Water Res. Vol. 19, No. 8, pp. 941–945, 1985
Printed in Great Britain.

0043-1354/85 $3.00 + 0.00

THE DILUTION OF WASTE IN THE WAKE OF A SHIP

R. E. Lewis
The Brixham Laboratory, ICI plc, Freshwater Quarry, Brixham, South Devon, England

(*Received January* 1983)

Abstract—This paper considers the problem of predicting the dilution of waste which has been discharged into the wake of a moving ship. A theoretical model is developed which relates the dispersion of the effluent field to the intensity of the turbulence created by the movement of the vessel. Specifically, the model describes the dependence of the dilution on the speed, dimensions and specific resistance coefficient of the ship.

The new theory predicts that the rate of dilution decreases with time after discharge and, as with the empirical IMCO formula, indicates that dilution is more sensitive to the speed of the ship than the volume rate of discharge of waste.

Key words—turbulence, wake, dilution, energy, slipstream, effluent

NOMENCLATURE

B = beam of ship
c_e = initial effluent concentration
c_p = effluent concentration in wake
c_F = specific resistance coefficient
d = draught of ship
D = dilution in wake
E = turbulent kinetic energy per unit mass
h = depth of layer through which effluent is fully mixed
i = intensity of turbulence spectrum
L = ship's length
P = constant derived from dimensions of vessel
P_E = power developed by engine at speed U
P_F = power used to overcome friction
P_w = power used in formation of waves
Q_M = mass rate of effluent discharge
Q_v = volume rate of effluent discharge
R_e = Reynolds number
R_F = frictional resistance force
R_L = Lagrangian correlation coefficient
s = wetted surface area of ship
t = time from discharge
$\overline{u'^2_\sigma}$ = mean square of turbulent velocity fluctuations at spread σ
$\overline{u^2_x}$ = mean square of fluctuations of turbulent velocity component in full wake spectrum
U = ship's speed
β = ratio of Lagrangian to Eulerian time scales
σ = standard deviation of concentration distribution
τ_σ = Lagrangian time scale for plume with spread σ
τ_x = Lagrangian time scale for full turbulence spectrum
ν = kinematic coefficient of viscosity.

INTRODUCTION

There is a continuing interest in the rate of dilution of waste which is discharged into the wake of a moving ship.

Theoretical work on dilution rates in turbulent wakes (Reichardt, 1964) was followed by model studies at the Delft Hydraulics Laboratory (Abraham and Hilberts, 1967) and field trails using a small coaster (Delft, 1970). In these studies emphasis was laid on the effect of density differences between the effluent and seawater, since the potential energy change assists dilution of the waste. Kinetic energy for mixing was presumed to arise from the rotation of the propeller. A summary of these considerations was given by Abraham and Eysink (1970).

Model tests in the U.S.A. (Mercier *et al.*, 1973) suggested that the boundary layer created by the hull of a ship was the principal reason for the rapid intermixing of waste with the sea. The turbulence generated by the ship's propeller was thought to have a relatively small beneficial influence on dilution. Further model studies undertaken by the Delft Hydraulics Laboratory (Delft, 1975) indicated that the dilution of the waste could be improved by discharging it through orifices positioned along both sides of the ship.

In order to derive an empirical formula to make dilution predictions on a time scale of a few minutes, Tromp (1976) used a dimensionless approach to combine the results from the model and full-scale studies. This formed the basis for the IMCO formula which predicts dilution of effluent in the wake of a ship for a period of some 5–10 min after discharge. Dilution on longer time scales has been examined by other investigators (Kullenberg, 1974; Farmer and Lemon, 1975; Ball and Reynolds, 1976).

An empirical approach provides only a limited understanding of the processes responsible for dilution and, therefore, a theoretical model of the wake dispersion is more useful.

It is reasonable to presume that the rapid dilution of waste in the wake of ship is primarily due to the disturbance created by the passage of the vessel and, therefore, a theory was developed to relate dilution to the energy of the wake turbulence.

W.R. 19/8—A

THEORY

Time dependence of dilution

The movement of a dumping vessel through the sea will create a disturbance, termed the wake, and into this wake effluent is discharged (Fig. 1). The lower boundary of the disturbance caused by the passage of the ship will be a little deeper than the draught of the vessel.

Let it be assumed that:

(a) The average properties of the turbulence created by the ship are uniform in space and steady in time (i.e. homogeneous and stationary). In practice, wake turbulence will decay but observations of wakes suggest that the rate of decay is slow.

(b) The initial dilution of the discharged waste is so rapid that the difference in density between the effluent and that of seawater is not important.

(c) Vertical mixing through the depth of the boundary layer is very rapid.

(d) Vertical mixing by natural turbulence below the boundary layer is relatively slow so that, at intermediate and longer times, dilution is principally due to lateral mixing.

Batchelor (1952) made predictions regarding the spread of clusters of particles for various times of diffusion in homogeneous and stationary turbulence:

Small times

$$\sigma = k_1 t^2 \tag{1}$$

Intermediate times

$$\sigma = k_2 t \tag{2}$$

Long times

$$\sigma = k_3 t^{1/2} \tag{3}$$

where σ is the standard deviation of the spread at time, t.

Smith and Hay (1961) considered the spread of a cluster of particles in terms of the energy of the turbulent field. They concluded that over a wide range of values of σ, the rate of expansion of a cluster should be almost constant and have a maximum value given by,

$$(\mathrm{d}\sigma/\mathrm{d}x)_{\mathrm{MAX}} = \tfrac{2}{3}\beta i^2 \tag{4}$$

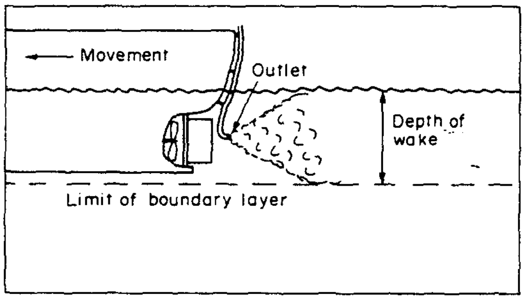

Fig. 1. The formation of a plume of waste in the wake of a discharging tanker.

where β is the ratio of the Lagrangian to Eulerian time scales and i is the intensity of turbulence.

$$i^2 = \overline{u_x^2}/U^2 \tag{5}$$

where $\overline{u_x^2}$ is the mean square of the fluctuations of one component of the turbulent velocity in the full wake spectrum and U is the speed of the discharging tanker.

Since $x = Ut$, it follows from equation (4) that,

$$(\mathrm{d}\sigma/\mathrm{d}t)_{\mathrm{MAX}} = \tfrac{2}{3}\beta i^2 U. \tag{6}$$

Integration gives,

$$\sigma = \tfrac{2}{3}\beta i^2 U t + \text{constant}. \tag{7}$$

Taking $\sigma = 0$, when $t = 0$,

$$\sigma = \tfrac{2}{3}\beta i^2 U t. \tag{8}$$

Comparing equations (2) and (8), it can be seen that,

$$k_2 = \tfrac{2}{3}\beta i^2 U \tag{9}$$

for intermediate diffusion times.

At long times, the mean square separation of a pair of particles tends to twice the mean square displacement of particles released serially from a fixed position (Batchelor, 1952). Thus, at long times, the rate of spread can be deduced from the formula for a serial release of particles (Taylor, 1921).

i.e.

$$\mathrm{d}\sigma^2/\mathrm{d}t \approx 4\,\overline{u'^2_\sigma}\,\tau_\sigma \tag{10}$$

where $\overline{u'^2_\sigma}$ is the mean square velocity fluctuation of all the particles in the cluster and

$$\tau_\sigma = \int_0^\infty R_L(t)\mathrm{d}t$$

the Lagrangian time scale of the cluster.

For large σ, $\overline{u'^2}$ and τ_σ become constants equal to those for the full turbulence spectrum so that,

$$\sigma^2 \approx 4\,\overline{u'^2_\infty}\cdot\tau_\infty\cdot t. \tag{11}$$

Note that long times correspond to $t \gg \tau_\sigma$.

Substituting for i into equation (11),

$$\sigma^2 = 4\, i^2 U^2 \tau_\infty t$$

and, therefore,

$$\sigma \approx 2\,\tau_\infty^{1/2}\, i U t^{1/2}. \tag{12}$$

Comparing equations (3) and (12), it can be seen that,

$$k_3 = 2\,\tau_\infty^{1/2}\, i U \tag{13}$$

for long diffusion times.

Equations (8) and (12) show the relation between dispersion and time of travel in homogeneous turbulence for intermediate and long times. Various forms for the correlation coefficient were tried to examine this relation in the atmosphere (Pasquill, 1974).

The peak concentration along the axis of a discharge plume which is mixed through a layer of depth, h, is given by,

$$c_P = Q_M/\{\sqrt{2\pi}\,Uh\sigma_y\} \tag{14}$$

where Q_M is the mass discharge rate, U is the ship's speed and σ_y is the standard deviation of the transverse spread.

If the volume discharge rate of the effluent is Q_V at concentration, c_e, it follows that,

$$Q_M = Q_V \cdot c_e. \tag{15}$$

Taking the dilution, D, to be given by,

$$D = c_e/c_P \tag{16}$$

equations (15) and (16) can be substituted into (14) to give,

$$D = \sqrt{2\pi}\,Uh\sigma_y/Q_V. \tag{17}$$

From equations (8) and (12), it can be seen that at: intermediate times

$$D = \frac{2\sqrt{2\pi}}{3\,Q_V}\cdot h\beta i^2 U^2 t \tag{18}$$

long times

$$D = \frac{2\sqrt{2\pi}}{Q_V}\cdot h\tau_\infty^{1/2} i U^2 t^{1/2}. \tag{19}$$

Estimate of turbulent intensity

The energy used to drive the ship's propeller is used to:

(a) Overcome the frictional resistance of the water.

(b) Generate bow and stern waves.

Thus, the power of the engine P_E, can be expressed as:

$$P_E = P_F + P_W \tag{20}$$

where P_F is the power required to overcome friction and P_W is the power used in the formation of waves.

The waves spread out from the vessel carrying energy with them. The frictional energy goes into the turbulence formed in the wake of the vessel at a rate given by,

$$P_F = R_F \cdot U \tag{21}$$

where R_F is the frictional resistance force.

The magnitude of R_F can be estimated from an expression proposed by the International Towing Tank Conference, 1957 (Muckle, 1975, p. 215)

$$R_F = \tfrac{1}{2}\rho c_F S U^2 \tag{22}$$

where ρ is the density of seawater, c_F is the specific resistance coefficient given by $c_F = 0.075/\{\log_{10} R_e - 2\}^2$, S is the wetted surface area of the ship and U is the ship's speed.

The Reynolds number, R_e, is given by,

$$R_e = U \cdot L/\nu \tag{23}$$

where L is the ship's length and ν is the kinematic coefficient of viscosity.

To a first approximation, the initial area of the turbulence field created by the ship is given by the cross-section of the immersed stern. Thus, using equation (21), the energy per unit mass, E, supplied to the turbulence by the ship is given by,

$$E = P_F/\rho BdU = R_F/\rho Bd \tag{24}$$

where B is the beam of the ship and d is the draught. The turbulent kinetic energy per unit mass is given by,

$$E = \tfrac{1}{2}\{\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\} \tag{25}$$

where $\overline{u'^2}$, $\overline{v'^2}$ and $\overline{w'^2}$ are the mean square velocities of the turbulent fluctuations in the coordinate directions.

Assuming that the turbulence is isotropic,

$$E = \tfrac{3}{2}\overline{u'^2}. \tag{26}$$

For the whole turbulent field, let it be assumed that natural turbulence is small compared with that due to the wake so that $\overline{u'^2} = \overline{u_\infty^2}$.

It follows from equations (24) and (26) that,

$$\overline{u_\infty^2} = \tfrac{2}{3} R_F/\rho Bd. \tag{27}$$

Using equation (22),

$$\overline{u_\infty^2} = \tfrac{1}{3} c_F S U^2/Bd. \tag{28}$$

From equation (5), the intensity of turbulence is given by,

$$i^2 = \tfrac{1}{3} c_F S/Bd. \tag{29}$$

The wetted surface area of a ship is given by,

$$S = L(c_B \cdot B + 1.7\,d) \tag{30}$$

where c_B is a block coefficient (Muckle, 1975, p. 222). Typically c_B lies in the range 0.65–0.80.

From equations (29) and (30),

$$i^2 = \tfrac{1}{3} c_F L(c_B B + 1.7\,d)/Bd \tag{31}$$

or

$$i^2 = \tfrac{1}{3} c_F L P \tag{32}$$

where

$$P = (c_B B + 1.7\,d)/Bd.$$

Dilution formulae

Substituting equation (32) into (18) gives the dilution at intermediate times as,

$$D = \frac{2\sqrt{2\pi}}{9\,Q_v}\cdot c_F L P h\beta U^2 t. \tag{33}$$

From equations (32) and (19), the dilution at long times is,

$$D = \frac{2\sqrt{2\pi}}{\sqrt{3}\,Q_v}(c_F L P)^{1/2} h\tau_\infty^{1/2} U^2 t^{1/2}, \tag{34}$$

EVALUATION

Equations (33) and (34) can be re-expressed at intermediate times as,

$$D = \frac{0.56}{Q_r}(c_F \beta \gamma) U^2 L t \qquad (35)$$

and at long times as,

$$D = \frac{2.89}{Q_r} c_F^{1/2} \gamma^{1/2} \delta U^{3/2} L^{3/2} t^{1/2} \qquad (36)$$

where $\gamma = P \cdot h$ and $\delta = (\tau_\infty h U)^{1/2}/L$.

These equations suggest that increasing the speed of the vessel is a more effective means of increasing the dilution compared with reduction of the volume rate of effluent discharge.

Equations (35) and (36) expressed in dimensionless form become,

$$\log_{10}\left(\frac{Q_r \cdot D}{UL^2}\right) = \log_{10}\left(\frac{Ut}{L}\right) + \log_{10}(0.56\, c_F \beta \gamma) \qquad (37)$$

and

$$\log_{10}\left(\frac{Q_r \cdot D}{UL^2}\right) = \frac{1}{2}\log_{10}\left(\frac{Ut}{L}\right) + \log_{10}(2.89\, c_F^{1/2} \gamma^{1/2} \delta). \qquad (38)$$

To determine the dilution at a given time after discharge, it is necessary to make estimates of the parameters β, τ_∞ and h. The magnitude of β was taken to be 4.0 from studies in the atmosphere (Pasquill, 1974). The Lagrangian time scale of the full turbulence spectrum, τ_∞, is difficult to estimate and a value of one minute was arbitrarily selected. Figure 2 shows plots of equations (37) and (38) for various values for γ. In evaluating equation (38), the mixing at long times was assumed to extend to the full depth of the boundary layer of the ship and a value for h of 5 m was chosen as being typical for a small tanker.

Figure 2 demonstrates the theoretical prediction that, at intermediate times, the dilution increases linearly with time and, at long times, the dilution increases with the square root of the time. "Small" times correspond to the initial mixing when the effluent plume is still very narrow and this stage is not considered in the theory.

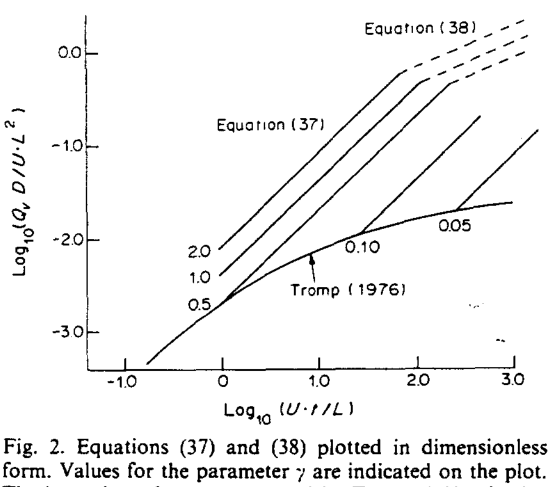

Fig. 2. Equations (37) and (38) plotted in dimensionless form. Values for the parameter γ are indicated on the plot. The lower bound curve suggested by Tromp (1976) is also shown in the figure.

At long times, most of the wake energy would be dissipated leaving only the energy of the natural turbulence and the wake dilution theory would be expected to give overestimates of the dilution of discharged waste. Figure 2 shows that the dilution improves as the magnitude of γ increases. Since P is a constant which depends upon the dimensions of the vessel, higher values of γ correspond to thicker mixing layers for a given tanker.

DISCUSSION

In order to draw together the results of dilution studies using models and full-size ships, Tromp (1976) expressed the available data in terms of dimensionless parameters and used a lower bound curve on a log–log plot to determine dilution minima. The curve suggested by Tromp is shown in Fig. 2. To a first approximation, the curve corresponds to the dilution formula,

$$D = \frac{0.0024}{Q_r} U^{1.71} L^{1.29} t^{0.71}\, 10^{-0.14[\log_{10}(Ut/L)]^2}. \qquad (39)$$

By taking a tangent to this curve, the general formula adopted by IMCO was derived i.e.

$$D = \frac{c_1}{Q_r} U^{1.4} L^{1.6} t^{0.4} \qquad (40)$$

where the constant, c_1, is estimated to be 0.0030 for a single discharge orifice and 0.0045 for discharges symmetrical to the ship's axis.

However, the lower bound curve may not be the most satisfactory approach to estimating dilutions because the lowest values obtained in the individual experiments had a dependence on the duration of the mixing time which depended, in turn, on the sampling procedure. Thus, the curve is not based on a clearly defined set of circumstances and empirical formulae derived from it are likely to give only a very general representation of the effect of parameters such as discharge rate, speed and ship's length on dilution.

It can be seen from Fig. 2 that the results used in the derivation of the lower bound curve corresponded to values of γ between 0.05 and 0.5 in the wake energy model. Assuming a typical value for P of 0.3 m^{-1}, such a range would correspond to depths of the mixed layer ranging between 0.15 and 1.7 m. These values for h are appreciably less than the draughts of the discharge tankers used in the field trials. This suggests that either the dilutions predicted by the wake energy model are better than those obtainable or the discharge systems used in the experiments did not permit rapid mixing through the boundary layers.

There is some circumstantial evidence that the latter situation could have occurred in that the field trails were generally either releases onto the sea surface from trailed pipes or through orifices along the sides of vessels. Both techniques could have resulted in the initial formation of thin mixed layers. This is supported by the U.S.A. model tests in which waste released from amidships on each side of the vessel was "swept upward and outward" and did not immediately spread through the boundary layer (Mercier *et al.*, 1973).

The value of the wake energy model is that it takes account of the mechanisms which cause wake dilution and is, therefore, potentially capable of application to a variety of discharge conditions. The model does suggest that if waste could be rapidly mixed through the depth of the boundary layer, possibly by discharging through an orifice immediately above the propeller, then higher dilutions than those predicted by the IMCO formula could be obtained. However, the existing experimental data are limited and validation of the wake energy model will have to await the results of further field trials.

SUMMARY AND CONCLUSIONS

A wake energy model provides some insight into the way in which dilution in the wake of a discharging tanker may be influenced by the characteristics of the vessel. These characteristics include the speed of the ship, its dimensions and its specific resistance coefficient.

The model indicates that at intermediate times after discharge, dilution is linearly dependent on time [equation (33)] and, at longer times, the dilution becomes dependent on the square root of time [equation (34)]. As with the IMCO formula, the wake energy model indicates that increasing the speed of the ship is a more effective means of increasing dilution compared with reduction of the volume rate of effluent discharge.

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