

SUPPLEMENTARY APPENDICES FOR:

Quantifying biological carbon pump pathways with a data-constrained mechanistic model ensemble approach

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Supplement 1: Core NEMURO_{BCP} equations

Equations for small phytoplankton (SP): The rate of change of small (i.e., non-diatom) phytoplankton is equal to the gross production of small phytoplankton on nitrate plus the gross production of small phytoplankton on ammonium minus the sum of small phytoplankton respiration, small phytoplankton excretion, grazing on small phytoplankton by all five zooplankton groups, and non-grazer mediated mortality.

$$\frac{dSP}{dt} = GPP_{NO3SP} + GPP_{NH4SP} - RES_{SP} - EXC_{SP} - GRAZ_{SP2SZ} - GRAZ_{SP2LZRES} - GRAZ_{SP2LZDVM} - GRAZ_{SP2PZRES} - GRAZ_{SP2PZDVM} - MORT_{SP}$$

$$GPP_{NO3SP} = V_{max,SP} \times TLIM_{SP} \times LLIM_{SP} \times NOLIM_{SP} \times SP$$

$$GPP_{NH4SP} = V_{max,SP} \times TLIM_{SP} \times LLIM_{SP} \times NHLIM_{SP} \times SP$$

$$TLIM_{SP} = e^{tc_{V,SP} \times T}$$

$$LLIM_{SP} = \left(1 - e^{-\alpha_{SP}/V_{max,SP} \times PAR}\right) \left(e^{-\beta_{SP}/V_{max,SP} \times PAR}\right)$$

$$NOLIM_{SP} = \frac{NO3}{NO3 + K_{NO3,SP}} e^{-(NH4 \times inh_{NH,NO,SP})}$$

$$NHLIM_{SP} = \frac{NH4}{NO3 + K_{NH4,SP}}$$

$$GPP_{NO2SP} = TLIM_{SP} \times LLIM_{SP} \times NOLIM_{SP}$$

$$RES_{SP} = ref_{resp,SP} \times e^{tc_r,SP \times T} \times SP$$

$$EXC_{SP} = ext_{exc,sp} \times (GPP_{NO3SP} + GPP_{NH4SP})$$

$$GRAZ_{SP2SZ} = g_{max,SZ,SP} \times TLIM_{g,SZ,SP} \times OxyLIM_{Mic} \times GLIM_{SZ,SP} \times SZ$$

$$TLIM_{g,SZ,SP} = e^{tc_{g,SZ,SP} \times T}$$

$$OxyLIM_{Mic} = \frac{Oxy}{Oxy + k_{oxy,mic}}$$

$$GLIM_{SZ,SP} = \max(0, 1 - e^{iv_{SZ,SP} \times (thresh_{SZ,SP} - SP)})$$

$$GRAZ_{SP2LZRES} = g_{max,LZRES,SP} \times TLIM_{g,LZRES,SP} \times OxyLIM_{Met} \times GLIM_{LZRES,SP} \times LZ_{RES}$$

$$TLIM_{g,LZRES,SP} = e^{tc_{g,LZRES,SP} \times T}$$

$$OxyLIM_{Met} = \frac{Oxy}{Oxy + k_{oxy,met}}$$

$$GLIM_{LZRES,SP} = \max(0, 1 - e^{iv_{LZRES,SP} \times (thresh_{LZRES,SP} - SP)})$$

$$GRAZ_{SP2LZDVM} = g_{max,LZDVM,SP} \times TLIM_{g,LZDVM,SP} \times OxyLIM_{Met} \times GLIM_{LZDVM,SP} \times LZ_{DVM}$$

$$TLIM_{g,LZDVM,SP} = e^{tc_{g,LZDVM,SP} \times T}$$

$$GLIM_{LZDVM,SP} = \max(0, 1 - e^{iv_{LZDVM,SP} \times (thresh_{LZDVM,SP} - SP)})$$

$$GRAZ_{SP2PZRES} = g_{max,PZRES,SP} \times TLIM_{g,PZRES,SP} \times OxyLIM_{Met} \times GLIM_{PZRES,SP} \times PZ_{RES}$$

$$TLIM_{g,PZRES,SP} = e^{tc_{g,PZRES,SP} \times T}$$

$$GLIM_{PZRES,SP} = \max(0, 1 - e^{iv_{PZRES,SP} \times (thresh_{PZRES,SP} - SP)}) \times e^{-\left(inh_{SZLZLP,SP} \times (LP + SZ + LZRES + LZDVM)\right)}$$

$$GRAZ_{SP2PZDVM} = g_{max,PZDVM,SP} \times TLIM_{g,PZDVM,SP} \times OxyLIM_{Met} \times GLIM_{PZDVM,SP} \times PZ_{DVM}$$

$$TLIM_{g,PZDVM,SP} = e^{tc_{g,PZDVM,SP} \times T}$$

$$GLIM_{PZDVM,SP} = \max(0, 1 - e^{iv_{PZDVM,SP} \times (thresh_{PZDVM,SP} - SP)}) \times e^{-\left(inh_{SZLZLP,SP} \times (LP + SZ + LZRES + LZDVM)\right)}$$

$$MORT_{SP} = ref_{mort,SP} \times e^{tc_m,SP \times T} \times SP$$

Equations for large phytoplankton (LP): The rate of change of large phytoplankton (i.e., diatoms) is equal to the gross production of large phytoplankton on nitrate plus the gross production of large phytoplankton on ammonium minus the sum of large phytoplankton respiration, large phytoplankton excretion, grazing on large phytoplankton by all five zooplankton groups, and non-grazer mediated mortality.

$$\frac{dLP}{dt} = GPP_{NO2LP} + GPP_{NH2LP} - RES_{LP} - EXC_{LP} - GRAZ_{LP2SZ} - GRAZ_{LP2LZRES} - GRAZ_{LP2LZDVM} - GRAZ_{LP2PZRES} - GRAZ_{LP2PZDVM} - MORT_{LP}$$

$$GPP_{NO3LP} = V_{max,LP} \times TLIM_{LP} \times LLIM_{LP} \times \min(NOLIM_{LP} + NHLIM_{LP}, SiLIM_{LP}) \times \frac{NOLIM_{LP}}{NOLIM_{LP} + NHLIM_{LP}} \times LP$$

$$GPP_{NH4LP} = V_{max,LP} \times TLIM_{LP} \times LLIM_{LP} \times \min(NOLIM_{LP} + NHLIM_{LP}, SiLIM_{LP}) \times \frac{NHLIM_{LP}}{NOLIM_{LP} + NHLIM_{LP}} \times LP$$

$$TLIM_{LP} = e^{tc_{V,LP} \times T}$$

$$LLIM_{LP} = (1 - e^{-\alpha_{LP}/V_{max,LP} \times PAR}) (e^{-\beta_{LP}/V_{max,LP} \times PAR})$$

$$NOLIM_{LP} = \frac{NO3}{NO3 + K_{NO3,SP}} e^{-(NH4 \times inh_{NH,NO,LP})}$$

$$NHLIM_{LP} = \frac{NH4}{NO3 + K_{NH4,LP}}$$

$$SiLIM_{LP} = \frac{Si}{Si + K_{Si,LP}}$$

$$GPP_{NO2LP} = TLIM_{LP} \times LLIM_{LP} \times NOLIM_{LP}$$

$$RES_{LP} = ref_{resp,LP} \times e^{tc_{r,LP} \times T} \times LP$$

$$EXC_{LP} = ext_{exc,LP} \times (GPP_{NO3LP} + GPP_{NH4LP})$$

$$GRAZ_{LP2SZ} = g_{max,SZ,LP} \times TLIM_{g,SZ,LP} \times OxyLIM_{Mic} \times GLIM_{SZ,LP} \times SZ$$

$$TLIM_{g,SZ,LP} = e^{tc_{g,SZ,LP} \times T}$$

$$GLIM_{SZ,LP} = \max(0, 1 - e^{iv_{SZ,LP} \times (thresh_{SZ,LP} - LP)})$$

$$GRAZ_{LP2LZRES} = g_{max,LZRES,LP} \times TLIM_{g,LZRES,LP} \times OxyLIM_{Met} \times GLIM_{LZRES,LP} \times LZ_{RES}$$

$$TLIM_{g,LZRES,LP} = e^{tc_{g,LZRES,LP} \times T}$$

$$GLIM_{LZRES,LP} = \max(0, 1 - e^{iv_{LZRES,LP} \times (thresh_{LZRES,LP} - LP)})$$

$$GRAZ_{LP2LZDVM} = g_{max,LZDVM,LP} \times TLIM_{g,LZDVM,LP} \times OxyLIM_{Met} \times GLIM_{LZDVM,LP} \times LZ_{DVM}$$

$$TLIM_{g,LZDVM,LP} = e^{tc_{g,LZDVM,LP} \times T}$$

$$GLIM_{LZDVM,LP} = \max(0, 1 - e^{iv_{LZDVM,LP} \times (thresh_{LZDVM,LP} - LP)})$$

$$GRAZ_{LP2PZRES} = g_{max,PZRES,LP} \times TLIM_{g,PZRES,LP} \times OxyLIM_{Met} \times GLIM_{PZRES,LP} \times PZ_{RES}$$

$$TLIM_{g,PZRES,LP} = e^{tc_{g,PZRES,LP} \times T}$$

$$GLIM_{PZRES,LP} = \max(0, 1 - e^{iv_{PZRES,LP} \times (thresh_{PZRES,LP} - LP)}) \times e^{-\left(inh_{SZLZ,LP} \times (SZ + LZRES + LZDVM)\right)}$$

$$GRAZ_{LP2PZDVM} = g_{max,PZDVM,LP} \times TLIM_{g,PZDVM,LP} \times OxyLIM_{Met} \times GLIM_{PZDVM,LP} \times PZ_{DVM}$$

$$TLIM_{g,PZDVM,LP} = e^{tc_{g,PZDVM,LP} \times T}$$

$$GLIM_{PZDVM,LP} = \max(0, 1 - e^{iv_{PZDVM,LP} \times (thresh_{PZDVM,LP} - LP)}) \times e^{-\left(inh_{SZLZ,LP} \times (SZ + LZRES + LZDVM)\right)}$$

$$MORT_{LP} = ref_{mort,LP} \times e^{tc_{m,LP} \times T} \times LP$$

Equations for small zooplankton (SZ): The rate of change of small zooplankton (i.e., heterotrophic protists) is equal to the grazing of small zooplankton on small and large phytoplankton, minus a linear mortality term, minus an excretion term that is proportional to ingestion, minus an egestion term that is proportional to ingestion, minus mortality to the four classes of metazoan grazers.

$$\frac{dSZ}{dt} = (GRAZ_{SP2SZ} + GRAZ_{LP2SZ}) - MORT_{SZ} - EXC_{SZ} - EGE_{SZ} - GRAZ_{SZ2LZRES} - GRAZ_{SZ2LZDVM} - GRAZ_{SZ2PZRES} - GRAZ_{SZ2PZDVM}$$

$$MORT_{SZ} = ref_{mort,SZ} \times e^{tc_{m,SZ} \times T} \times OxyLIM_{Mic} \times SZ$$

$$EXC_{SZ} = (GRAZ_{SP2SZ} + GRAZ_{LP2SZ}) \times (AE_{SZ} - GGE_{SZ})$$

$$EGE_{SZ} = (GRAZ_{SP2SZ} + GRAZ_{LP2SZ}) \times (1 - AE_{SZ})$$

$$GRAZ_{SZ2LZRES} = g_{max,LZRES,SZ} \times TLIM_{g,LZRES,SZ} \times OxyLIM_{Met} \times GLIM_{LZRES,SZ} \times LZ_{RES}$$

$$TLIM_{g,LZRES,SZ} = e^{tc_{g,LZRES,SZ} \times T}$$

$$GLIM_{LZRES,SZ} = \max(0, 1 - e^{iv_{LZRES,SZ} \times (thresh_{LZRES,SZ} - SZ)})$$

$$GRAZ_{SZ2LZDVM} = g_{max,LZDVM,SZ} \times TLIM_{g,LZDVM,SZ} \times OxyLIM_{Met} \times GLIM_{LZDVM,SZ} \times LZ_{DVM}$$

$$TLIM_{g,LZDVM,SZ} = e^{tc_{g,LZDVM,SZ} \times T}$$

$$GLIM_{LZDVM,SZ} = \max(0, 1 - e^{iv_{LZDVM,SZ} \times (thresh_{LZDVM,SZ} - SZ)})$$

$$GRAZ_{SZ2PZRES} = g_{max,PZRES,SZ} \times TLIM_{g,PZRES,SZ} \times OxyLIM_{Met} \times GLIM_{PZRES,SZ} \times PZ_{RES}$$

$$TLIM_{g,PZRES,SZ} = e^{tc_{g,PZRES,SZ} \times T}$$

$$GLIM_{PZRES,SZ} = \max(0, 1 - e^{iv_{PZRES,SZ} \times (thresh_{PZRES,SZ} - SZ)}) \times e^{-(inh_{LZ,SZ} \times (LZ_{RES} + LZ_{DVM}))}$$

$$GRAZ_{SZ2PZDVM} = g_{max,PZDVM,SZ} \times TLIM_{g,PZDVM,SZ} \times OxyLIM_{Met} \times GLIM_{PZDVM,SZ} \times PZ_{DVM}$$

$$TLIM_{g,PZDVM,SZ} = e^{tc_{g,PZDVM,SZ} \times T}$$

$$GLIM_{PZDVM,SZ} = \max(0, 1 - e^{iv_{PZDVM,SZ} \times (thresh_{PZDVM,SZ} - SZ)}) \times e^{-(inh_{LZ,SZ} \times (LZ_{RES} + LZ_{DVM}))}$$

Equations for epipelagic-resident large zooplankton (LZ_{RES}): The rate of change of epipelagic-resident large zooplankton (i.e., non-vertically migrating small mesozooplankton that are <1-mm) is equal to the grazing of epipelagic-resident large zooplankton on small and large phytoplankton and small zooplankton, minus a quadratic mortality term (that depends on time of day to account for day-night differences in abundances of vertically-migrating predators and activity of visual predators), minus an excretion term that is proportional to ingestion, minus a basal excretion term, minus an egestion term that is proportional to ingestion, minus mortality to the two classes of predatory zooplankton.

$$\begin{aligned}\frac{dLZ_{RES}}{dt} &= (GRAZ_{LP2LZRES} + GRAZ_{SP2LZRES} + GRAZ_{SZ2LZRES}) \times GGE_{LZRES} - MORT_{LZRES} - EXC_{active,LZRES} - EXC_{basal,LZRES} - EGE_{LZRES} - GRAZ_{LZRES2PZRES} - GRAZ_{LZRES2PZDVM} \\ MORT_{LZRES} &= \begin{cases} ref_{mort,night,LZRES} \times e^{tc_m,LZRES \times T} \times OxyLIM_{Met} \times LZ_{RES}^2 & \text{during the night} \\ ref_{mort,day,LZRES} \times e^{tc_m,LZRES \times T} \times OxyLIM_{Met} \times LZ_{RES}^2 & \text{during the day} \end{cases} \\ EXC_{active,LZRES} &= (GRAZ_{LP2LZRES} + GRAZ_{SP2LZRES} + GRAZ_{SZ2LZRES}) \times act_{res,LZRES} \\ EXC_{basal,LZRES} &= Ik_{LZ} \times e^{Ik_{a2} \times T} \times OxyLIM_{Met} \times LZ_{RES} \\ EGE_{LZRES} &= (GRAZ_{LP2LZRES} + GRAZ_{SP2LZRES} + GRAZ_{SZ2LZRES}) \times (1 - AE_{LZRES}) \\ GRAZ_{LZRES2PZRES} &= g_{max,PZRES,LZRES} \times TLIM_{g,PZRES,LZRES} \times OxyLIM_{Met} \times GLIM_{PZRES,LZRES} \times PZ_{RES} \\ TLIM_{g,PZRES,LZRES} &= e^{tc_{g,PZRES,LZRES} \times T} \\ GLIM_{PZRES,LZRES} &= \max(0, 1 - e^{iv_{PZRES,LZRES} \times (thresh_{PZRES,LZRES} - LZ_{RES})}) \\ GRAZ_{LZRES2PZDVM} &= g_{max,PZDVM,LZRES} \times TLIM_{g,PZDVM,LZRES} \times OxyLIM_{Met} \times GLIM_{PZDVM,LZRES} \times PZ_{DVM} \\ TLIM_{g,PZDVM,LZRES} &= e^{tc_{g,PZDVM,LZRES} \times T} \\ GLIM_{PZDVM,LZRES} &= \max(0, 1 - e^{iv_{PZDVM,LZRES} \times (thresh_{PZDVM,LZRES} - LZ_{RES})})\end{aligned}$$

Equations for vertically-migrating large zooplankton (LZ_{DVM}): The rate of change of vertically-migrating large zooplankton (i.e., diel vertically migrating small mesozooplankton that are <1-mm) is equal to the grazing of vertically-migrating large zooplankton on small and large phytoplankton and small zooplankton, minus a quadratic mortality term (that depends on time of day to account for day-night differences in abundances of vertically-migrating predators and activity of visual predators), minus an excretion term that is proportional to ingestion, minus a basal excretion term, minus an egestion term that is proportional to ingestion, minus mortality to the two classes of predatory zooplankton.

$$\frac{dLZ_{DVM}}{dt} = (GRAZ_{LP2LZDVM} + GRAZ_{SP2LZDVM} + GRAZ_{SZ2LZDVM}) \times GGE_{LZDVM} - MORT_{LZDVM} - EXC_{active,LZDVM} - EXC_{basal,LZDVM} - EGE_{LZDVM} - GRAZ_{LZDVM2PZRES} - GRAZ_{LZDVM2PZDVM}$$

$$MORT_{LZDVM} = \begin{cases} ref_{mort,night,LZDVM} \times e^{tc_{m,LZDVM} \times T} \times OxyLIM_{Met} \times LZ_{DVM}^2 & \text{during the night} \\ ref_{mort,day,LZDVM} \times e^{tc_{m,LZDVM} \times T} \times OxyLIM_{Met} \times LZ_{DVM}^2 & \text{during the day} \end{cases}$$

$$EXC_{active,LZDVM} = (GRAZ_{LP2LZDVM} + GRAZ_{SP2LZDVM} + GRAZ_{SZ2LZDVM}) \times act_{res,LZDVM}$$

$$EXC_{basal,LZDVM} = Ik_{LZ} \times e^{Ik_{a2} \times T} \times OxyLIM_{Met} \times LZ_{DVM}$$

$$EGE_{LZDVM} = (GRAZ_{LP2LZDVM} + GRAZ_{SP2LZDVM} + GRAZ_{SZ2LZDVM}) \times (1 - AE_{LZDVM})$$

$$GRAZ_{LZDVM2PZRES} = g_{max,PZRES,LZDVM} \times TLIM_{g,PZRES,LZDVM} \times OxyLIM_{Met} \times GLIM_{PZRES,LZDVM} \times PZ_{RES}$$

$$TLIM_{g,PZRES,LZDVM} = e^{tc_{g,PZRES,LZDVM} \times T}$$

$$GLIM_{PZRES,LZDVM} = \max(0, 1 - e^{iv_{PZRES,LZDVM} \times (thresh_{PZRES,LZDVM} - LZ_{DVM})})$$

$$GRAZ_{LZDVM2PZDVM} = g_{max,PZDVM,LZDVM} \times TLIM_{g,PZDVM,LZDVM} \times OxyLIM_{Met} \times GLIM_{PZDVM,LZDVM} \times PZ_{DVM}$$

$$TLIM_{g,PZDVM,LZDVM} = e^{tc_{g,PZDVM,LZDVM} \times T}$$

$$GLIM_{PZDVM,LZDVM} = \max(0, 1 - e^{iv_{PZDVM,LZDVM} \times (thresh_{PZDVM,LZDVM} - LZ_{DVM})})$$

Equations for epipelagic-resident predatory zooplankton (PZ_{RES}): The rate of change of epipelagic-resident predatory zooplankton (i.e., non-vertically migrating large mesozooplankton that are >1-mm, please note that the term “predatory” zooplankton is used to be consistent with prior versions of NEMURO but these taxa are functionally omnivorous) is equal to the grazing of epipelagic-resident predatory zooplankton on small and large phytoplankton and small and large zooplankton, minus a quadratic mortality term (that depends on time of day to account for day-night differences in abundances of vertically-migrating predators and activity of visual predators), minus an excretion term that is proportional to ingestion, minus a basal excretion term, minus an egestion term that is proportional to ingestion, minus mortality to the two classes of predatory zooplankton.

$$\frac{dPZ_{RES}}{dt} = (GRAZ_{SP2PZRES} + GRAZ_{LP2PZRES} + GRAZ_{SZ2PZRES} + GRAZ_{LZRES2PZRES} + GRAZ_{LZDVM2PZRES}) \times GGE_{PZRES} - MORT_{PZRES} - EXC_{activ,PZRES} - EXC_{basal,PZRES} - EGE_{PZRES}$$

$$MORT_{PZRES} = \begin{cases} ref_{mort,night,PZRES} \times e^{tc_{m,PZRES} \times T} \times OxyLIM_{Met} \times PZ_{RES}^2 & \text{during the night} \\ ref_{mort,day,PZRES} \times e^{tc_{m,PZRES} \times T} \times OxyLIM_{Met} \times PZ_{RES}^2 & \text{during the day} \end{cases}$$

$$EXC_{active,PZRES} = (GRAZ_{LP2PZRES} + GRAZ_{SP2PZRES} + GRAZ_{SZ2PZRES}) \times act_{res,PZRES}$$

$$EXC_{basal,PZRES} = Ik_{PZ} \times e^{Ik_{a2} \times T} \times OxyLIM_{Met} \times PZ_{RES}$$

$$EGE_{PZRES} = (GRAZ_{LP2PZRES} + GRAZ_{SP2PZRES} + GRAZ_{SZ2PZRES}) \times (1 - AE_{PZRES})$$

Equations for vertically migrating predatory zooplankton (PZ_{DVM}): The rate of change of vertically migrating predatory zooplankton (i.e., diel-vertically migrating large mesozooplankton that are >1-mm, please note that the term “predatory” zooplankton is used to be consistent with prior versions of NEMURO but these taxa are functionally omnivorous) is equal to the grazing of epipelagic-resident predatory zooplankton on small and large phytoplankton and small and large zooplankton, minus a quadratic mortality term (that depends on time of day to account for day-night differences in abundances of vertically-migrating predators and activity of visual predators), minus an excretion term that is proportional to ingestion, minus a basal excretion term, minus an egestion term that is proportional to ingestion, minus mortality to the two classes of predatory zooplankton.

$$\frac{dPZ_{DVM}}{dt} = (GRAZ_{SP2PZDVM} + GRAZ_{LP2PZDVM} + GRAZ_{SZ2PZDVM} + GRAZ_{LZRES2PZDVM} + GRAZ_{LZDVM2PZDVM}) \times GGE_{PZDVM} - MORT_{PZDVM} - EXC_{activ,PZDVM} - EXC_{basal,PZDVM} - EGE_{PZDVM}$$

$$MORT_{PZDVM} = \begin{cases} ref_{mort,night,PZDVM} \times e^{t_{cm,PZDVM} \times T} \times OxyLIM_{Met} \times PZ_{DVM}^2 & \text{during the night} \\ ref_{mort,day,PZDVM} \times e^{t_{cm,PZDVM} \times T} \times OxyLIM_{Met} \times PZ_{DVM}^2 & \text{during the day} \end{cases}$$

$$EXC_{active,PZDVM} = (GRAZ_{LP2PZDVM} + GRAZ_{SP2PZDVM} + GRAZ_{SZ2PZDVM}) \times act_{res,PZDVM}$$

$$EXC_{basal,PZDVM} = Ik_{PZ} \times e^{Ik_{a2} \times T} \times OxyLIM_{Met} \times PZ_{DVM}$$

$$EGE_{PZDVM} = (GRAZ_{LP2PZDVM} + GRAZ_{SP2PZDVM} + GRAZ_{SZ2PZDVM}) \times (1 - AE_{PZDVM})$$

Equations for nitrate (NO₃): The rate of change of nitrate is equal to nitrification rates minus uptake by large and small phytoplankton plus the respiration of small and large phytoplankton multiplied by the effective *f*-ratio of these taxa (note that this latter term simulates the fact that respiration actually means that they take up less nutrients than their net growth rate would require; it is not meant to simulate the phytoplankton taking up nutrients and then regenerating those nutrients).

$$\frac{dNO_3}{dt} = NITRIFICATION - GPP_{NO_3SP} + RES_{SP} \times NP_{frac,SP} - GPP_{NO_3LP} + RES_{LP} \times NP_{frac,LP}$$

$$NP_{frac,SP} = f_{SP} \times \min(gpp_{SP}/RES_{SP}) \quad \text{Note that this formulation is different from default NEMURO and fixes an oddity that allows phytoplankton to create NO}_3 \text{ when respiration exceeds GPP}$$

$$gpp_{SP} = GPP_{NO_3SP} + GPP_{NH_4SP}$$

$$f_{SP} = \frac{GPP_{NO_3SP}}{GPP_{NO_3SP} + GPP_{NH_4SP}}$$

$$NP_{frac,LP} = f_{SP} \times \min(gpp_{LP}/RES_{LP}) \quad \text{Note that this formulation is different from default NEMURO and fixes an oddity that allows phytoplankton to create NO}_3 \text{ when respiration exceeds GPP}$$

$$gpp_{LP} = GPP_{NO3LP} + GPP_{NH4LP}$$

$$f_{LP} = \frac{GPP_{NO3LP}}{GPP_{NO3LP} + GPP_{NH4LP}}$$

$$NITRIFICATION = ref_{nitr} \times e^{tc_{nitr} \times T} \times OxyLIM_{Mic} \times NH4$$

Equations for ammonium (NH₄): The rate of change of ammonium is equal to the sum of all excretion terms for protists and metazoan zooplankton plus remineralization of particulate organic nitrogen (detritus) plus remineralization of dissolved organic nitrogen to ammonium minus uptake to small and large phytoplankton plus the respiration of small and large phytoplankton multiplied by one minus the effective f -ratio of these taxa minus nitrification.

$$\begin{aligned} \frac{dNH_4}{dt} = & \sum_{i=protists} \sum_{j=prey} GRAZ_{i2j} \times (AE_i - GGE_i) + \sum_{i=mesozoo} (EXC_{activ,i} + EXC_{basal,i}) + REMIN_{PON,NH_4} + REMIN_{LPON,NH_4} + REMIN_{DON,NH_4} \times (1 - f_{DON,DONref}) + REMIN_{DONref,NH_4} \\ & - GPP_{NO3SP} + RES_{SP} \times (1 - NP_{frac,SP}) - GPP_{NO3LP} + RES_{LP} \times (1 - NP_{frac,LP}) - NITRIFICATION \end{aligned}$$

$$REMIN_{PON,NH_4} = ref_{dec,pon,nh} \times e^{tc_{dec,pon,nh} \times T} \times OxyLIM_{Mic} \times PON$$

$$REMIN_{LPON,NH_4} = ref_{dec,lpon,nh} \times e^{tc_{dec,lpon,nh} \times T} \times OxyLIM_{Mic} \times LPON$$

$$REMIN_{DON,NH_4} = ref_{dec,don,nh} \times e^{tc_{dec,don,nh} \times T} \times OxyLIM_{Mic} \times (1 - f_{DON,DONref}) \times DON$$

$$REMIN_{DONref,NH_4} = ref_{dec,donref,nh} \times e^{tc_{dec,donref,nh} \times T} \times OxyLIM_{Mic} \times DON_{ref}$$

Equations for particulate organic nitrogen (PON): The rate of change of particulate organic nitrogen (i.e., slowly sinking detritus) is equal to linear mortality of small and large phytoplankton, plus egestion of all small zooplankton minus aggregation to large particulate organic nitrogen minus remineralization to ammonium and dissolved organic nitrogen.

$$\frac{dPON}{dt} = MORT_{SP} + MORT_{LP} + \sum_{i=protists} \sum_{j=prey} GRAZ_{i2j} \times (1 - AE_i) - AGGREGATION - REMIN_{PON,NH_4} - REMIN_{PON,DON}$$

$$AGGREGATION = ref_{dec,pon,lpon} \times e^{tc_{dec,PON,LPON} \times T} \times PON^2$$

$$REMIN_{PON,DON} = ref_{dec,pon,don} \times e^{tc_{dec,pon,don} \times T} \times PON$$

Equations for large particulate organic nitrogen (LPON): The rate of change of large particulate organic nitrogen (i.e., rapidly sinking detritus) is equal to non-predation mortality of all zooplankton groups, plus egestion of all four metazoan zooplankton groups plus aggregation of small particulate organic nitrogen minus remineralization to ammonium and dissolved organic nitrogen.

$$\frac{dLPON}{dt} = MORT_{SZ} + MORT_{LZres} + MORT_{LZdvm} + MORT_{PZres} + MORT_{PZdvm} + \sum_{i=mesozoo} \sum_{j=prey} GRAZ_{i2j} \times (1 - AE_i) + AGGREGATION - REMIN_{LPON,NH4} - REMIN_{LPON,DON}$$

$$REMIN_{LPON,DON} = ref_{dec,lpon,don} \times e^{tc_{dec,lpon,don} \times T} \times LPON$$

Equations for dissolved organic nitrogen (DON): The rate of change of dissolved organic nitrogen (i.e., labile DON) is equal to excretion of large and small phytoplankton plus remineralization of detritus to DON minus remineralization of DON to ammonium and conversion of labile DON to refractory DON.

$$\frac{dDON}{dt} = EXC_{SP} + EXC_{LP} + REMIN_{PON,DON} + REMIN_{LPON,DON} - REMIN_{DON,NH4} - REMIN_{DON,DONref}$$

$$REMIN_{DON,DONref} = ref_{dec,don,nh} \times e^{tc_{dec,don,nh} \times T} \times f_{DON,DONref} \times DON$$

Equations for refractory dissolved organic nitrogen (DON_{ref}): The rate of change of refractory DON is equal to excretion of large and small phytoplankton plus remineralization of detritus to DON minus remineralization of DON to ammonium and conversion of labile DON to refractory DON.

$$\frac{dDON_{ref}}{dt} = REMIN_{DON,NH4} \times f_{DON,DONref} - REMIN_{DONref,NH4}$$

$$REMIN_{DONref,NH4} = ref_{dec,donref,nh4} \times e^{tc_{dec,donref,nh4} \times T} \times DONref$$

Equations for silicic acid (Si): The rate of change of silicic acid is equal to excretion of large phytoplankton times the silicon:nitrogen ratio of large phytoplankton ($R_{Si:N}$) plus dissolution of small opal to silicic acid plus dissolution of large opal to silicic acid minus growth of large phytoplankton times $R_{Si:N}$

$$\frac{dSi}{dt} = R_{Si:N} \times EXC_{LP} + DISSOLUTION_{Opal, Si} + DISSOLUTION_{LOpal, Si} - R_{Si:N} \times (GPP_{NO2LP} + GPP_{NH2LP} - RES_{LP})$$

$$DISSOLUTION_{Opal, Si} = ref_{dec, Opal, Si} \times e^{tc_{dec, Opal, Si} \times T} \times OP$$

$$DISSOLUTION_{LOpal, Si} = ref_{dec, LOpal, Si} \times e^{tc_{dec, LOpal, Si} \times T} \times LOP$$

Equations for opal (OP): The rate of change of opal (i.e., small detrital silicon particles) is equal to mortality of large phytoplankton times the silicon:nitrogen ratio of large phytoplankton ($R_{Si:N}$) plus grazing of small zooplankton (protists) on large phytoplankton times $R_{Si:N}$ minus dissolution of opal to silicic acid.

$$\frac{dOP}{dt} = MORT_{LP} \times R_{Si:N} + GRAZ_{LP2SZ} \times R_{Si:N} - DISSOLUTION_{Opal, Si}$$

Equations for large opal (LOP): The rate of change of large opal (i.e., large detrital silicon particles) is equal to grazing by all four metazoan zooplankton groups on large phytoplankton times the silicon:nitrogen ratio of large phytoplankton ($R_{Si:N}$) minus dissolution of large opal to silicic acid.

$$\frac{dLOP}{dt} = \sum_{i=mesozoo} GRAZ_{LP2i} \times R_{Si:N} - DISSOLUTION_{LOpal, Si}$$

Equations for chlorophyll *a* (Chl_{SP} and Chl_{LP} for chl associated with small and large phytoplankton, respectively): Chlorophyll *a* associated with small and large phytoplankton are state variables calculated as a function of temperature, nutrients, and light availability for phytoplankton.

$$Chl_{SP} = SP \times R_{C:N} \times 12 \times Chl: C_{SP}$$

$$Chl:C_{SP} = \max \left(\frac{Chl:C_{SP,max}}{1+0.5 \times Chl:C_{SP,max} \times \frac{\alpha_{chl,sp}}{P_{m,SP}^C} \times PAR}, Chl:C_{SP,min} \right)$$

$$\alpha_{chl,sp} = \alpha_{sp} / Chl:C_{SP,max}$$

$$P_{m,SP}^C = V_{max,SP} \times TLIM_{SP} \times (NOLIM_{SP} + NHLIM_{SP}) \times \frac{\alpha_{sp}}{\alpha_{sp} + \beta_{sp}} \times \left(\frac{\beta_{sp}}{\alpha_{sp} + \beta_{sp}} \right)^{\beta_{sp}/\alpha_{sp}}$$

$$Chl_{LP} = LP \times R_{C:N} \times 12 \times Chl:C_{LP}$$

$$Chl:C_{LP} = \max \left(\frac{Chl:C_{LP,max}}{1+0.5 \times Chl:C_{LP,max} \times \frac{\alpha_{chl,lp}}{P_{m,LP}^C} \times PAR}, Chl:C_{LP,min} \right)$$

$$\alpha_{chl,lp} = \alpha_{lp} / Chl:C_{LP,max}$$

$$P_{m,LP}^C = V_{max,LP} \times TLIM_{LP} \times \min(NOLIM_{LP} + NHLIM_{LP}, SiLIM_{LP}) \times \frac{\alpha_{lp}}{\alpha_{lp} + \beta_{lp}} \times \left(\frac{\beta_{lp}}{\alpha_{lp} + \beta_{lp}} \right)^{\beta_{lp}/\alpha_{lp}}$$

Equations for oxygen (Oxy): Organisms are assumed to use fixed oxygen:nitrogen stoichiometry, hence changes in O are diagnosed from changes in N.

$$\Delta Oxy = \Delta N_{org} \times O:N_{NH4} - \Delta NO3 \times (O:N_{NO3} - O:N_{NH4})$$

$$N_{org} = SP + LP + SZ + LZ_{Res} + LZ_{DVM} + PZ_{Res} + PZ_{DVM} + PON + LPON + DON + DON_{ref}$$

N_{org} is the sum of organic nitrogen in a model layer.

$$O:N_{NO3} = 150/16$$

$$O:N_{NH4} = 118/16$$

Supplement 2: Optional Carbon system equations

NEMURO_{BCP} has an optional carbon system module that can be toggled on or off. It includes alkalinity (Alk) and dissolved inorganic carbon (DIC) as state variables. It uses the model of Follows et al. (2006) to diagnose pH and pCO₂ from Alk and DIC and to mediate air-sea gas exchange. The coupling of the carbon system model with core NEMURO_{BCP} assumes fixed C:N ratios and uses the equations below:

$$N_{org} = SP + LP + SZ + LZ_{Res} + LZ_{DVM} + PZ_{Res} + PZ_{DVM} + PON + LPON + DON + DON_{ref}$$

N_{org} is the sum of organic nitrogen in a model layer.

$$\Delta DIC = -\Delta N_{org} \times C:N$$

$$C:N = 106/16$$

$$\Delta Alk = -\Delta NO3 \times C:N + \Delta NH4 - \frac{\Delta NO3 + \Delta NH4}{N:P}$$

$$N:P = 16$$

Supplement 3: Thorium equations

NEMURO_{BCP} has an optional ^{234}Th module that can be toggled on or off. It includes ten state variables. One is for dissolved ^{234}Th . The others are for ^{234}Th adsorbed onto each of the particulate-nitrogen-containing state variables in core NEMURO_{BCP} (SP, LP, SZ, LZ_{RES}, LZ_{DVM}, PZ_{RES}, PZ_{DVM}, PON, and LPON). ^{234}Th is produced from ^{238}U (which is assumed to be proportional to salinity) and adsorbs onto particulate matter with second-order sorption kinetics following Resplandy et al. (2012). When particulate nitrogen pools are transformed from one state variable to another (e.g., phytoplankton dying and becoming detrital PON) the ^{234}Th is transformed as well. When particulate nitrogen pools are transformed into dissolved pools (e.g., DON or dissolved nutrients) the associated ^{234}Th is converted back into dissolved ^{234}Th . All particulate ^{234}Th pools also undergo desorption and radioactive decay. Because substantial evidence shows that metazoan zooplankton have much higher C: ^{234}Th ratios than seston (Rodriguez Y Baena et al., 2007; Stukel et al., 2019), we assign a lower adsorption factor to metazoans than is used for other particulate matter. Equations governing the thorium submodule are:

$k_{\text{for}} = 0.013 \times \text{C:N}$ ($\text{m}^3 / \text{mmol C} / \text{day}$) thorium sorption coefficient (Stukel et al., 2019)

$k_{\text{back}} = 3$ (yr^{-1}) thorium desorption coefficient (Resplandy et al., 2012)

$\lambda_{234} = 0.02876$ (d^{-1}) thorium decay constant

$f_{\text{LZ}} = 0.04$ fraction that adsorption to LZ is decreased relative to seston

$f_{\text{PZ}} = 0.01$ fraction that adsorption to PZ is decreased relative to seston

$$\begin{aligned}
\frac{dTh}{dt} = & \lambda_{234} \times {}^{238}U + GRAZ_{SP2SZ} \times (AE_{SZ} - GGE_{SZ}) \frac{SP_{Th}}{SP} + GRAZ_{LP2SZ} \times (AE_{SZ} - GGE_{SZ}) \frac{LP_{Th}}{LP} + GRAZ_{SP2LZRES} \times act_{res,LZRES} \frac{SP_{Th}}{SP} + GRAZ_{SP2LZDVM} \times act_{res,LZDVM} \frac{SP_{Th}}{SP} \\
& + GRAZ_{SP2PZRES} \times act_{res,PZRES} \frac{SP_{Th}}{SP} + GRAZ_{SP2PZDVM} \times act_{res,PZDVM} \frac{SP_{Th}}{SP} + GRAZ_{LP2LZRES} \times act_{res,LZRES} \frac{LP_{Th}}{LP} + GRAZ_{LP2LZDVM} \times act_{res,LZDVM} \frac{LP_{Th}}{LP} \\
& + GRAZ_{LP2PZRES} \times act_{res,PZRES} \frac{LP_{Th}}{LP} + GRAZ_{LP2PZDVM} \times act_{res,PZDVM} \frac{LP_{Th}}{LP} + GRAZ_{SZ2LZRES} \times act_{res,LZRES} \frac{SZ_{Th}}{SZ} + GRAZ_{SZ2LZDVM} \times act_{res,LZDVM} \frac{SZ_{Th}}{SZ} \\
& + GRAZ_{SZ2PZRES} \times act_{res,PZRES} \frac{SZ_{Th}}{SZ} + GRAZ_{SZ2PZDVM} \times act_{res,PZDVM} \frac{SZ_{Th}}{SZ} + GRAZ_{LZRES2PZRES} \times act_{res,PZRES} \frac{LZres_{Th}}{LZres} \\
& + GRAZ_{LZDVM2PZRES} \times act_{res,PZRES} \frac{LZdvm_{Th}}{LZdvm} + GRAZ_{LZRES2PZDVM} \times act_{res,PZDVM} \frac{LZres_{Th}}{LZres} + GRAZ_{LZDVM2PZDVM} \times act_{res,PZDVM} \frac{LZdvm_{Th}}{LZdvm} + RES_{SP} \frac{SP_{Th}}{SP} \\
& + RES_{LP} \frac{LP_{Th}}{LP} + EXC_{basal,LZRES} \frac{LZres_{Th}}{LZres} + EXC_{basal,LZDVM} \frac{LZdvm_{Th}}{LZdvm} + EXC_{basal,PZRES} \frac{PZres_{Th}}{PZres} + EXC_{basal,PZDVM} \frac{PZdvm_{Th}}{PZdvm} \\
& + (REMIN_{PON,NH4} + REMIN_{PON,DON}) \frac{PON_{Th}}{PON} + (REMIN_{LPON,NH4} + REMIN_{LPON,DON}) \frac{LPON_{Th}}{LPON} + k_{back} SP_{Th} + k_{back} LP_{Th} + k_{back} SZ_{Th} + k_{back} LZres_{Th} \\
& + k_{back} LZdvm_{Th} + k_{back} PZres_{Th} + k_{back} PZdvm_{Th} + k_{back} PON_{Th} + k_{back} LPON_{Th} - k_{for} \times SP \times Th - k_{for} \times LP \times Th - k_{for} \times SZ \times Th \\
& - k_{for} \times LZres \times f_{LZ} \times Th - k_{for} \times LZdvm \times f_{LZ} \times Th - k_{for} \times PZres \times f_{PZ} \times Th - k_{for} \times PZdvm \times f_{PZ} \times Th - k_{for} \times PON \times Th \\
& - k_{for} \times LPON \times Th - \lambda_{234} \times Th
\end{aligned}$$

$$\begin{aligned}
\frac{dSP_{Th}}{dt} = & k_{for} \times SP \times Th - GRAZ_{SP2SZ} \frac{SP_{Th}}{SP} - GRAZ_{SP2LZRES} \frac{SP_{Th}}{SP} - GRAZ_{SP2LZDVM} \frac{SP_{Th}}{SP} - GRAZ_{SP2PZRES} \frac{SP_{Th}}{SP} - GRAZ_{SP2PZDVM} \frac{SP_{Th}}{SP} - MORT_{SP} \frac{SP_{Th}}{SP} - RES_{SP} \frac{SP_{Th}}{SP} \\
& - k_{back} SP_{Th} - \lambda_{234} SP_{Th}
\end{aligned}$$

$$\begin{aligned}
\frac{dLP_{Th}}{dt} = & k_{for} \times LP \times Th - GRAZ_{LP2SZ} \frac{LP_{Th}}{LP} - GRAZ_{LP2LZRES} \frac{LP_{Th}}{LP} - GRAZ_{LP2LZDVM} \frac{LP_{Th}}{LP} - GRAZ_{LP2PZRES} \frac{LP_{Th}}{LP} - GRAZ_{LP2PZDVM} \frac{LP_{Th}}{LP} - MORT_{LP} \frac{LP_{Th}}{LP} - RES_{LP} \frac{LP_{Th}}{LP} \\
& - k_{back} LP_{Th} - \lambda_{234} LP_{Th}
\end{aligned}$$

$$\begin{aligned} \frac{dSZ_{Th}}{dt} = & k_{for} \times SZ \times Th + GRAZ_{SP2SZ} \times GGE_{SZ} \frac{SP_{Th}}{SP} + GRAZ_{LP2SZ} \times GGE_{SZ} \frac{LP_{Th}}{LP} - GRAZ_{SZ2LZRES} \frac{SZ_{Th}}{SZ} - GRAZ_{SZ2LZDVM} \frac{SZ_{Th}}{SZ} - GRAZ_{SZ2PZRES} \frac{SZ_{Th}}{SZ} - GRAZ_{SZ2PZDVM} \frac{SZ_{Th}}{SZ} \\ & - MORT_{SZ} \frac{SZ_{Th}}{SZ} - k_{back} SZ_{Th} - \lambda_{234} SZ_{Th} \end{aligned}$$

$$\begin{aligned} \frac{dLZres_{Th}}{dt} = & k_{for} \times LZres \times f_{LZ} \times Th - GRAZ_{LZRES2PZRES} \frac{LZres_{Th}}{LZres} - GRAZ_{LZRES2PZDVM} \frac{LZres_{Th}}{LZres} - MORT_{LZRES} \frac{LZres_{Th}}{LZres} - EXC_{basal,LZRES} \frac{LZres_{Th}}{LZres} - k_{back} LZres_{Th} \\ & - \lambda_{234} LZres_{Th} \end{aligned}$$

$$\begin{aligned} \frac{dLZdvm_{Th}}{dt} = & k_{for} \times LZdvm \times f_{LZ} \times Th - GRAZ_{LZDVM2PZRES} \frac{LZdvm_{Th}}{LZdvm} - GRAZ_{LZDVM2PZDVM} \frac{LZdvm_{Th}}{LZdvm} - MORT_{LZDVM} \frac{LZdvm_{Th}}{LZdvm} - EXC_{basal,LZDVM} \frac{LZdvm_{Th}}{LZdvm} \\ & - k_{back} LZdvm_{Th} - \lambda_{234} LZdvm_{Th} \end{aligned}$$

$$\frac{dPZres_{Th}}{dt} = k_{for} \times PZres \times f_{PZ} \times Th - MORT_{PZRES} \frac{PZres_{Th}}{PZres} - EXC_{basal,PZRES} \frac{PZres_{Th}}{PZres} - k_{back} PZres_{Th} - \lambda_{234} PZres_{Th}$$

$$\frac{dPZdvm_{Th}}{dt} = k_{for} \times PZdvm \times f_{PZ} \times Th - MORT_{PZDVM} \frac{PZdvm_{Th}}{PZdvm} - EXC_{basal,PZDVM} \frac{PZdvm_{Th}}{PZdvm} - k_{back} PZdvm_{Th} - \lambda_{234} PZdvm_{Th}$$

$$\begin{aligned} \frac{dPON_{Th}}{dt} = & k_{for} \times PON \times Th + GRAZ_{SP2SZ} \times (1 - AE_{SZ}) \frac{SP_{Th}}{SP} + GRAZ_{LP2SZ} \times (1 - AE_{SZ}) \frac{LP_{Th}}{LP} + MORT_{SP} \frac{SP_{Th}}{SP} + MORT_{LP} \frac{LP_{Th}}{LP} - AGGREGATION \frac{PON_{Th}}{PON} \\ & - (REMIN_{PON,NH_4} + REMIN_{PON,DON}) \frac{PON_{Th}}{PON} - k_{back} PON_{Th} - \lambda_{234} PON_{Th} \end{aligned}$$

$$\begin{aligned} \frac{dLPON_{Th}}{dt} = & k_{for} \times PON \times Th + GRAZ_{SP2LZRES} \times (1 - act_{res,LZRES}) \frac{SP_{Th}}{SP} + GRAZ_{SP2LZDVM} \times (1 - act_{res,LZDVM}) \frac{SP_{Th}}{SP} + GRAZ_{SP2PZRES} \times (1 - act_{res,PZRES}) \frac{SP_{Th}}{SP} \\ & + GRAZ_{SP2PZDVM} \times (1 - act_{res,PZDVM}) \frac{SP_{Th}}{SP} + GRAZ_{LP2LZRES} \times (1 - act_{res,LZRES}) \frac{LP_{Th}}{LP} + GRAZ_{LP2LZDVM} \times (1 - act_{res,LZDVM}) \frac{LP_{Th}}{LP} \\ & + GRAZ_{LP2PZRES} \times (1 - act_{res,PZRES}) \frac{LP_{Th}}{LP} + GRAZ_{LP2PZDVM} \times (1 - act_{res,PZDVM}) \frac{LP_{Th}}{LP} + GRAZ_{SZ2LZRES} \times (1 - act_{res,LZRES}) \frac{SZ_{Th}}{SZ} \\ & + GRAZ_{SZ2LZDVM} \times (1 - act_{res,LZDVM}) \frac{SZ_{Th}}{SZ} + GRAZ_{SZ2PZRES} \times (1 - act_{res,PZRES}) \frac{SZ_{Th}}{SZ} + GRAZ_{SZ2PZDVM} \times (1 - act_{res,PZDVM}) \frac{SZ_{Th}}{SZ} \\ & + GRAZ_{LZRES2PZRES} \times (1 - act_{res,PZRES}) \frac{LZres_{Th}}{LZres} + GRAZ_{LZDVM2PZRES} \times (1 - act_{res,PZRES}) \frac{LZdvm_{Th}}{LZdvm} + GRAZ_{LZRES2PZDVM} \times (1 - act_{res,PZDVM}) \frac{LZres_{Th}}{LZres} \\ & + GRAZ_{LZDVM2PZDVM} \times (1 - act_{res,PZDVM}) \frac{LZdvm_{Th}}{LZdvm} + MORT_{SZ} \frac{SZ_{Th}}{SZ} + MORT_{LZRES} \frac{LZres_{Th}}{LZres} + MORT_{LZDVM} \frac{LZdvm_{Th}}{LZdvm} + MORT_{PZRES} \frac{PZres_{Th}}{PZres} \\ & + MORT_{PZDVM} \frac{PZdvm_{Th}}{PZdvm} + AGGREGATION \frac{PON_{Th}}{PON} - (REMIN_{LPON,NH_4} + REMIN_{LPON,DON}) \frac{LPON_{Th}}{LPON} - k_{back} LPON_{Th} - \lambda_{234} LPON_{Th} \end{aligned}$$

Supplement 4: ¹⁵Nitrogen equations

NEMURO_{BCP} has an optional ¹⁵N module that can be toggled on or off. It includes thirteen state variables each of which represents the amount of ¹⁵N contained within one of the nitrogen-containing state variables in core NEMUROBCP (NO₃, NH₄, SP, LP, SZ, LZ_{RES}, LZ_{DVM}, PZ_{RES}, PZ_{DVM}, PON, LPON, DON, and DON_{ref}). The 15N submodule is based on the NEMURO+¹⁵N model of Stukel et al. (2018). The 15N submodule is 15N-conserving and includes isotopic fractionation associated with most biological processes. Fractionation parameters are given in Supp. Table 7. The equations governing the ¹⁵N submodule are presented here:

$$R_{SP} = SP_{N15} / SP$$

$$R_{LP} = LP_{N15} / LP$$

$$R_{SZ} = SZ_{N15} / SZ$$

$$R_{LZres} = LZres_{N15} / LZ_{RES}$$

$$R_{LZdvm} = LZdvm_{N15} / LZ_{DVM}$$

$$R_{PZres} = PZres_{N15} / PZ_{RES}$$

$$R_{PZdvm} = PZdvm_{N15} / PZ_{DVM}$$

$$R_{NO3} = NO3_{N15} / NO3$$

$$R_{NH4} = NH4_{N15} / NH4$$

$$R_{PON} = PON_{N15} / PON$$

$$R_{LPON} = LPON_{N15} / LPON$$

$$R_{DON} = DON_{N15} / DON$$

$$R_{DONREF} = DON_{refN15} / DON_{REF}$$

$$\begin{aligned} \frac{dSP_{N15}}{dt} = & \left(GPP_{NO3SP} - RES_{SP} \times NP_{frac,SP} \right) \times R_{NO3} \times \alpha_{NO3up} + \left(GPP_{NH4SP} - RES_{SP} \times \left(1 - NP_{frac,SP} \right) \right) \times R_{NH4} \times \alpha_{NH4up} - EXC_{SP} \times R_{SP} \times \alpha_{Exu} - MORT_{SP} \times R_{SP} \\ & - GRAZ_{SP2SZ} \times R_{SP} - GRAZ_{SP2LZRES} \times R_{SP} - GRAZ_{SP2LZDVM} \times R_{SP} - GRAZ_{SP2PZRES} \times R_{SP} - GRAZ_{SP2PZDVM} \times R_{SP} \end{aligned}$$

$$\begin{aligned} \frac{dLP_{N15}}{dt} = & \left(GPP_{NO3LP} - RES_{LP} \times NP_{frac,LP} \right) \times R_{NO3} \times \alpha_{NO3up} + \left(GPP_{NH4LP} - RES_{LP} \times \left(1 - NP_{frac,LP} \right) \right) \times R_{NH4} \times \alpha_{NH4up} - EXC_{LP} \times R_{LP} \times \alpha_{Exu} - MORT_{LP} \times R_{LP} \\ & - GRAZ_{LP2SZ} \times R_{LP} - GRAZ_{LP2LZRES} \times R_{LP} - GRAZ_{LP2LZDVM} \times R_{LP} - GRAZ_{LP2PZRES} \times R_{LP} - GRAZ_{LP2PZDVM} \times R_{LP} \end{aligned}$$

$$\begin{aligned} \frac{dSZ_{N15}}{dt} = & GRAZ_{SP2SZ} \times R_{SP} + GRAZ_{LP2SZ} \times R_{LP} - (1 - AE_{SZ})(GRAZ_{SP2SZ} + GRAZ_{LP2SZ})R_{SZ}\alpha_{SZ,Eg} - (AE_{SZ} - GGE_{SZ})(GRAZ_{SP2SZ} + GRAZ_{LP2SZ})R_{SZ}\alpha_{SZ,Exc} \\ & - GRAZ_{SZ2LZRES} \times R_{SZ} - GRAZ_{SZ2LZDVM} \times R_{SZ} - GRAZ_{SZ2PZRES} \times R_{SZ} - GRAZ_{SZ2PZDVM} \times R_{SZ} - MORT_{SZ} \times R_{SZ} \end{aligned}$$

$$\begin{aligned} \frac{dLZres_{N15}}{dt} = & GRAZ_{SP2LZRES} \times R_{SP} + GRAZ_{LP2LZRES} \times R_{LP} + GRAZ_{SZ2LZRES} \times R_{SZ} - (1 - AE_{LZRES})(GRAZ_{SP2LZRES} + GRAZ_{LP2LZRES} + GRAZ_{SZ2LZRES})R_{LZRES}\alpha_{LZ,Eg} \\ & - act_{res,LZRES}(GRAZ_{SP2LZRES} + GRAZ_{LP2LZRES} + GRAZ_{SZ2LZRES})R_{LZRES}\alpha_{LZ,Exc} - GRAZ_{LZRES2PZRES} \times R_{LZRES} - GRAZ_{LZRES2PZDVM} \times R_{LZRES} - MORT_{LZRES} \times R_{LZRES} \\ & - EXC_{basal,LZRES}R_{LZRES}\alpha_{LZ,Exc} \end{aligned}$$

$$\begin{aligned} \frac{dLZdvm_{N15}}{dt} = & GRAZ_{SP2LZDVM} \times R_{SP} + GRAZ_{LP2LZDVM} \times R_{LP} + GRAZ_{SZ2LZDVM} \times R_{SZ} - (1 - AE_{LZDVM})(GRAZ_{SP2LZDVM} + GRAZ_{LP2LZDVM} + GRAZ_{SZ2LZDVM})R_{LZDVM}\alpha_{LZ,Eg} \\ & - act_{res,LZDVM}(GRAZ_{SP2LZDVM} + GRAZ_{LP2LZDVM} + GRAZ_{SZ2LZDVM})R_{LZDVM}\alpha_{LZ,Exc} - GRAZ_{LZDVM2PZRES} \times R_{LZDVM} - GRAZ_{LZDVM2PZDVM} \times R_{LZDVM} \\ & - MORT_{LZDVM} \times R_{LZDVM} - EXC_{basal,LZDVM}R_{LZDVM}\alpha_{LZ,Exc} \end{aligned}$$

$$\begin{aligned}
\frac{dPZres_{N15}}{dt} = & GRAZ_{SP2PZRES} \times R_{SP} + GRAZ_{LP2PZRES} \times R_{LP} + GRAZ_{SZ2PZRES} \times R_{SZ} + GRAZ_{LZRES2PZRES} \times R_{LZRES} + GRAZ_{LZDVM2PZRES} \times R_{LZDVM} \\
& - (1 - AE_{PZRES})(GRAZ_{SP2PZRES} + GRAZ_{LP2PZRES} + GRAZ_{SZ2PZRES} + GRAZ_{LZRES2PZRES} + GRAZ_{LZDVM2PZRES})R_{PZRES}\alpha_{PZ,Eg} \\
& - act_{res,PZRES}(GRAZ_{SP2PZRES} + GRAZ_{LP2PZRES} + GRAZ_{SZ2PZRES} + GRAZ_{LZRES2PZRES} + GRAZ_{LZDVM2PZRES})R_{PZRES}\alpha_{PZ,Exc} - MORT_{PZRES} \times R_{PZRES} \\
& - EXC_{basal,PZRES}R_{PZRES}\alpha_{PZ,Exc}
\end{aligned}$$

$$\begin{aligned}
\frac{dPZdvm_{N15}}{dt} = & GRAZ_{SP2PZDVM} \times R_{SP} + GRAZ_{LP2PZDVM} \times R_{LP} + GRAZ_{SZ2PZDVM} \times R_{SZ} + GRAZ_{LZRES2PZDVM} \times R_{LZRES} + GRAZ_{LZDVM2PZDVM} \times R_{LZDVM} \\
& - (1 - AE_{PZDVM})(GRAZ_{SP2PZDVM} + GRAZ_{LP2PZDVM} + GRAZ_{SZ2PZDVM} + GRAZ_{LZRES2PZDVM} + GRAZ_{LZDVM2PZDVM})R_{PZDVM}\alpha_{PZ,Eg} \\
& - act_{res,PZDVM}(GRAZ_{SP2PZDVM} + GRAZ_{LP2PZDVM} + GRAZ_{SZ2PZDVM} + GRAZ_{LZRES2PZDVM} + GRAZ_{LZDVM2PZDVM})R_{PZDVM}\alpha_{PZ,Exc} - MORT_{PZDVM} \times R_{PZDVM} \\
& - EXC_{basal,PZDVM}R_{PZDVM}\alpha_{PZ,Exc}
\end{aligned}$$

$$\frac{dNO3_{N15}}{dt} = NITRIFICATION \times R_{NH4}\alpha_{Nit} - (GPP_{NO3SP} - RES_{SP} \times NP_{frac,SP} + GPP_{NO3LP} - RES_{LP} \times NP_{frac,LP}) \times R_{NO3} \times \alpha_{NO3up}$$

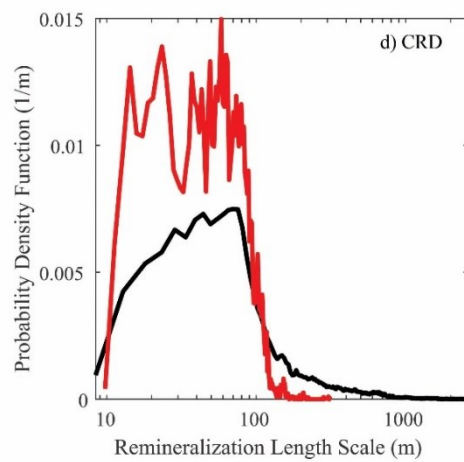
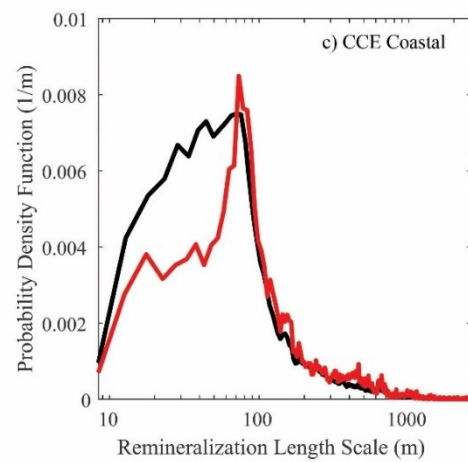
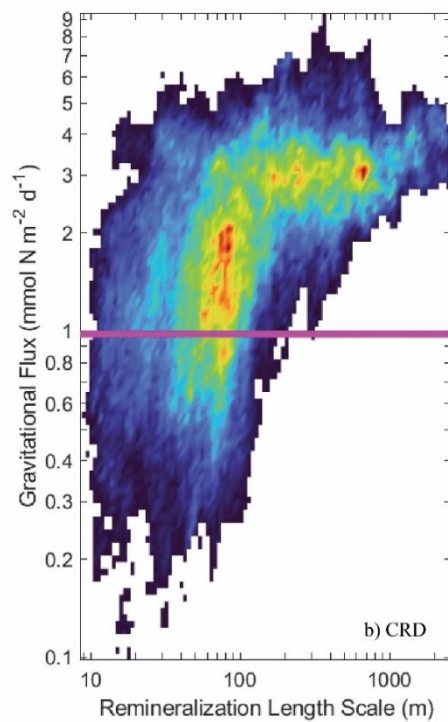
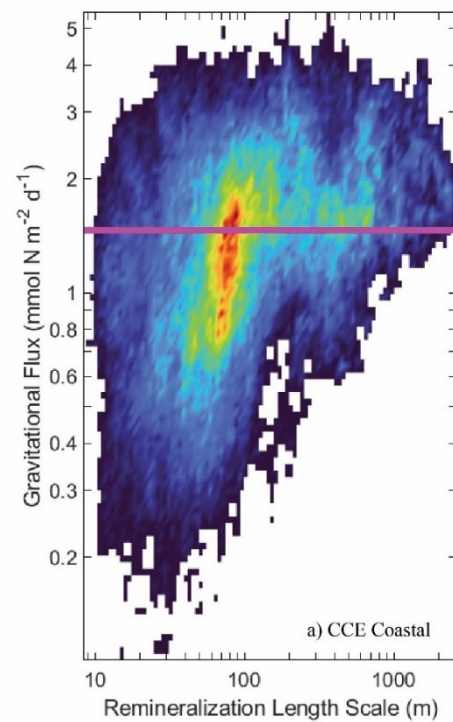
$$\begin{aligned}
\frac{dNH4_{N15}}{dt} = & - \left(GPP_{NH4SP} - RES_{SP} \times (1 - NP_{frac,SP}) + GPP_{NH4LP} - RES_{LP} \times (1 - NP_{frac,LP}) \right) \times R_{NH4} \times \alpha_{NH4up} + (AE_{SZ} - GGE_{SZ})(GRAZ_{SP2SZ} + GRAZ_{LP2SZ})R_{SZ}\alpha_{SZ,Exc} \\
& + act_{res,LZRES}(GRAZ_{SP2LZRES} + GRAZ_{LP2LZRES} + GRAZ_{SZ2LZRES})R_{LZRES}\alpha_{LZ,Exc} + act_{res,LZDVM}(GRAZ_{SP2LZDVM} + GRAZ_{LP2LZDVM} + GRAZ_{SZ2LZDVM})R_{LZDVM}\alpha_{LZ,Exc} \\
& + act_{res,PZRES}(GRAZ_{SP2PZRES} + GRAZ_{LP2PZRES} + GRAZ_{SZ2PZRES} + GRAZ_{LZRES2PZRES} + GRAZ_{LZDVM2PZRES})R_{PZRES}\alpha_{PZ,Exc} \\
& + act_{res,PZDVM}(GRAZ_{SP2PZDVM} + GRAZ_{LP2PZDVM} + GRAZ_{SZ2PZDVM} + GRAZ_{LZRES2PZDVM} + GRAZ_{LZDVM2PZDVM})R_{PZDVM}\alpha_{PZ,Exc} + EXC_{basal,LZRES}R_{LZRES}\alpha_{LZ,Exc} \\
& + EXC_{basal,LZDVM}R_{LZDVM}\alpha_{LZ,Exc} + EXC_{basal,PZRES}R_{PZRES}\alpha_{PZ,Exc} + EXC_{basal,PZDVM}R_{PZDVM}\alpha_{PZ,Exc} - NITRIFICATION \times R_{NH4}\alpha_{Nit} + REMIN_{PON,NH4} \times R_{PON}\alpha_{P2N} \\
& + REMIN_{LPON,NH4} \times R_{LPON}\alpha_{P2N} + REMIN_{DON,NH4} \times R_{DON}\alpha_{D2N} + REMIN_{DONref,NH4} \times R_{DONref}\alpha_{D2N}
\end{aligned}$$

$$\begin{aligned} \frac{dPON_{N15}}{dt} = & MORT_{SP} \times R_{SP} + MORT_{LP} \times R_{LP} + (1 - AE_{SZ})(GRAZ_{SP2SZ} + GRAZ_{LP2SZ})R_{SZ}\alpha_{SZ,Eg} - AGGREGATION \times R_{PON} - REMIN_{PON,NH4} \times R_{PON}\alpha_{P2N} \\ & - REMIN_{PON,DON} \times R_{PON}\alpha_{P2D} \end{aligned}$$

$$\begin{aligned} \frac{dLPON_{N15}}{dt} = & (1 - AE_{LZRES})(GRAZ_{SP2LZRES} + GRAZ_{LP2LZRES} + GRAZ_{SZ2LZRES})R_{LZRES}\alpha_{LZ,Eg} + (1 - AE_{LZDVM})(GRAZ_{SP2LZDVM} + GRAZ_{LP2LZDVM} + GRAZ_{SZ2LZDVM})R_{LZDVM}\alpha_{LZ,Eg} \\ & + (1 - AE_{PZRES})(GRAZ_{SP2PZRES} + GRAZ_{LP2PZRES} + GRAZ_{SZ2PZRES} + GRAZ_{LZRES2PZRES} + GRAZ_{LZDVM2PZRES})R_{PZRES}\alpha_{PZ,Eg} \\ & + (1 - AE_{PZDVM})(GRAZ_{SP2PZDVM} + GRAZ_{LP2PZDVM} + GRAZ_{SZ2PZDVM} + GRAZ_{LZRES2PZDVM} + GRAZ_{LZDVM2PZDVM})R_{PZDVM}\alpha_{PZ,Eg} + MORT_{SZ} \times R_{SZ} \\ & + MORT_{LZRES} \times R_{LZRES} + MORT_{LZDVM} \times R_{LZDVM} + MORT_{PZRES} \times R_{PZRES} + MORT_{PZDVM} \times R_{PZDVM} + AGGREGATION \times R_{PON} - REMIN_{LPON,NH4} \times R_{LPON}\alpha_{P2N} \\ & - REMIN_{LPON,DON} \times R_{LPON}\alpha_{P2D} \end{aligned}$$

$$\begin{aligned} \frac{dDON_{N15}}{dt} = & EXC_{SP} \times R_{SP} \times \alpha_{Exu} + EXC_{LP} \times R_{LP} \times \alpha_{Exu} + REMIN_{PON,DON} \times R_{PON}\alpha_{P2D} + REMIN_{LPON,DON} \times R_{LPON}\alpha_{P2D} - REMIN_{DON,NH4} \times R_{DON}\alpha_{D2N} \\ & - REMIN_{DON,DONref} \times R_{DON} \end{aligned}$$

$$\frac{dDONref_{N15}}{dt} = REMIN_{DON,DONref} \times R_{DON} - REMIN_{DONref,NH4} \times R_{DONref}\alpha_{D2N}$$



Supp. Fig. S1 – Heatmap showing the relationship between modeled gravitational flux and the remineralization length scale of fast-sinking detritus ($RLS = L_{\text{sink}} / (\text{ref}_{\text{dec}, \text{LPON}, \text{DON}} + \text{ref}_{\text{dec}, \text{LPON}, \text{NH}_4})$) for a representative CCE upwelling-influenced experiment (a, 1604-3) and a representative Costa Rica Dome experiment (b, CRD-1). Horizontal magenta lines show the sediment-trap measured gravitational flux. Probability density function for RLS determined by the OEP_{MCMC} procedure (black lines in c and d) and for only those solutions that predicted export flux within ± 1 standard deviation of the mean sediment trap measurement for CCE 1604-3 and CRD-1 (red lines in c and d, respectively).

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