**Figure S1.** Simulated downcore, discrete 1 cm depth 1σ SST values of simulated single foraminifera from various 5 cm ka⁻¹ SAR scenarios with 10 cm BD, each with 100 ensembles of SEAMUS runs. In each panel, each ensemble is shown using a coloured line. The solid black lines represent the 95% interval of the ensemble runs at each discrete 1 cm depth. Also shown for reference as a thick grey line is the 1000 year (12000 month) moving 1σ of the 1.5-7 year filtered monthly SST data (as also shown in Fig. 1c.) The left panels (a, c, e and g) show the output of scenarios with 50, 100, 500 and $10^4$ randomly picked foraminifera per discrete 1 cm depth, all with constant species abundance and no assumed analytical error. The right panels (b, d, f and h) show the output of scenarios with 50, 100, 500 and $10^4$ randomly picked foraminifera per discrete 1 cm depth with constant species abundance and an assumed analytical error of ±1°C in SST.
Figure S2. Simulated single foraminifera SST distributions from 100 ensembles of SEAMUS runs, with SAR of 5 cm ka⁻¹, BD of 10 cm, no analytical error and constant abundance. In each ensemble, the single foraminifera SST distribution from a single discrete depth with a simulated median age of 17.5 ka is shown, and compared to the TRACE-21ka SST distribution for the 18 ka to 17 ka period. The left panels (a, d, g and j) show the 100 SEAMUS ensembles as coloured lines in the case of 50, 100, 500 and $10^4$ randomly picked foraminifera, with the TRACE-21ka SST distribution is shown as a black line. The middle panels (b, e, h and k) show the the rate of over/undersampling for each of the 100 SEAMUS ensembles (coloured lines) relative to the TRACE-21ka SST distribution (black line) in the case of 50, 100, 500 and $10^4$ randomly picked foraminifera. The right panels (c, f, i and l) show Q-Q plots of the 100 SEAMUS ensemble quantiles vs the TRACE-21ka quantiles as coloured lines in the case of 50, 100, 500 and $10^4$ randomly picked foraminifera, with a perfect 1:1 correspondence to TRACE-21ka shown for reference as a black line.
Figure S3. Simulated single foraminifera SST distributions from 100 ensembles of SEAMUS runs, with SAR of 5 cm ka$^{-1}$, BD of 10 cm, ±1 °C analytical error and constant abundance. In each ensemble, the single foraminifera SST distribution from a single discrete depth with a simulated median age of 17.5 ka is shown, and compared to the TRACE-21ka SST distribution for the 18 ka to 17 ka period. The left panels (a, d, g, and j) show the 100 SEAMUS ensembles as coloured lines in the case of 50, 100, 500 and $10^4$ randomly picked foraminifera, with the TRACE-21ka SST distribution shown as a black line. The middle panels (b, e, h, and k) show the rate of over/undersampling for each of the 100 SEAMUS ensembles (coloured lines) relative to the TRACE-21ka SST distribution (black line) in the case of 50, 100, 500 and $10^4$ randomly picked foraminifera. The right panels (c, f, i, and l) show Q-Q plots of the 100 SEAMUS ensemble quantiles vs the TRACE-21ka quantiles as coloured lines in the case of 50, 100, 500 and $10^4$ randomly picked foraminifera, with a perfect 1:1 correspondence to TRACE-21ka shown for reference as a black line.
Figure S4. Simulated downcore, discrete 1 cm depth 1σ SST values of simulated single foraminifera from various 5 cm ka⁻¹ SAR scenarios with 10 cm BD, each with 100 ensembles of SEAMUS runs. In each panel, each ensemble is shown using a coloured line. The solid black lines represent the 95% interval of the ensemble runs at each discrete 1 cm depth. Also shown for reference as a thick grey line is the 1000 year (12000 month) moving 1σ of the 1.5-7 year filtered monthly SST data (as also shown in Fig. 1c. The left panels (a, c, e and g) show the output of scenarios with 50, 100, 500 and 10⁴ randomly picked foraminifera per discrete 1 cm depth, all with a SAR of 40 cm ka⁻¹, bioturbation depth of 10 cm, dynamic species abundance (following Fig. 2a) and no assumed analytical error. The right panels (b, d, f and h) show the output of scenarios with 50, 100, 500 and 10⁴ randomly picked foraminifera per discrete 1 cm depth, all with dynamic species abundance (following Fig. 2a) and an assumed analytical error of ±1°C in SST.
Figure S5. Simulated single foraminifera SST distributions from 100 ensembles of SEAMUS runs, with SAR of 5 cm ka\(^{-1}\), BD of 10 cm, no analytical error and dynamic abundance (following Fig. 2a). In each ensemble, the single foraminifera SST distribution from a single discrete depth with a simulated median age of 17.5 ka is shown, and compared to the TRACE-21ka SST distribution for the 18 ka to 17 ka period. The left panels (a, d, g and j) show the 100 SEAMUS ensembles as coloured lines in the case of 50, 100, 500 and \(10^4\) randomly picked foraminifera, with the TRACE-21ka SST distribution shown as a black line. The middle panels (b, e, h and k) show the rate of over/undersampling for each of the 100 SEAMUS ensembles (coloured lines) relative to the TRACE-21ka SST distribution (black line) in the case of 50, 100, 500 and \(10^4\) randomly picked foraminifera. The right panels (c, f, i and l) show Q-Q plots of the 100 SEAMUS ensemble quantiles vs the TRACE-21ka quantiles as coloured lines in the case of 50, 100, 500 and \(10^4\) randomly picked foraminifera, with a perfect 1:1 correspondence to TRACE-21ka shown for reference as a black line.
Figure S6. Simulated single foraminifera SST distributions from 100 ensembles of SEAMUS runs, with SAR of 5 cm ka⁻¹, BD of 10 cm, ±1 °C analytical error and dynamic abundance (following Fig. 2a). In each ensemble, the single foraminifera SST distribution from a single discrete depth with a simulated median age of 17.5 ka is shown, and compared to the TRACE-21ka SST distribution for the 18 ka to 17 ka period. The left panels (a, d, g and j) show the 100 SEAMUS ensembles as coloured lines in the case of 50, 100, 500 and $10^4$ randomly picked foraminifera, with the TRACE-21ka SST distribution shown as a black line. The middle panels (b, e, h and k) show the rate of over/undersampling for each of the 100 SEAMUS ensembles (coloured lines) relative to the TRACE-21ka SST distribution (black line) in the case of 50, 100, 500 and $10^4$ randomly picked foraminifera. The right panels (c, f, i and l) show Q-Q plots of the 100 SEAMUS ensemble quantiles vs the TRACE-21ka quantiles as coloured lines in the case of 50, 100, 500 and $10^4$ randomly picked foraminifera, with a perfect 1:1 correspondence to TRACE-21ka shown for reference as a black line.
Figure S7. Simulated downcore, discrete 1 cm depth 1σ SST values of simulated single foraminifera from various 40 cm ka\(^{-1}\) SAR scenarios with 10 cm BD, each with 100 ensembles of SEAMUS runs. In each panel, each ensemble is shown using a coloured line. The solid black lines represent the 95% interval of the ensemble runs at each discrete 1 cm depth. Also shown for reference as a thick grey line is the 1000 year (12000 month) moving 1σ of the 1.5-7 year filtered monthly SST data (as also shown in Fig. 1c.) The left panels (a, c, e and g) show the output of scenarios with 50, 100, 500 and \(10^4\) randomly picked foraminifera per discrete 1 cm depth, all with constant species abundance and no assumed analytical error. The right panels (b, d, f and h) show the output of scenarios with 50, 100, 500 and \(10^4\) randomly picked foraminifera per discrete 1 cm depth with constant species abundance and an assumed analytical error of ±1°C in SST.
Figure S8. Simulated single foraminifera SST distributions from 100 ensembles of SEAMUS runs, with SAR of 40 cm ka$^{-1}$, BD of 10 cm, ±1 °C analytical error and constant abundance. In each ensemble, the single foraminifera SST distribution from a single discrete depth with a simulated median age of 17.5 ka is shown, and compared to the TRACE-21ka SST distribution for the 18 ka to 17 ka period. The left panels (a, d, g and j) show the 100 SEAMUS ensembles as coloured lines in the case of 50, 100, 500 and $10^4$ randomly picked foraminifera, with the TRACE-21ka SST distribution shown as a black line. The middle panels (b, e, h and k) show the rate of over/undersampling for each of the 100 SEAMUS ensembles (coloured lines) relative to the TRACE-21ka SST distribution (black line) in the case of 50, 100, 500 and $10^4$ randomly picked foraminifera. The right panels (c, f, i and l) show Q-Q plots of the 100 SEAMUS ensemble quantiles vs the TRACE-21ka quantiles as coloured lines in the case of 50, 100, 500 and $10^4$ randomly picked foraminifera, with a perfect 1:1 correspondence to TRACE-21ka shown for reference as a black line.
Figure S9. Simulated downcore, discrete 1 cm depth 1σ SST values of simulated single foraminifera from various 40 cm ka⁻¹ SAR scenarios with 10 cm BD, each with 100 ensembles of SEAMUS runs. In each panel, each ensemble is shown using a coloured line. The solid black lines represent the 95% interval of the ensemble runs at each discrete 1 cm depth. Also shown for reference as a thick grey line is the 1000 year (12000 month) moving 1σ of the 1.5-7 year filtered monthly SST data (as also shown in Fig. 1c). The left panels (a, c, e and g) show the output of scenarios with 50, 100, 500 and $10^4$ randomly picked foraminifera per discrete 1 cm depth, all with a SAR of 40 cm ka⁻¹, bioturbation depth of 10 cm, dynamic species abundance (following Fig. 2a) and no assumed analytical error. The right panels (b, d, f and h) show the output of scenarios with 50, 100, 500 and $10^4$ randomly picked foraminifera per discrete 1 cm depth, all with dynamic species abundance (following Fig. 2a) and an assumed analytical error of ±1°C in SST.
Figure S10. Simulated downcore, discrete 1 cm depth 1σ SST values of simulated single foraminifera from various 40 cm ka\(^{-1}\) SAR scenarios with 10 cm BD, each with 100 ensembles of SEAMUS runs. In each panel, each ensemble is shown using a coloured line. The solid black lines represent the 95% interval of the ensemble runs at each discrete 1 cm depth. Also shown for reference as a thick grey line is the 1000 year (12000 month) moving 1σ of the 1.5-7 year filtered monthly SST data (as also shown in Fig. 1c. The left panels (a, c, e and g) show the output of scenarios with 50, 100, 500 and \(10^4\) randomly picked foraminifera per discrete 1 cm depth, all with a SAR of 40 cm ka\(^{-1}\), bioturbation depth of 10 cm, dynamic species abundance (following Fig. 2a) and no assumed analytical error. The right panels (b, d, f and h) show the output of scenarios with 50, 100, 500 and \(10^4\) randomly picked foraminifera per discrete 1 cm depth, all with dynamic species abundance (following Fig. 2a) and an assumed analytical error of ±1°C in SST.
Figure S11. Simulated single foraminifera SST distributions from 100 ensembles of SEAMUS runs, with SAR of 40 cm ka\(^{-1}\), BD of 10 cm, no analytical error and dynamic abundance (following Fig. 2a). In each ensemble, the single foraminifera SST distribution from a single discrete depth with a simulated median age of 17.5 ka is shown, and compared to the TRACE-21ka SST distribution for the 18 ka to 17 ka period. The left panels (a, d, g and j) show the 100 SEAMUS ensembles as coloured lines in the case of 50, 100, 500 and \(10^4\) randomly picked foraminifera, with the TRACE-21ka SST distribution shown as a black line. The middle panels (b, e, h and k) show the rate of over/undersampling for each of the 100 SEAMUS ensembles (coloured lines) relative to the TRACE-21ka SST distribution (black line) in the case of 50, 100, 500 and \(10^4\) randomly picked foraminifera. The right panels (c, f, i and l) show Q-Q plots of the 100 SEAMUS ensemble quantiles vs the TRACE-21ka quantiles as coloured lines in the case of 50, 100, 500 and \(10^4\) randomly picked foraminifera, with a perfect 1:1 correspondence to TRACE-21ka shown for reference as a black line.
Figure S12. Simulated single foraminifera SST distributions from 100 ensembles of SEAMUS runs, with SAR of 40 cm ka\(^{-1}\), BD of 10 cm, ±1 °C analytical error and dynamic abundance (following Fig. 2a). In each ensemble, the single foraminifera SST distribution from a single discrete depth with a simulated median age of 17.5 ka is shown, and compared to the TRACE-21ka SST distribution for the 18 ka to 17 ka period. The left panels (a, d, g and j) show the 100 SEAMUS ensembles as coloured lines in the case of 50, 100, 500 and \(10^4\) randomly picked foraminifera, with the TRACE-21ka SST distribution is shown as a black line. The middle panels (b, e, h and k) show the rate of over/undersampling for each of the 100 SEAMUS ensembles (coloured lines) relative to the TRACE-21ka SST distribution (black line) in the case of 50, 100, 500 and \(10^4\) randomly picked foraminifera. The right panels (c, f, i and l) show Q-Q plots of the 100 SEAMUS ensemble quantiles vs the TRACE-21ka quantiles as coloured lines in the case of 50, 100, 500 and \(10^4\) randomly picked foraminifera, with a perfect 1:1 correspondence to TRACE-21ka shown for reference as a black line.