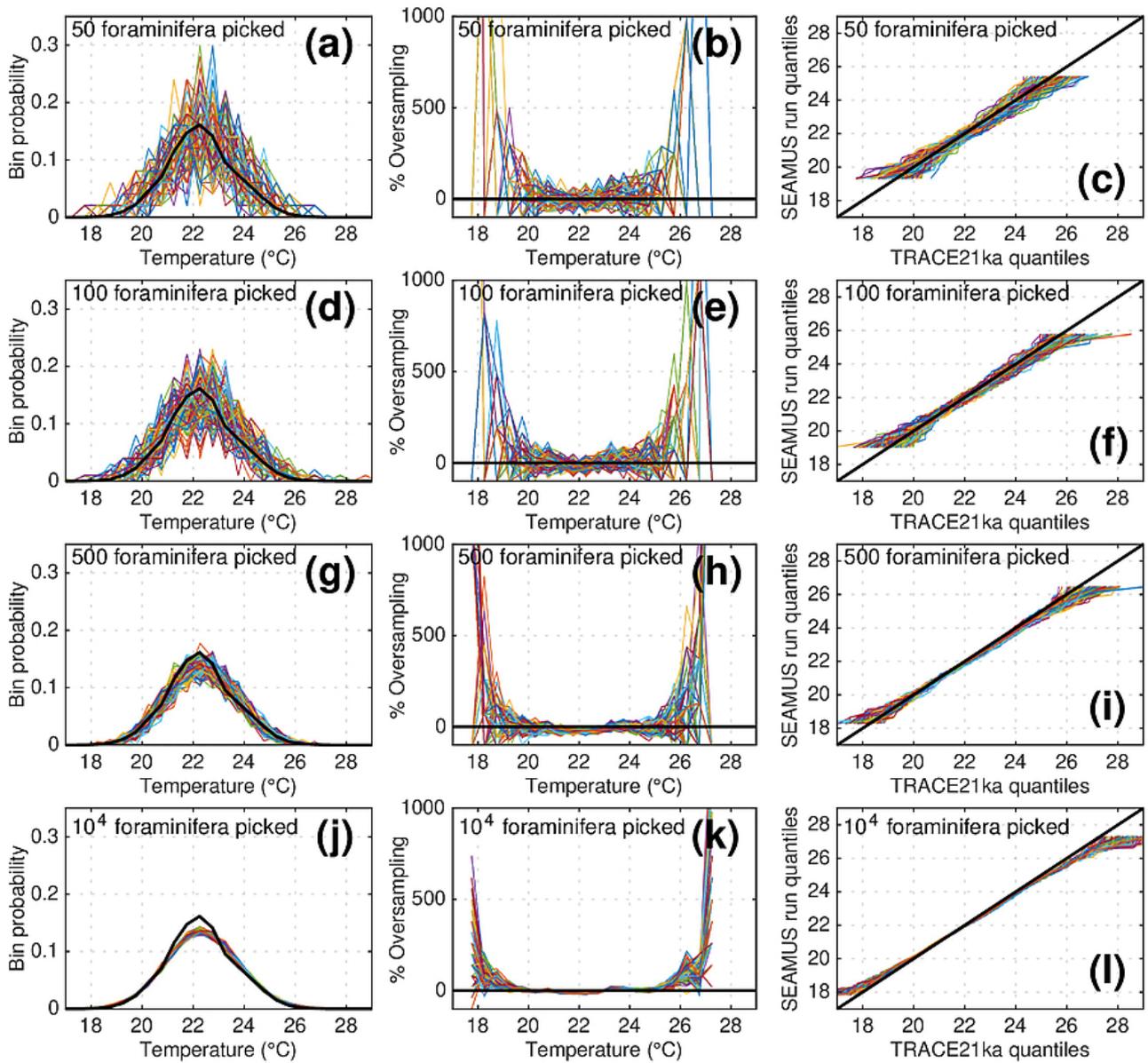
**Figure S1.** Simulated downcore, discrete 1 cm depth  $1\sigma$  SST values of simulated single foraminifera from various  $5 \text{ cm ka}^{-1}$  SAR scenarios with 10 cm BD, each with 100 ensembles of SEAMUS runs. In each panel, each ensemble is shown using a coloured line. The solid black lines represent the 95% interval of the ensemble runs at each discrete 1 cm depth. Also shown for reference as a thick grey line is the 1000 year (12000 month) moving  $1\sigma$  of the 1.5-7 year filtered monthly SST data (as also shown in Fig. 1c.) The left panels (a, c, e and g) show the output of scenarios with 50, 100, 500 and  $10^4$  randomly picked foraminifera per discrete 1 cm depth, all with constant species abundance and no assumed analytical error. The right panels (b, d, f and h) show the output of scenarios with 50, 100, 500 and  $10^4$  randomly picked foraminifera per discrete 1 cm depth with constant species abundance and an assumed analytical error of  $\pm 1^\circ\text{C}$  in SST.

**Ensemble of 100 SEAMUS runs compared to TRACE21ka for period spanning 18 ka to 17 ka.  
SAR: 5 cm ka<sup>-1</sup>, BD: 10 cm, no analysis error and constant abundance**

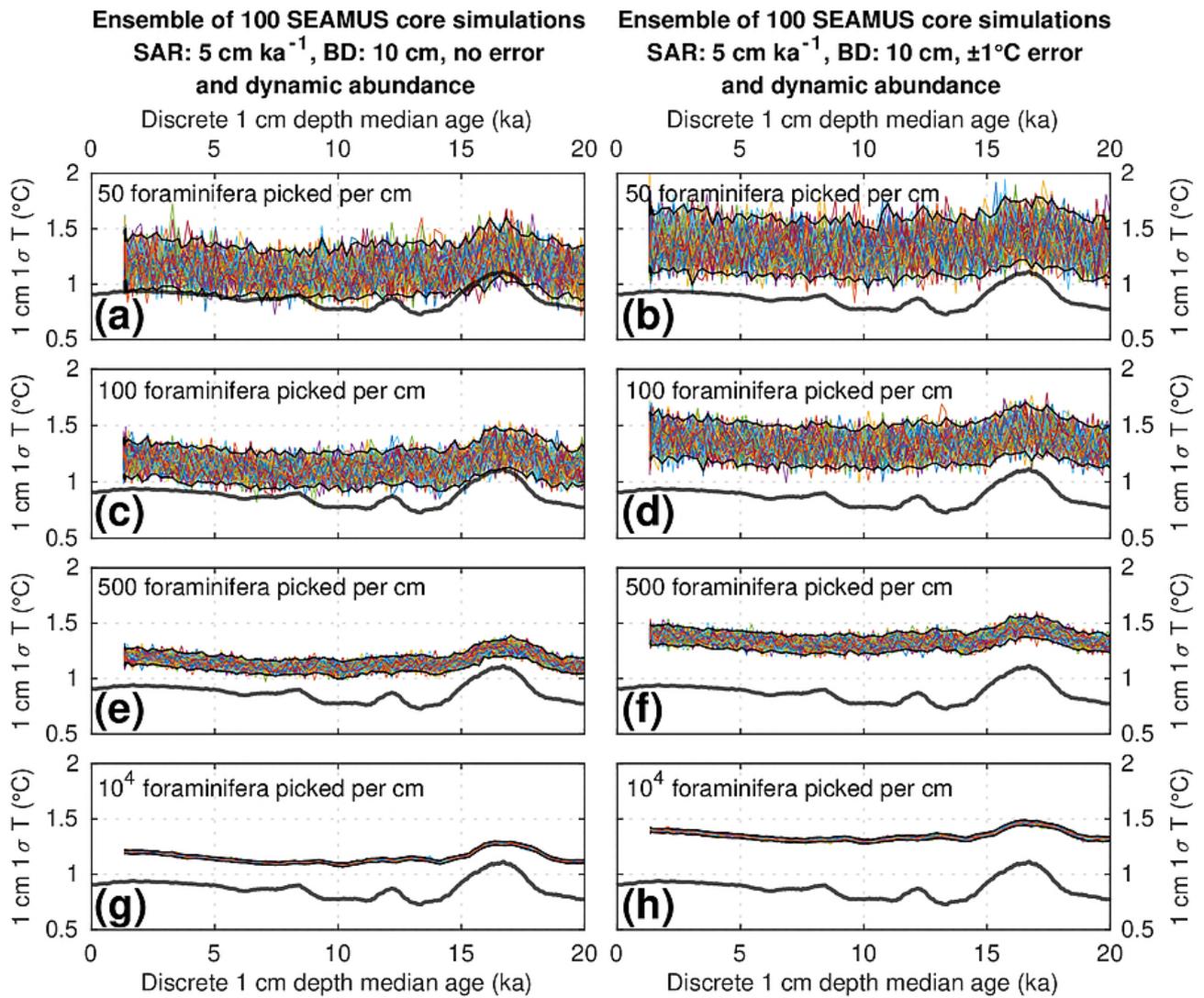


**Figure S2.** Simulated single foraminifera SST distributions from 100 ensembles of SEAMUS runs, with SAR of 5 cm ka<sup>-1</sup>, BD of 10 cm, no analytical error and constant abundance. In each ensemble, the single foraminifera SST distribution from a single discrete depth with a simulated median age of 17.5 ka is shown, and compared to the TRACE-21ka SST distribution for the 18 ka to 17 ka period. The left panels (a, d, g and j) show the 100 SEAMUS ensembles as coloured lines in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera, with the TRACE-21ka SST distribution is shown as a black line. The middle panels (b, e, h and k) show the the rate of over/undersampling for each of the 100 SEAMUS ensembles (coloured lines) relative to the TRACE-21ka SST distribution (black line) in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera. The right panels (c, f, i and l) show Q-Q plots of the 100 SEAMUS ensemble quantiles vs the TRACE-21ka quantiles as coloured lines in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera, with a perfect 1:1 correspondence to TRACE-21ka shown for reference as a black line.

**Ensemble of 100 SEAMUS runs compared to TRACE21ka for period spanning 18 ka to 17 ka.  
SAR: 5 cm ka<sup>-1</sup>, BD: 10 cm, ±1°C analysis error and constant abundance**

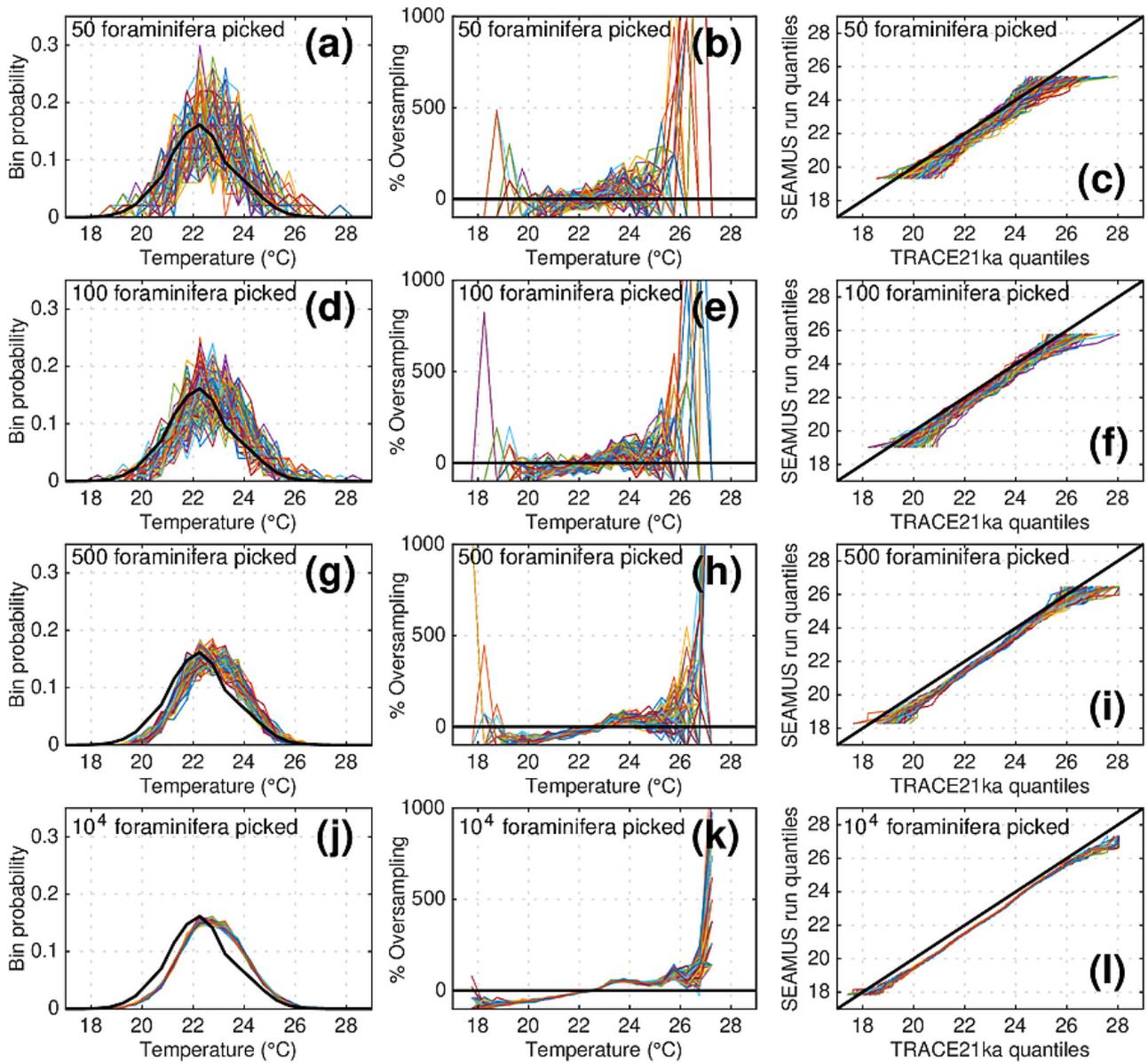


**Figure S3.** Simulated single foraminifera SST distributions from 100 ensembles of SEAMUS runs, with SAR of 5 cm ka<sup>-1</sup>, BD of 10 cm, ±1 °C analytical error and constant abundance. In each ensemble, the single foraminifera SST distribution from a single discrete depth with a simulated median age of 17.5 ka is shown, and compared to the TRACE-21ka SST distribution for the 18 ka to 17 ka period. The left panels (a, d, g and j) show the 100 SEAMUS ensembles as coloured lines in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera, with the TRACE-21ka SST distribution is shown as a black line. The middle panels (b, e, h and k) show the the rate of over/undersampling for each of the 100 SEAMUS ensembles (coloured lines) relative to the TRACE-21ka SST distribution (black line) in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera. The right panels (c, f, i and l) show Q-Q plots of the 100 SEAMUS ensemble quantiles vs the TRACE-21ka quantiles as coloured lines in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera, with a perfect 1:1 correspondence to TRACE-21ka shown for reference as a black line.



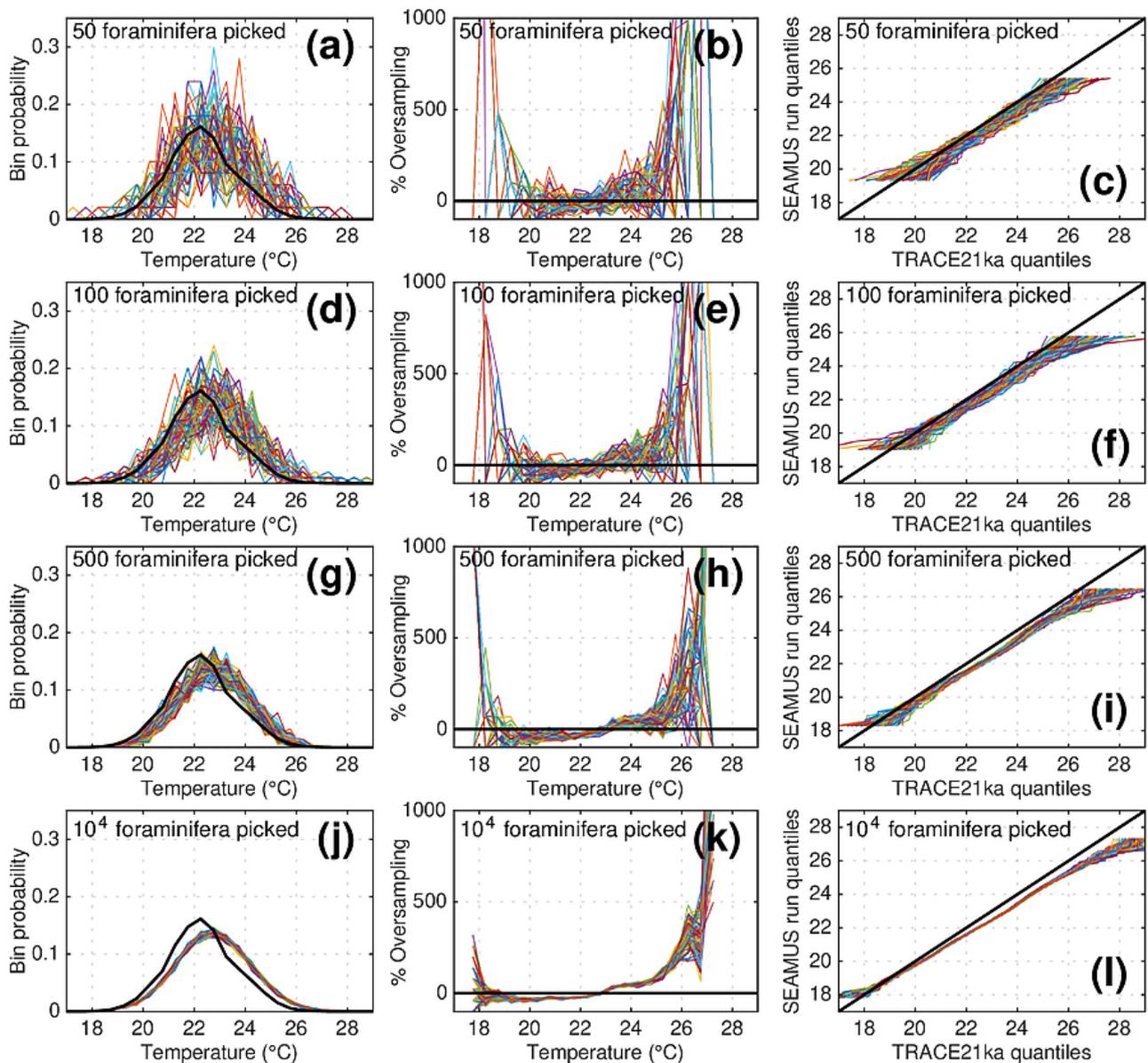
**Figure S4.** Simulated downcore, discrete 1 cm depth  $1\sigma$  SST values of simulated single foraminifera from various  $5 \text{ cm ka}^{-1}$  SAR scenarios with 10 cm BD, each with 100 ensembles of SEAMUS runs. In each panel, each ensemble is shown using a coloured line. The solid black lines represent the 95% interval of the ensemble runs at each discrete 1 cm depth. Also shown for reference as a thick grey line is the 1000 year (12000 month) moving  $1\sigma$  of the 1.5-7 year filtered monthly SST data (as also shown in Fig. 1c. The left panels (a, c, e and g) show the output of scenarios with 50, 100, 500 and  $10^4$  randomly picked foraminifera per discrete 1 cm depth, all with a SAR of  $40 \text{ cm ka}^{-1}$ , bioturbation depth of 10 cm, dynamic species abundance (following Fig. 2a) and no assumed analytical error. The right panels (b, d, f and h) show the output of scenarios with 50, 100, 500 and  $10^4$  randomly picked foraminifera per discrete 1 cm depth, all with dynamic species abundance (following Fig. 2a) and an assumed analytical error of  $\pm 1^\circ\text{C}$  in SST.

**Ensemble of 100 SEAMUS runs compared to TRACE21ka for period spanning 18 ka to 17 ka.  
SAR: 5 cm ka<sup>-1</sup>, BD: 10 cm, no analysis error and dynamic abundance**

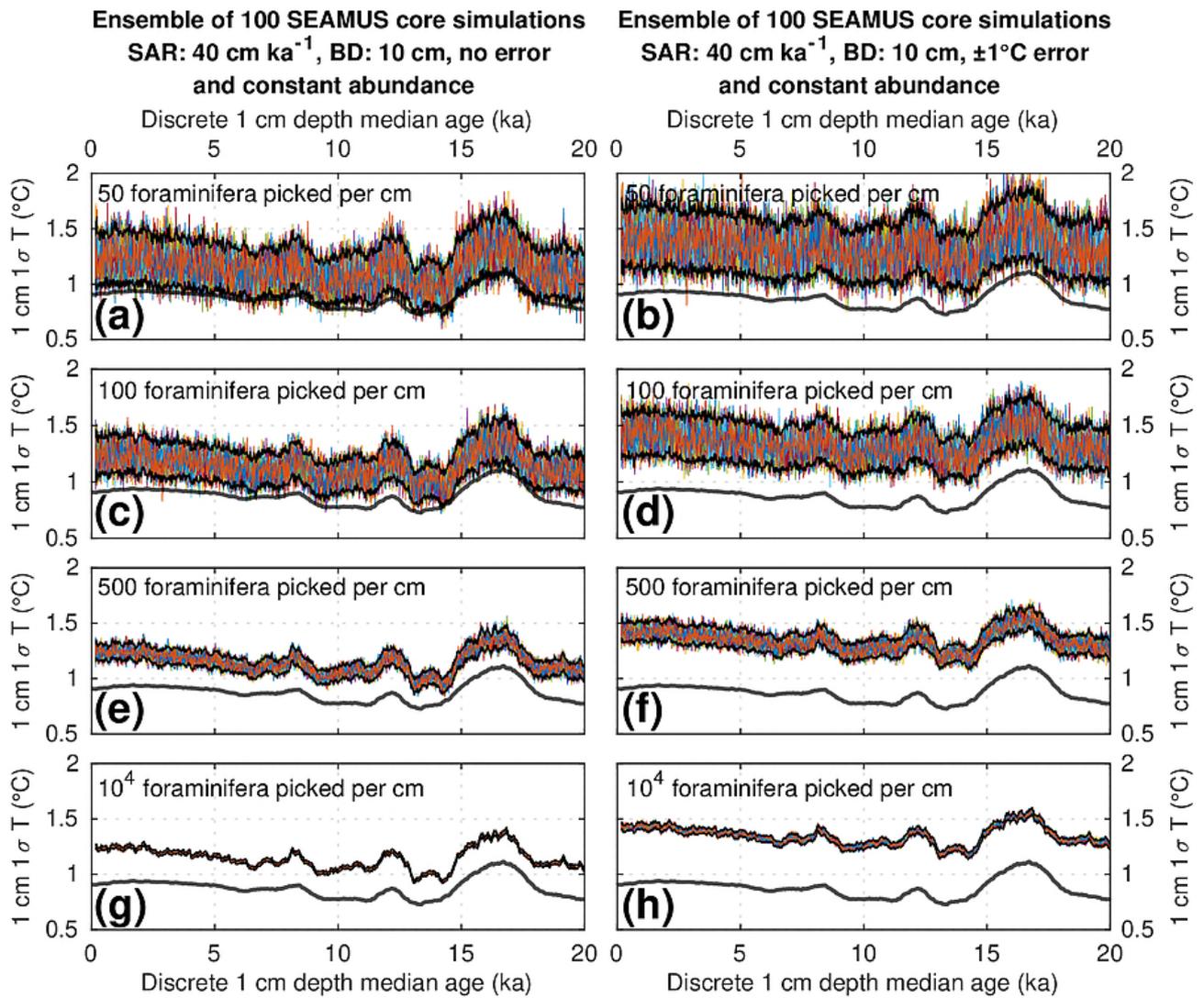


**Figure S5.** Simulated single foraminifera SST distributions from 100 ensembles of SEAMUS runs, with SAR of 5 cm ka<sup>-1</sup>, BD of 10 cm, no analytical error and dynamic abundance (following Fig. 2a). In each ensemble, the single foraminifera SST distribution from a single discrete depth with a simulated median age of 17.5 ka is shown, and compared to the TRACE-21ka SST distribution for the 18 ka to 17 ka period. The left panels (a, d, g and j) show the 100 SEAMUS ensembles as coloured lines in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera, with the TRACE-21ka SST distribution is shown as a black line. The middle panels (b, e, h and k) show the the rate of over/undersampling for each of the 100 SEAMUS ensembles (coloured lines) relative to the TRACE-21ka SST distribution (black line) in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera. The right panels (c, f, i and l) show Q-Q plots of the 100 SEAMUS ensemble quantiles vs the TRACE-21ka quantiles as coloured lines in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera, with a perfect 1:1 correspondence to TRACE-21ka shown for reference as a black line.

**Ensemble of 100 SEAMUS runs compared to TRACE21ka for period spanning 18 ka to 17 ka.  
SAR: 5 cm ka<sup>-1</sup>, BD: 10 cm, ±1°C analysis error and dynamic abundance**

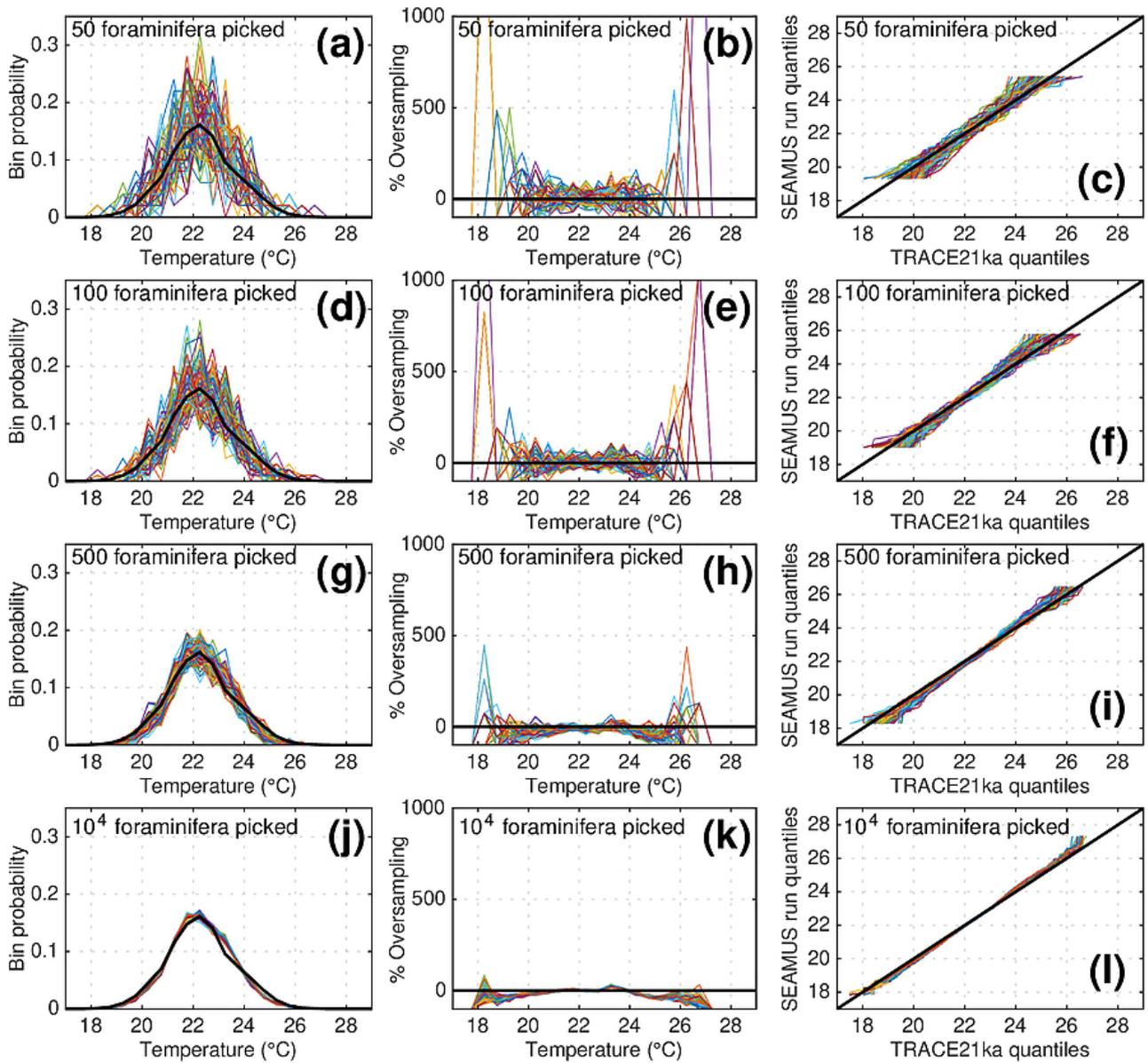


**Figure S6.** Simulated single foraminifera SST distributions from 100 ensembles of SEAMUS runs, with SAR of 5 cm ka<sup>-1</sup>, BD of 10 cm, ±1 °C analytical error and dynamic abundance (following Fig. 2a). In each ensemble, the single foraminifera SST distribution from a single discrete depth with a simulated median age of 17.5 ka is shown, and compared to the TRACE-21ka SST distribution for the 18 ka to 17 ka period. The left panels (a, d, g and j) show the 100 SEAMUS ensembles as coloured lines in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera, with the TRACE-21ka SST distribution is shown as a black line. The middle panels (b, e, h and k) show the the rate of over/undersampling for each of the 100 SEAMUS ensembles (coloured lines) relative to the TRACE-21ka SST distribution (black line) in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera. The right panels (c, f, i and l) show Q-Q plots of the 100 SEAMUS ensemble quantiles vs the TRACE-21ka quantiles as coloured lines in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera, with a perfect 1:1 correspondence to TRACE-21ka shown for reference as a black line.



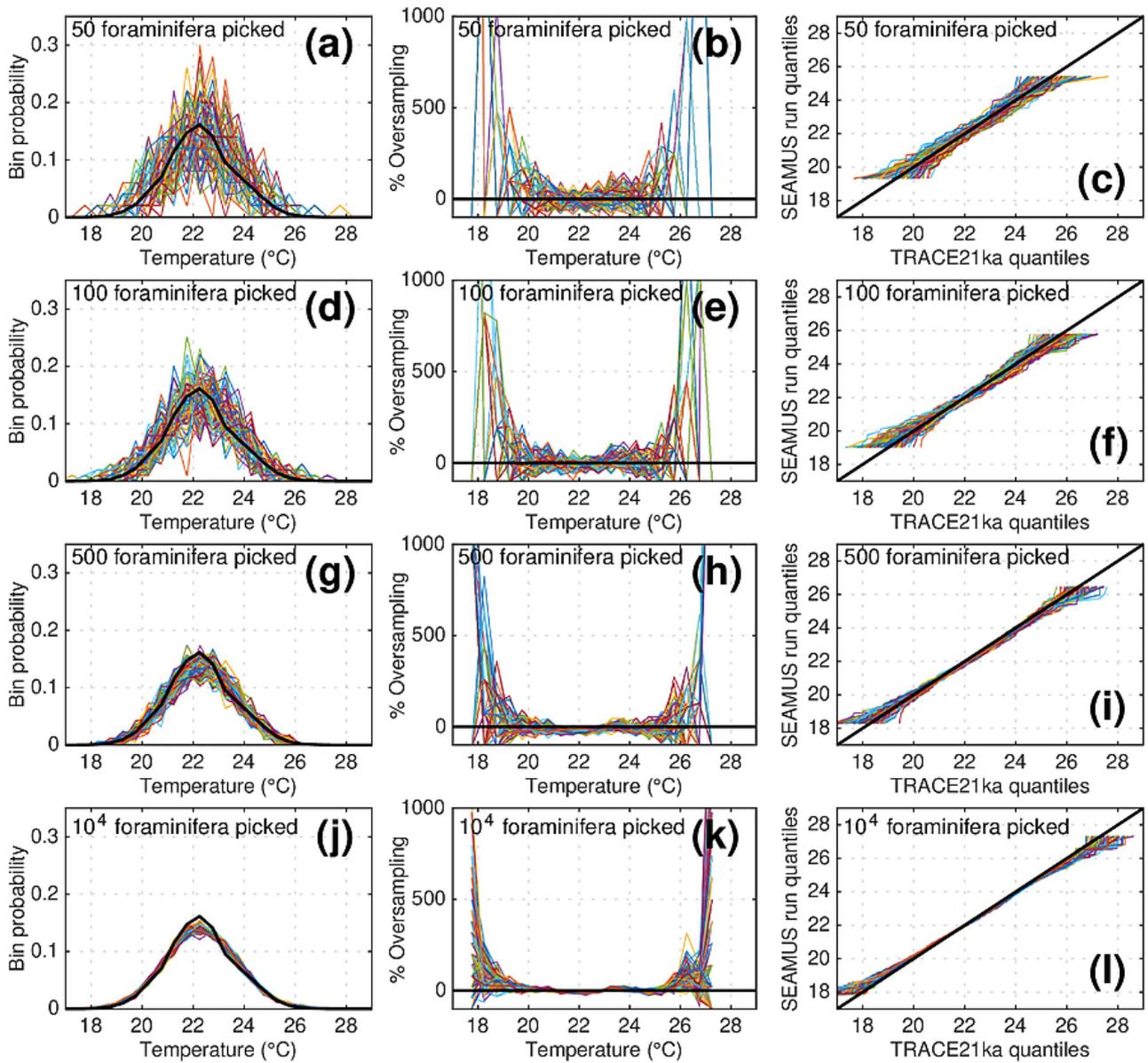
**Figure S7.** Simulated downcore, discrete 1 cm depth  $1\sigma$  SST values of simulated single foraminifera from various  $40 \text{ cm ka}^{-1}$  SAR scenarios with 10 cm BD, each with 100 ensembles of SEAMUS runs. In each panel, each ensemble is shown using a coloured line. The solid black lines represent the 95% interval of the ensemble runs at each discrete 1 cm depth. Also shown for reference as a thick grey line is the 1000 year (12000 month) moving  $1\sigma$  of the 1.5-7 year filtered monthly SST data (as also shown in Fig. 1c.) The left panels (a, c, e and g) show the output of scenarios with 50, 100, 500 and  $10^4$  randomly picked foraminifera per discrete 1 cm depth, all with constant species abundance and no assumed analytical error. The right panels (b, d, f and h) show the output of scenarios with 50, 100, 500 and  $10^4$  randomly picked foraminifera per discrete 1 cm depth with constant species abundance and an assumed analytical error of  $\pm 1^\circ\text{C}$  in SST.

**Ensemble of 100 SEAMUS runs compared to TRACE21ka for period spanning 18 ka to 17 ka.  
SAR: 40 cm ka<sup>-1</sup>, BD: 10 cm, no analysis error and constant abundance**

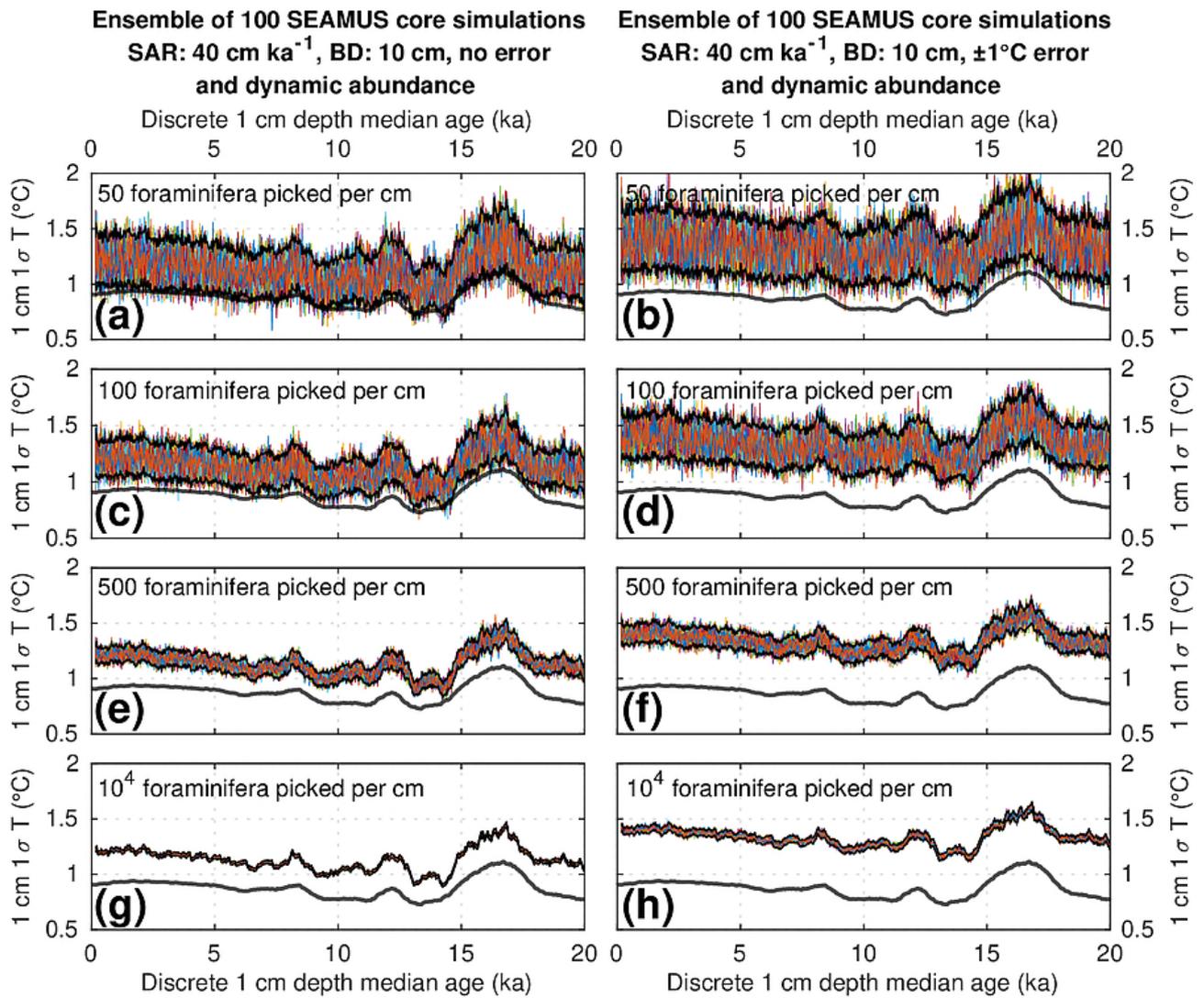


**Figure S8.** Simulated single foraminifera SST distributions from 100 ensembles of SEAMUS runs, with SAR of 40 cm ka<sup>-1</sup>, BD of 10 cm,  $\pm 1$  °C analytical error and constant abundance. In each ensemble, the single foraminifera SST distribution from a single discrete depth with a simulated median age of 17.5 ka is shown, and compared to the TRACE-21ka SST distribution for the 18 ka to 17 ka period. The left panels (a, d, g and j) show the 100 SEAMUS ensembles as coloured lines in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera, with the TRACE-21ka SST distribution is shown as a black line. The middle panels (b, e, h and k) show the the rate of over/undersampling for each of the 100 SEAMUS ensembles (coloured lines) relative to the TRACE-21ka SST distribution (black line) in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera. The right panels (c, f, i and l) show Q-Q plots of the 100 SEAMUS ensemble quantiles vs the TRACE-21ka quantiles as coloured lines in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera, with a perfect 1:1 correspondence to TRACE-21ka shown for reference as a black line.

**Ensemble of 100 SEAMUS runs compared to TRACE21ka for period spanning 18 ka to 17 ka.  
SAR: 40 cm ka<sup>-1</sup>, BD: 10 cm, ±1°C analysis error and constant abundance**

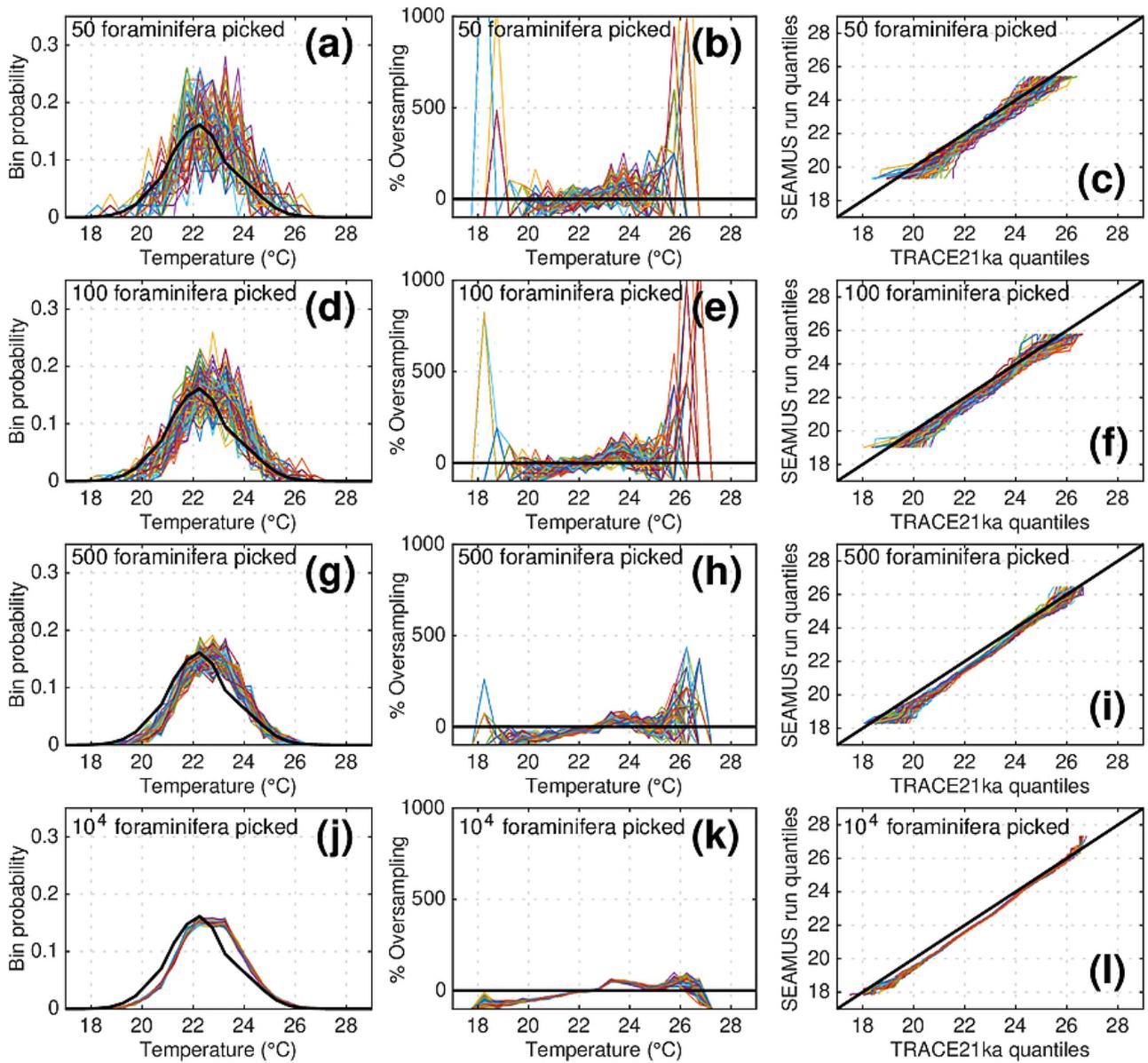


**Figure S9.** Simulated downcore, discrete 1 cm depth 1 $\sigma$  SST values of simulated single foraminifera from various 40 cm ka<sup>-1</sup> SAR scenarios with 10 cm BD, each with 100 ensembles of SEAMUS runs. In each panel, each ensemble is shown using a coloured line. The solid black lines represent the 95% interval of the ensemble runs at each discrete 1 cm depth. Also shown for reference as a thick grey line is the 1000 year (12000 month) moving 1 $\sigma$  of the 1.5-7 year filtered monthly SST data (as also shown in Fig. 1c). The left panels (a, c, e and g) show the output of scenarios with 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera per discrete 1 cm depth, all with a SAR of 40 cm ka<sup>-1</sup>, bioturbation depth of 10 cm, dynamic species abundance (following Fig. 2a) and no assumed analytical error. The right panels (b, d, f and h) show the output of scenarios with 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera per discrete 1 cm depth, all with dynamic species abundance (following Fig. 2a) and an assumed analytical error of ±1°C in SST.



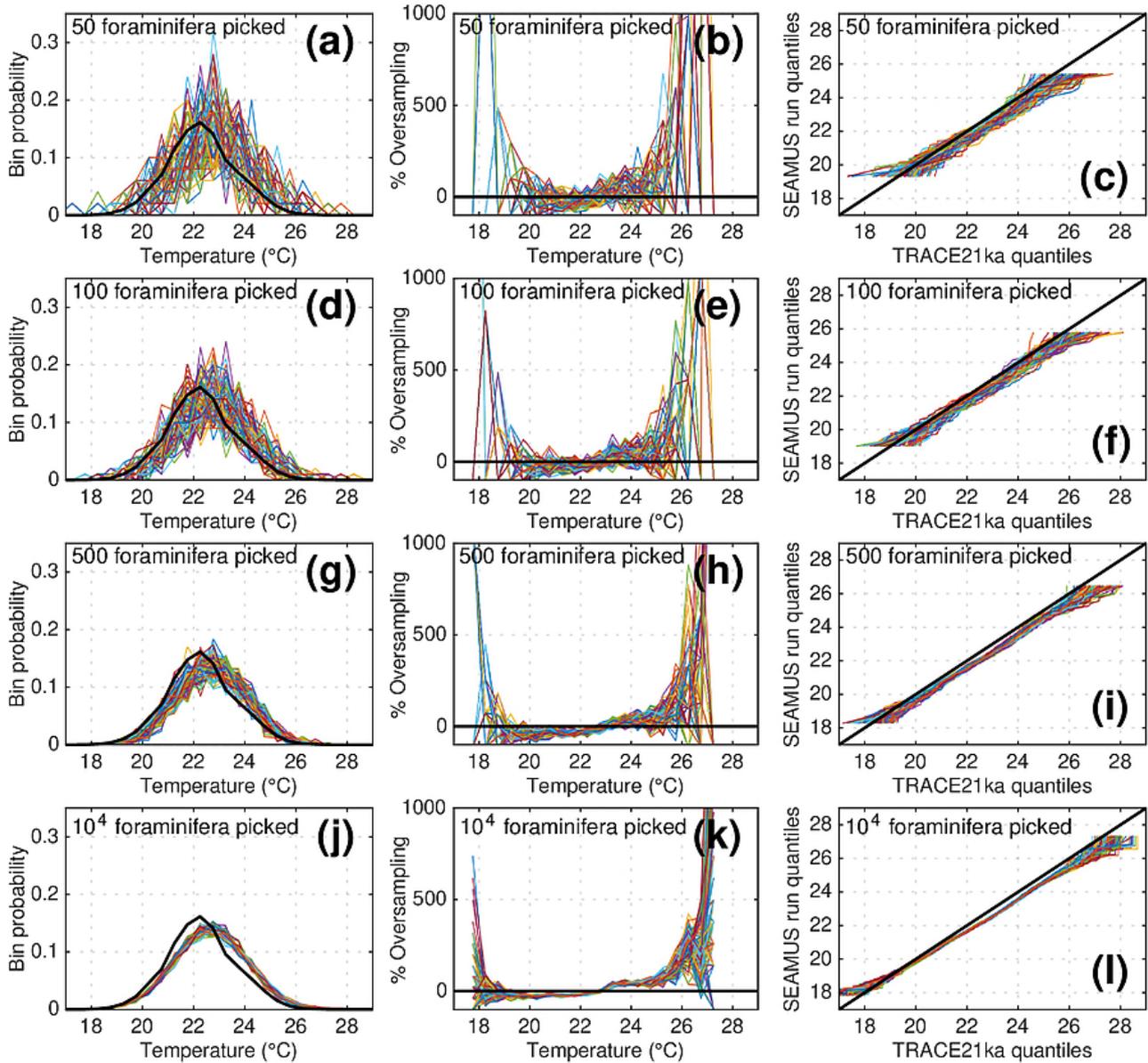
**Figure S10.** Simulated downcore, discrete 1 cm depth  $1\sigma$  SST values of simulated single foraminifera from various  $40 \text{ cm ka}^{-1}$  SAR scenarios with 10 cm BD, each with 100 ensembles of SEAMUS runs. In each panel, each ensemble is shown using a coloured line. The solid black lines represent the 95% interval of the ensemble runs at each discrete 1 cm depth. Also shown for reference as a thick grey line is the 1000 year (12000 month) moving  $1\sigma$  of the 1.5-7 year filtered monthly SST data (as also shown in Fig. 1c. The left panels (a, c, e and g) show the output of scenarios with 50, 100, 500 and  $10^4$  randomly picked foraminifera per discrete 1 cm depth, all with a SAR of  $40 \text{ cm ka}^{-1}$ , bioturbation depth of 10 cm, dynamic species abundance (following Fig. 2a) and no assumed analytical error. The right panels (b, d, f and h) show the output of scenarios with 50, 100, 500 and  $10^4$  randomly picked foraminifera per discrete 1 cm depth, all with dynamic species abundance (following Fig. 2a) and an assumed analytical error of  $\pm 1^\circ\text{C}$  in SST.

**Ensemble of 100 SEAMUS runs compared to TRACE21ka for period spanning 18 ka to 17 ka.  
SAR: 40 cm ka<sup>-1</sup>, BD: 10 cm, no analysis error and dynamic abundance**



**Figure S11.** Simulated single foraminifera SST distributions from 100 ensembles of SEAMUS runs, with SAR of 40 cm ka<sup>-1</sup>, BD of 10 cm, no analytical error and dynamic abundance (following Fig. 2a). In each ensemble, the single foraminifera SST distribution from a single discrete depth with a simulated median age of 17.5 ka is shown, and compared to the TRACE-21ka SST distribution for the 18 ka to 17 ka period. The left panels (a, d, g and j) show the 100 SEAMUS ensembles as coloured lines in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera, with the TRACE-21ka SST distribution is shown as a black line. The middle panels (b, e, h and k) show the the rate of over/undersampling for each of the 100 SEAMUS ensembles (coloured lines) relative to the TRACE-21ka SST distribution (black line) in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera. The right panels (c, f, i and l) show Q-Q plots of the 100 SEAMUS ensemble quantiles vs the TRACE-21ka quantiles as coloured lines in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera, with a perfect 1:1 correspondence to TRACE-21ka shown for reference as a black line.

**Ensemble of 100 SEAMUS runs compared to TRACE21ka for period spanning 18 ka to 17 ka.  
SAR: 40 cm ka<sup>-1</sup>, BD: 10 cm, ±1°C analysis error and dynamic abundance**



**Figure S12.** Simulated single foraminifera SST distributions from 100 ensembles of SEAMUS runs, with SAR of 40 cm ka<sup>-1</sup>, BD of 10 cm, ±1 °C analytical error and dynamic abundance (following Fig. 2a). In each ensemble, the single foraminifera SST distribution from a single discrete depth with a simulated median age of 17.5 ka is shown, and compared to the TRACE-21ka SST distribution for the 18 ka to 17 ka period. The left panels (a, d, g and j) show the 100 SEAMUS ensembles as coloured lines in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera, with the TRACE-21ka SST distribution is shown as a black line. The middle panels (b, e, h and k) show the the rate of over/undersampling for each of the 100 SEAMUS ensembles (coloured lines) relative to the TRACE-21ka SST distribution (black line) in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera. The right panels (c, f, i and l) show Q-Q plots of the 100 SEAMUS ensemble quantiles vs the TRACE-21ka quantiles as coloured lines in the case of 50, 100, 500 and 10<sup>4</sup> randomly picked foraminifera, with a perfect 1:1 correspondence to TRACE-21ka shown for reference as a black line.